Department of Electrical Engineering
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ELEG 5693 Wireless Communications
Ch. 8 CDMA

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OUTLINE

- **CDMA Fundamentals**
- **Pseudo-noise sequence**
- **Rake receiver**

FUNDAMENTALS

- **Code Division Multiple Access (CDMA)**
  - Multiple users share the same channel and transmit at the same time.
  - Different codes are used to separate the signals from different users.
  - The signal occupies a bandwidth much larger than the minimum bandwidth necessary to send the information.
    - Spread Spectrum Technologies.
- **Classifications**
  - Direct sequence CDMA (DS-CDMA)
    - Also called Direct sequence spread spectrum (DSSS)
    - Used in IS-95, WCDMA, cdma2000, IEEE 802.11, etc.
    - The most commonly used CDMA technique.
  - Frequency hopping CDMA (FH-CDMA)
    - Also called Frequency Hopping spread spectrum (FHSS)
    - Used in original IEEE 802.11, military applications, etc.

FUNDAMENTALS

- **Definitions**
  - Inner product (correlation between two sequences):
    - The inner product of two length *N* vectors **a** and **b** is defined as

$$\mathbf{a} = [a_1, a_2, \cdots, a_N] \qquad \mathbf{b} = [b_1, b_2, \cdots, b_N]$$

$$\mathbf{a} \bullet \mathbf{b} = \frac{1}{N}(a_1 b_1 + a_2 b_2 + \cdots + a_N b_N) = \frac{1}{N}\sum_{i=1}^{N} a_i b_i$$

    - For binary vector **a** (contains either -1 or 1)

$$\mathbf{a} \bullet \mathbf{a} = \frac{1}{N}\sum_{n=1}^{N} (\pm 1)^2 = \frac{1}{N} \cdot N = 1$$

  - Orthogonal:
    - Two vectors are orthogonal if their inner product is 0.

$\mathbf{a} \bullet \mathbf{b} = 0$ ➔ **a** and **b** are orthogonal

    - Example
      - a = [1 -1 1 -1], b = [1 1 -1 -1]

$$\mathbf{a} \bullet \mathbf{b} = \frac{1}{4}(1 - 1 - 1 + 1) = 0$$

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FUNDAMENTALS: OPERATION

- **CDMA operations: Transmitter**
  - Each user is assigned a unique code
    - Each bit in the code is called a "chip"
    - E.g. user 1, $\mathbf{c}_1 = [-1,-1,-1,+1,+1,-1,+1,+1]$
  - For each user, the modulation symbol *s* is further modulated by the code sequence as $s \times \mathbf{c}_i$ (spreading)
    - E.g. BPSK.
      - To transmit BPSK symbol '1', signal sent out by the user in one symbol period is

$$1 \times \mathbf{c}_1 = [-1,-1,-1,+1,+1,-1,+1,+1]$$

      - To transmit BPSK symbol '-1', signal sent out by user 1 in one symbol period is

$$-1 \times \mathbf{c}_1 = [+1,+1,+1,-1,-1,+1,-1,-1]$$

    - At each symbol period, instead of transmitting 1 symbol, *N* chips are transmitted
      - Chip period *Tc*: time duration of one chip
      - Symbol period $T_s = N \cdot T_c$

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FUNDAMNETALS: OPERATIONS

- **CDMA operations: receiver**
  - Despreading: at receiver, performing inner product between received signal and the code sequence of the user
    - Rx signal (after fading compensation)

$$\mathbf{y} = s \times \mathbf{c}_1 + \mathbf{n}$$

    - After despreading:

$$\mathbf{y} \bullet \mathbf{c}_1 = s \times \mathbf{c}_1 \bullet \mathbf{c}_1 + \mathbf{n} \bullet \mathbf{c}_1 = s + z$$

      - $z = \mathbf{n} \bullet \mathbf{c}_1$ is the noise component after despreading
  - Example (Assume no noise)
    - The Rx sequence is

$$\mathbf{y} = [+1,+1,+1,-1,-1,+1,-1,-1]$$

    - $$\mathbf{c}_1 = [-1,-1,-1,+1,+1,-1,+1,+1]$$
    - After despreading:

$$\mathbf{y} \bullet \mathbf{c}_1 = \frac{1}{8}(-1-1-1-1-1-1-1-1) = -1$$

FUNDAMENTALS: OPERATIONS

- **CDMA operations: multiple access**
  - Each user is assigned a unique sequence
    - Ideally, the sequences of different users are orthogonal to each other.
    - E.g user 1: $\mathbf{c}_1 = [-1,-1,-1,+1,+1,-1,+1,+1]$
    - User 2: $\mathbf{c}_2 = [-1,-1,+1,-1,+1,+1,+1,-1]$

