Department of Electrical Engineering
University of Arkansas

ELEG 5693 Wireless Communications
Ch. 6 Diversity

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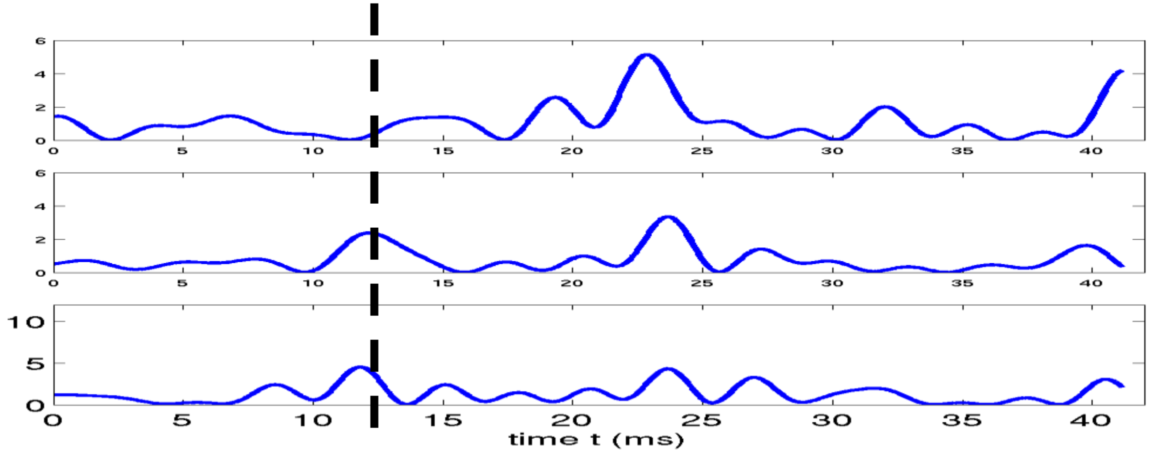
OUTLINE

- **Introduction**
- **Classification**
- **Space Diversity Techniques**
- **Space Time Block Code**

INTRODUCTION

- **What is diversity?**
  - E.g. Transmit the same signal several times.
    - Receiver has multiple copies of the same signal.
    - Each copy of the signal undergoes independent fading.
    - The probability that all the signals are severely distorted (deeply faded) simultaneously is small.



  - Diversity: Transmitting multiple replicas of the same signal at statistically independent fading channel to improve communication quality.
    - The probability that all the signals are distorted by channel is smaller compared to the non-diversity case.

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INTRODUCTION

- **Diversity can be achieved by utilizing the properties of fading**
  - So far, all of our discussions treat fading as a pure negative factor to reliable high speed wireless communication.
    - Time-varying, frequency-selective
    - Degrade system performance
  - If properly handled, fading can also benefit system performance!!!
  - E.g. Time-varying fading:
    - Coherent time
      - the time period over which the channel is strongly correlated.
    - Transmit the same signal at different time periods with interval larger than coherent time
    - Achieve diversity in the time domain ➔ Time diversity
- **Classifications:**
  - Time diversity
  - frequency diversity
  - space diversity.

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CLASSIFICATION: TIME DIVERSITY

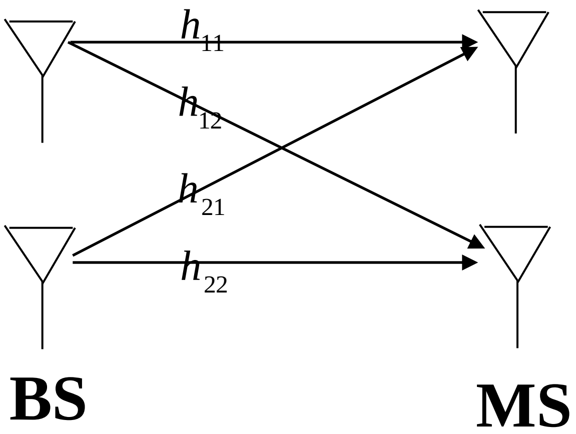
- **Time diversity**
  - Repeatedly transmit the information bearing signal at time spacing that exceeds the coherence time of the channel. (time varying fading)
    - Signals undergo statistically independent fading in the time domain.
    - If one copy of the signal is deeply faded, it's likely that the other copies can still be detected by the receiver.
  - Using redundancy in the time domain to tradeoff performance
    - Tradeoff between bandwidth efficiency and power efficiency!
  - Example: channel coding with interleaving
    - Channel coding ➔ information + redundancy
    - Interleaving ➔ spread out the codeword over time.
    - If the interleaving depth is large enough, every bit of the codeword undergoes independent fading
      - If one or more bit are in error, while the other bits are right, the information can still be recovered.

CLASSIFICATION: FREQUENCY DIVERSITY

- **Frequency diversity (multipath diversity)**
  - Transmitting the same information at different frequency bands with spacing larger than the coherent bandwidth (frequency selective fading)
    - Signals at different frequency undergo independent fading
  - Explanation from time domain:
    - Frequency selective fading ➔ multiple copies of the same signal arrive at the receiver at different time due to multipath fading.
    - Multipath components are independent ➔ signals propagating through different path are undergoing independent fading ➔ multipath diversity!
  - Frequency selective fading ➔ ISI ☹
  - Frequency selective fading ➔ mulitpath (frequency) diversity ☺
  - Frequency diversity can be achieved by using equalization in frequency selective fading
    - After ISI is partially removed, what is remaining is frequency diversity.

CLASSIFICATION: SPACE DIVERSITY

- **Space diversity (antenna diversity)**
  - Multiple antennas are used at transmitter and/or receiver, with spacing between two adjacent antennas being chosen to ensure the independence of fading.
    - Most popular diversity technique.
  - To ensure independence
    - Mobile station: antennas are separated half wavelength or more.
      - h_{11} and h_{12} are independent
    - Base station: antenna spacing need to be several tens of wavelengths.
      - h_{11} and h_{21} are independent



CLASSIFICATION: SPACE DIVERSITY

- **Space diversity classifications**
  - Receive diversity
    - Single transmit antenna, multiple receive antenna
    - SIMO (single input multiple output)
  - Transmit diversity
    - Multiple transmit antenna, single receive antenna
    - MISO (multiple input signal output)
  - Diversity on both transmitter and receiver
    - Multiple transmit antennas, multiple receive antennas
    - MIMO (multiple input multiple output)

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SPACE DIVERSITY

- **Instantaneous SNR of non-diversity system**
  - System model

$$y = h \cdot x + n$$

  - Signal power

$$|h \cdot x|^2 = |h|^2 E_s$$

Power of modulation symbol

  - Noise power

$$E\left[|n|^2\right] = \sigma_n^2$$