$$\mathbf{c}_1 \bullet \mathbf{c}_2 = \frac{1}{8}(1+1-1-1+1-1+1-1) = 0$$

  - Signals from multiple users are transmitted at the same time and same bandwidth.
    - E.g. user 1 “1”: $\mathbf{c}_1 = [-1,-1,-1,+1,+1,-1,+1,+1]$
    - User 2 “-1”: $-\mathbf{c}_2 = [+1,+1,-1,+1,-1,-1,-1,+1]$
    - Signal at receiver: $\mathbf{r} = \mathbf{c}_1 - \mathbf{c}_2 = [0,0,-2,+2,0,-2,0,+2]$
  - Detect signals from user 1

$$\mathbf{c}_1 \bullet \mathbf{r} = (0+0+2+2+0+2+0+2)/8 = 1$$

  - Detect signals from user 2

$$\mathbf{c}_2 \bullet \mathbf{r} = (0+0-2-2+0-2+0-2)/8 = -1$$

FUNDAMENTALS: OPERATIONS

- **Multiple access (Cont'd): why no interference among users?**

$$\mathbf{c}_1 \bullet \mathbf{r} = \mathbf{c}_1 \bullet (\mathbf{c}_1 - \mathbf{c}_2) = \mathbf{c}_1 \bullet \mathbf{c}_1 - \mathbf{c}_1 \bullet \mathbf{c}_2 = \mathbf{c}_1 \bullet \mathbf{c}_1 = 1$$
$$\mathbf{c}_2 \bullet \mathbf{r} = \mathbf{c}_2 \bullet (\mathbf{c}_1 - \mathbf{c}_2) = \mathbf{c}_2 \bullet \mathbf{c}_1 - \mathbf{c}_2 \bullet \mathbf{c}_2 = -\mathbf{c}_2 \bullet \mathbf{c}_2 = -1$$

  - Because the spreading codes are orthogonal with each other, there is no interference among users!
  - If there are M users

$$\mathbf{r} = \sum_{m=1}^{M} s_m \times \mathbf{c}_m + n$$

  - To detect the signal of the nth user

$$\mathbf{r} \bullet \mathbf{c}_n = s_n \times (\mathbf{c}_n \bullet \mathbf{c}_n) + \sum_{m \neq n} s_m \times (\mathbf{c}_m \bullet \mathbf{c}_n) + \mathbf{n} \bullet \mathbf{c}_n = s_n + z_n$$

noise after despreader

  - To guarantee the orthogonality among users, all the M users must be synchronized.
    - All users in system must transmit at exactly the same time!
    - This requirement can be lowered by using PN code.

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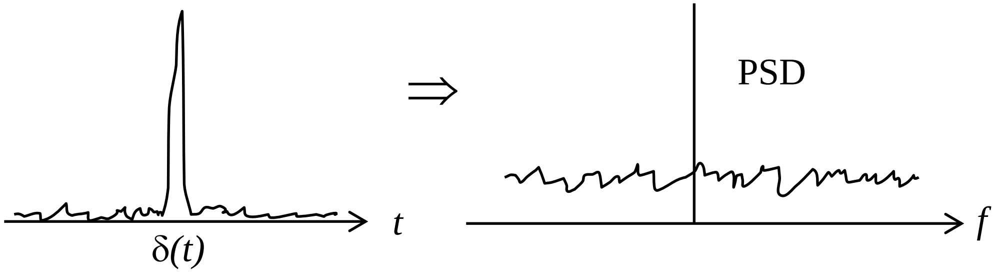
FUNDAMNETALS: PN CODE

- **PN Code (Pseudo-noise Code)**
  - Binary sequence with random properties ➔ noise like
  - Approximately the same number of 0s and 1s.
  - Very low correlation between time-shifted version of same sequence
    - Auto-correlation of PN code ➔ noise like!



    - E.g. [-1, -1, -1, +1, +1, -1, +1], time shift by 1: [+1, -1, -1, -1, +1, +1, -1]
      - Inner product: (-1+1+1-1+1-1-1)/7 = -1/7
  - Very low cross-correlation between different codes.

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FUNDAMNETALS: MAI

- **Multiple access interference (MAI)**
  - In practical CDMA systems
    - Users are not perfectly synchronized ➔ codes are time shifted ➔ codes are no longer orthogonal
    - The spreading codes among users are not perfectly orthogonal
  - MAI: interference among users in CDMA system because the codes among users are not perfectly orthogonal.

$$\mathbf{r} = \sum_{m=1}^{M} s_m \times \mathbf{c}_m + \mathbf{n}$$

  - To detect the signal of the *n*th user

$$\mathbf{r} \bullet \mathbf{c}_n = s_n \times (\mathbf{c}_n \bullet \mathbf{c}_n) + \boxed{\sum_{m \neq n} s_m \times (\mathbf{c}_m \bullet \mathbf{c}_n)} + \mathbf{n} \bullet \mathbf{c}_n$$

$$= s_n + \boxed{\sum_{m \neq n} I_m} + z_n$$

MAI

  - Usually: $\mathbf{c}_m \bullet \mathbf{c}_n << \mathbf{c}_n \bullet \mathbf{c}_n = 1$

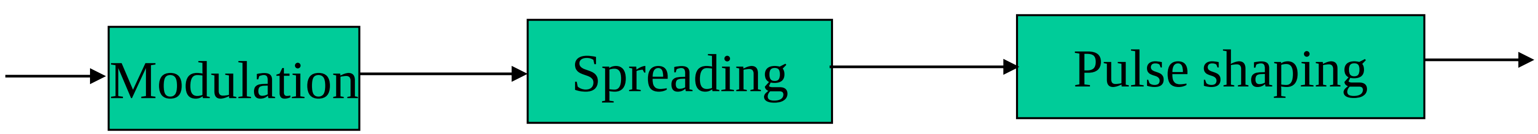
therefore $$E\left|\sum_{m \neq n} I_m\right|^2 << E|s_n|^2$$

FUNDAMENTALS: MAI

- **MAI (Cont'd)**
  - The power of MAI depends on two factors:
    - the cross-correlation between different PN codes
    - The signal power of interfering users
  - Near-far effect
    - Interfering users are close to BS ("near") ➔ high power at Rx
    - Desired user is far away from BS ("far") ➔ low power at Rx
    - the power of the interfering users is much higher than the power of the desired signal ➔ MAI is high ➔ signal to MAI ratio is low ➔ poor performance of desired user
    - Near-far effect will seriously affect the performance of CDMA system
    - To reduce the impairments of near-far effect, the power at Rx should be the same for all users
      - Close to BS: low Tx power. Far from BS: larger Tx power
    - Power control is very important for CDMA system.

FUNDAMENTALS: SPREAD SPRECTRUM

- **Bandwidth of CDMA signals**
  - TDMA or FDMA system: one symbol is transmitted at one symbol period (*Ts*)
    - Each symbol passes through pulse shaping filter with parameter (α, T_s)
    - Bandwidth: $BW = (1+\alpha)R_s = (1+\alpha)/T_s$
  - CDMA system: N chips are transmitted at one symbol period (*Ts*)
    - each chip passes through pulse shaping filter with parameter (α, T_c)
    - Bandwidth: $BW = (1+\alpha)R_c = (1+\alpha)/T_c$
    - $R_c = NR_s$: chip rate
      - The bandwidth is proportional to chip rate (N times of symbol rate)
    - The bandwidth of the signal is spreaded N times ➔ spread spectrum!
      - Within one symbol period, signals changes N times faster due to spreading code.
    - Due to the properties of PN code, signals look like noise ➔ high security



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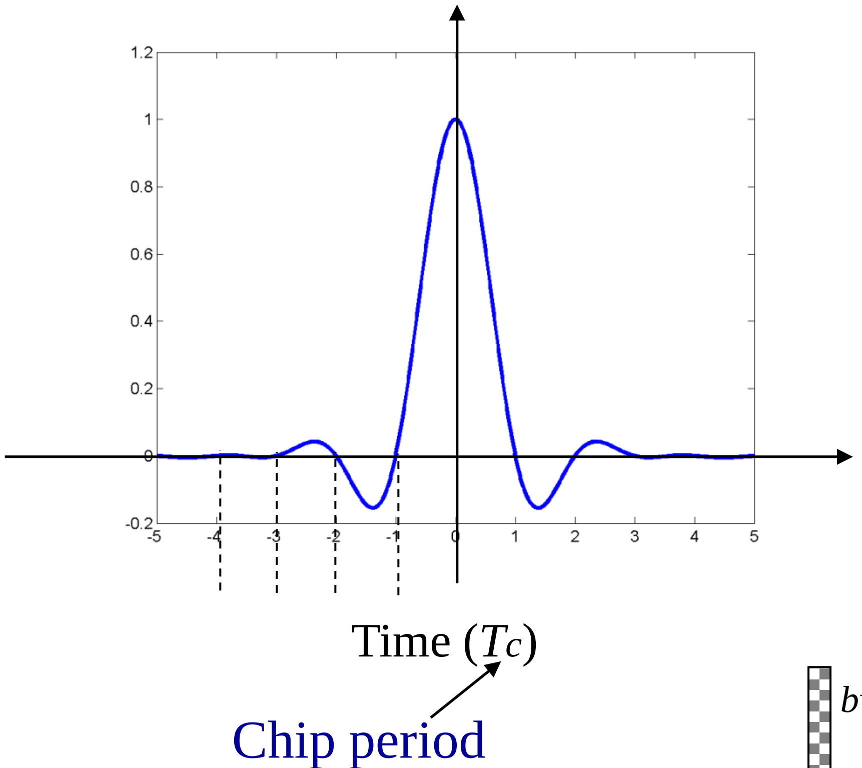
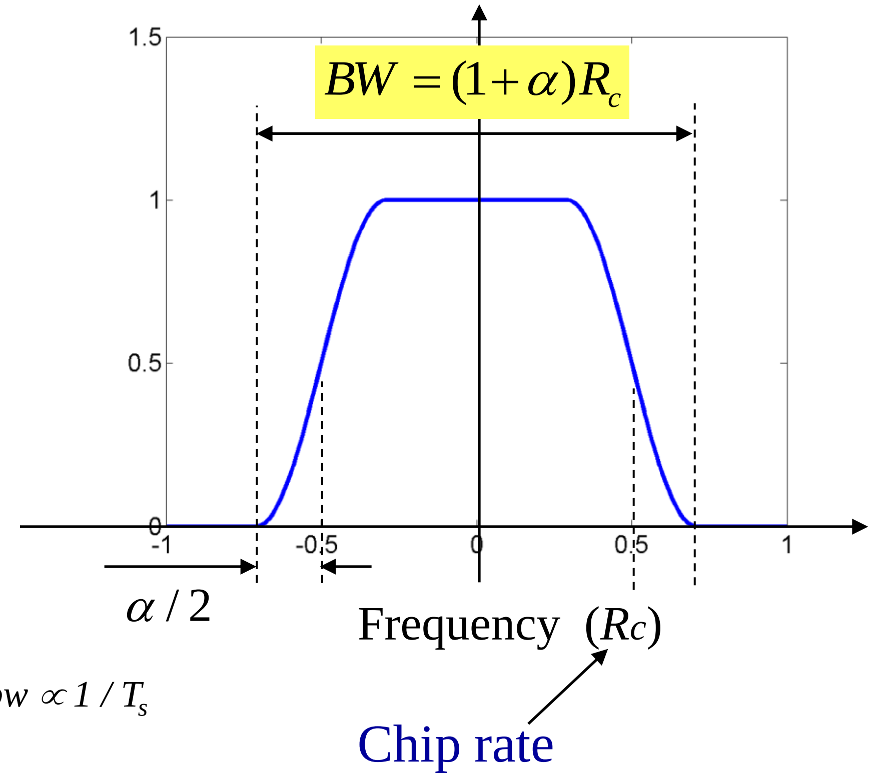
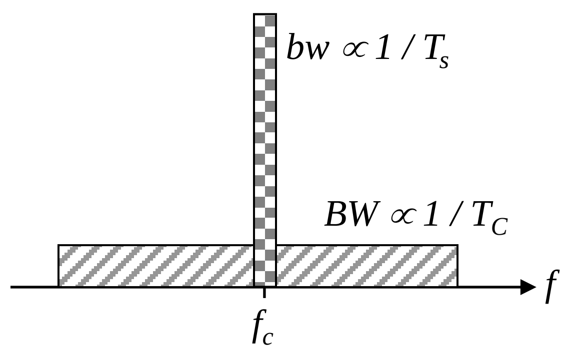
FUNDAMENTALS: SPREAD SPRECTRUM

- **Bandwidth of CDMA signals (Cont'd)**
  - Pulse shaping

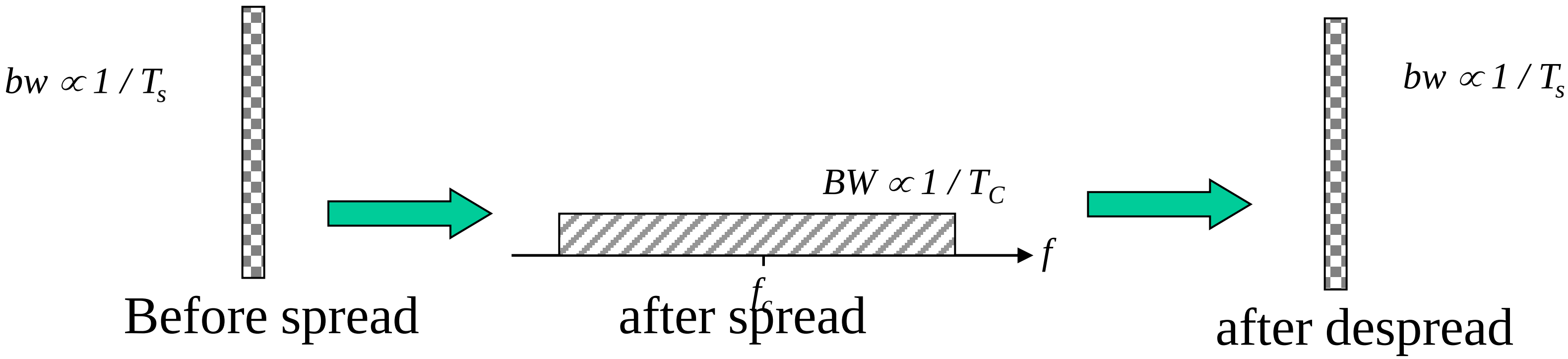






FUNDAMENTALS: SPREAD SPECTRUM

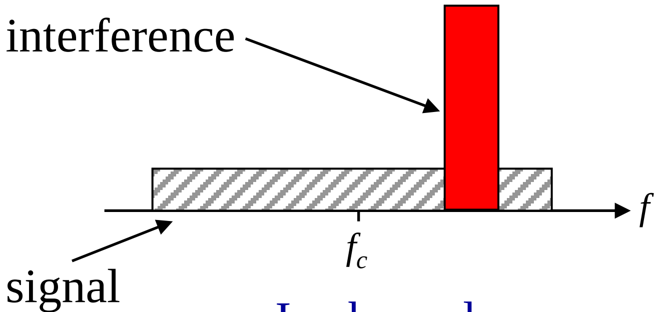
- **Bandwidth after despread**
  - After the operation of despread, each symbol period there is only one symbol ➔ BW $\propto 1 / Ts$
    - The bandwidth of the original signal is recovered after despread.



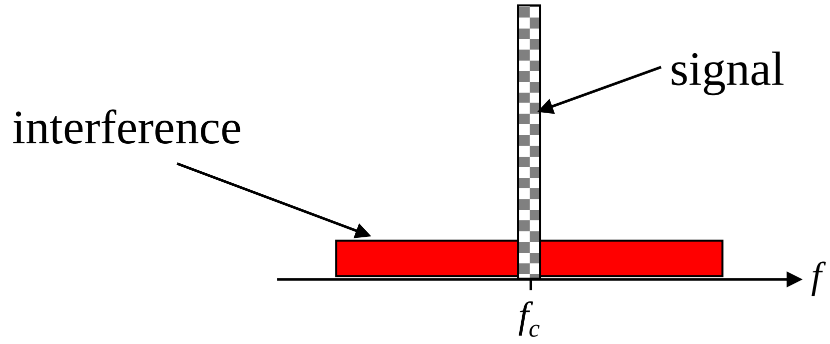
  - How about noise and signals from other users?
    - Signals from other users: they use different codes ➔ cross-correlation is low ➔ they remain to be wideband interference (MAI)
    - Noise: noise inner product with PN code ➔ noise properties remain the same.

FUNDAMENTALS: PROCESSING GAIN

- **Processing gain**
  - Narrow band interference



In channel



After despread

- At receiver, perform inner product between narrow band interference and PN code
  - Equivalent to perform spreading operation over interference
- Interference is spreaded, while signal is recovered !
  - Interference: BW $\propto 1 / Tc$
  - Signal: BW $\propto 1 / Ts$
  - Most of the interference are removed!

FUNDAMENTALS: PROCESSING GAIN

- **Process gain (Cont'd)**
  - Processing gain is an approximate measure of the interference rejection capability

$$PG = \frac{R_c}{R_s} = \frac{T_s}{T_c} = N$$

  - Process gain is equal to the number of chips per code.
  - The higher the processing gain, the greater the ability to suppress narrow band interference.

FUNDAMENTALS: FHSS

- **Frequency-hopping spectrum**
  - A wide spectrum is divided into *N* sub-channels
  - At each chip period, the carrier frequency of the signal is in one sub-channel
    - At next chip period, the carrier frequency of the signal is in another sub-channel
    - The signal "hops" from frequency to frequency
      - The signal itself is still narrow band.
  - At each symbol period, the signal hops *N* times
    - The order of hopping is determined by the PN code.
  - Very good security
    - It's hard to guess which sub-channel the signal is at without the knowledge of the PN code
    - Usually used in military applications.

OUTLINE

- CDMA Fundamentals
- **Pseudo-noise sequence**
- Rake receiver

PSEUDO-NOISE SEQUENCE

- **PN sequence**
  - A binary sequence with an autocorrelation resembles the autocorrelation of white noise.
  - Nearly equal number of 0s and 1s
  - Low correlation between time shifted versions of the same sequence
  - Very low cross-correlation between any two sequences.
- **Many methods exist for the generation of PN sequences**
  - Walsh code
  - m-sequence
  - Gold sequence
  - ……

PN SEQUENCE: WALSH CODE

- **Walsh code**

$$\mathbf{C}_1 = 1$$

$$\mathbf{C}_{i+1} = \begin{bmatrix} \mathbf{C}_i & \mathbf{C}_i \\ \mathbf{C}_i & \overline{\mathbf{C}_i} \end{bmatrix}$$

1	1
1	0

1	1	1	1
1	0	1	0
1	1	0	0
1	0	0	1

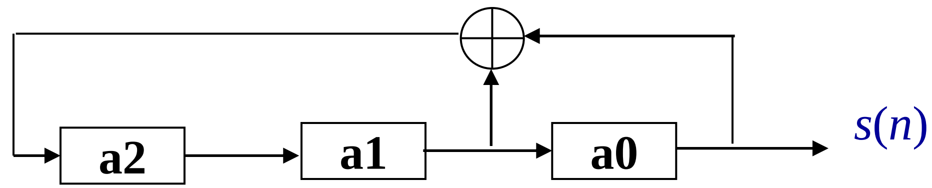
1	1	1	1	1	1	1	1
1	0	1	0	1	0	1	0
1	1	0	0	1	1	0	0
1	0	0	1	1	0	0	1
1	1	1	1	0	0	0	0
1	0	1	0	0	1	0	1
1	1	0	0	0	0	1	1
1	0	0	1	0	1	1	0

- Each row of the matrix is a PN sequence
- The PN sequences generated by using Walsh code are perfectly orthogonal with each other.
- Sequence length: $N = 2^m$
- There are N length-N sequences.

PN SEQUENCE: M-SEQUENCE

- **m-sequence**
  - Also called maximal length sequence
  - Sequence length: $N = 2^{m-1}$ (m: # of registers)
  - Generated by using shift register

a2 = a1+a0
s(n) = a0



  - E.g. $m = 3$ ($N = 7$)

a2	a1	a0	s(n)
0	0	1	1
1	0	0	0
0	1	0	0
1	0	1	1
1	1	0	0
1	1	1	1
0	1	1	1

1 0 0 1 0 1 1

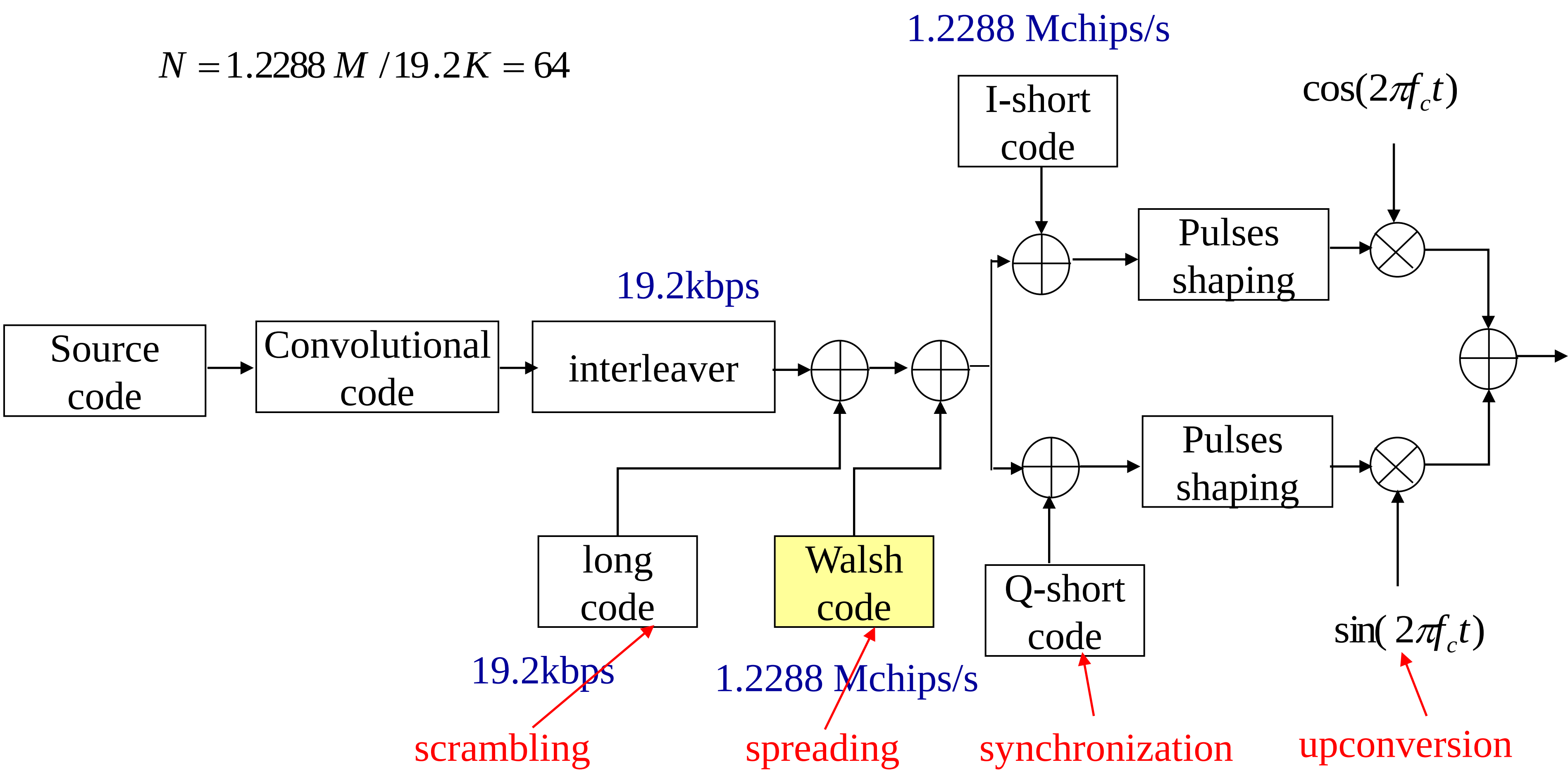
Periodic with period N = 7

PN SEQUENCE: M-SEQUENCE

- **m-sequence (Cont'd)**
  - 15 registers: m = 15 ➔ $N = 2^{15} - 1 = 32767$
    - The sequence will repeat itself for every 32767 chips.
    - Short code
    - Can be used for synchronization between Tx and Rx
      - Only when the clock between Tx and Rx are synchronized will the receiver gets strong signal ➔ adjust the receiver clock until a strong signal is obtained ➔ synchronization.
  - 42 registers: m = 42 ➔ $N = 2^{42} - 1 = 4398$ billions
    - The sequence will repeat itself for every 4398 billion chips
    - Long code
    - Can be used for scrambling (security)
      - Different users start the sequence from different offsets.
      - User data is XORed with chunks of long code with same length.
      - Impossible to detect the data without knowledge of long code.

PN SEQUENCE: IS-95 CDMA

$$N = 1.2288\,M / 19.2K = 64$$



OUTLINE

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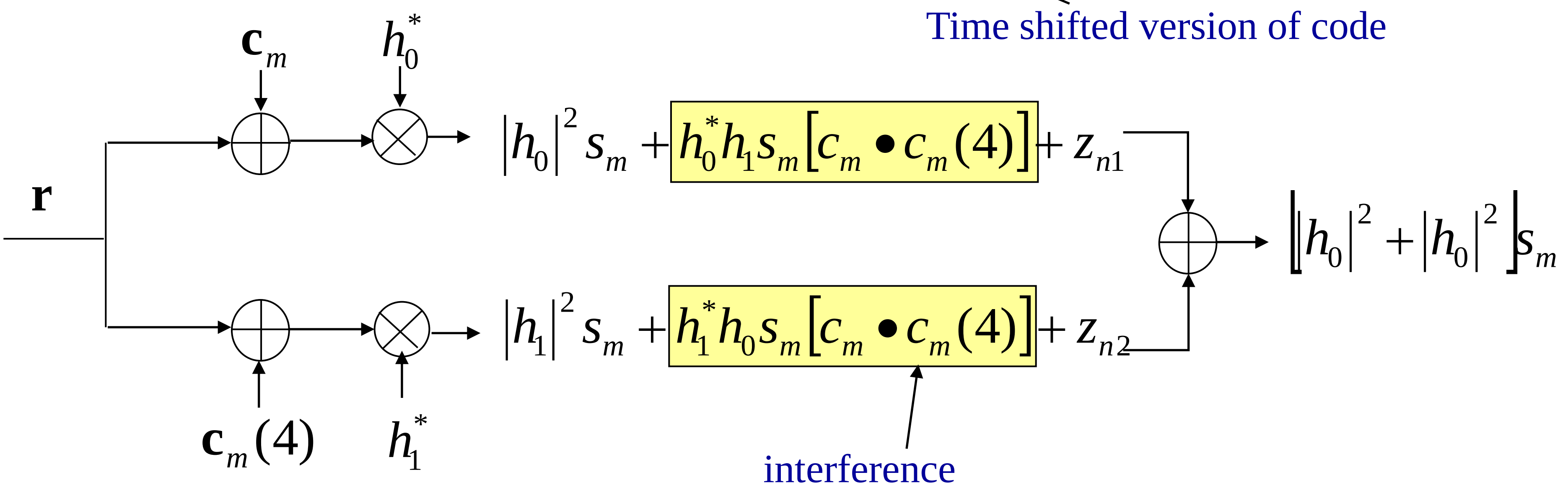
RAKE RECEIVER

- **Rake receiver**
  - Spread spectrum: signal bandwidth is very large ➔ signal BW >> channel coherence BW ➔ frequency selective fading
    - Relative delay among multipath components is no longer negligible.
  - The received signal is the combination of signals with different delays.
  - E.g. two path equal gain channel, space between two channel taps: 4 chips

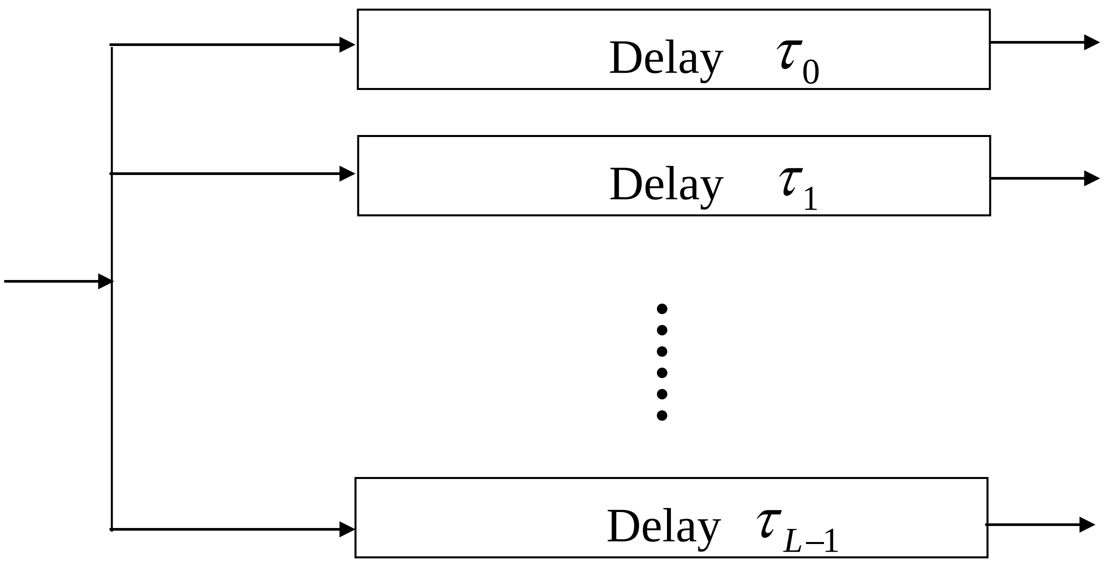
$$\mathbf{r} = h_0 \times s_m \times \mathbf{c}_m + h_1 \times s_m \times \mathbf{c}_m(4) + \mathbf{n}$$

Time shifted version of code



RAKE RECEIVER

- **Rake receiver**
  - Rake receiver is made up of several Rake fingers
    - Each finger corresponds to one of the multipath components.
  - Due to the low correlation between time shifted version of the same PN code, each finger can be treated as flat fading ➔ no equalization needed!



$$h(t,\tau) = \sum_{l=0}^{L-1} h_l(t) \times \delta(\tau - \tau_l)$$

CDMA

- **Major advantages:**
  - Resistant to narrow band interference
  - Combats multipath fading (no equalization required)
  - Multiple users can share the same spectrum
  - Good security
- **Major disadvantages**
  - MAI
    - Near-far effect.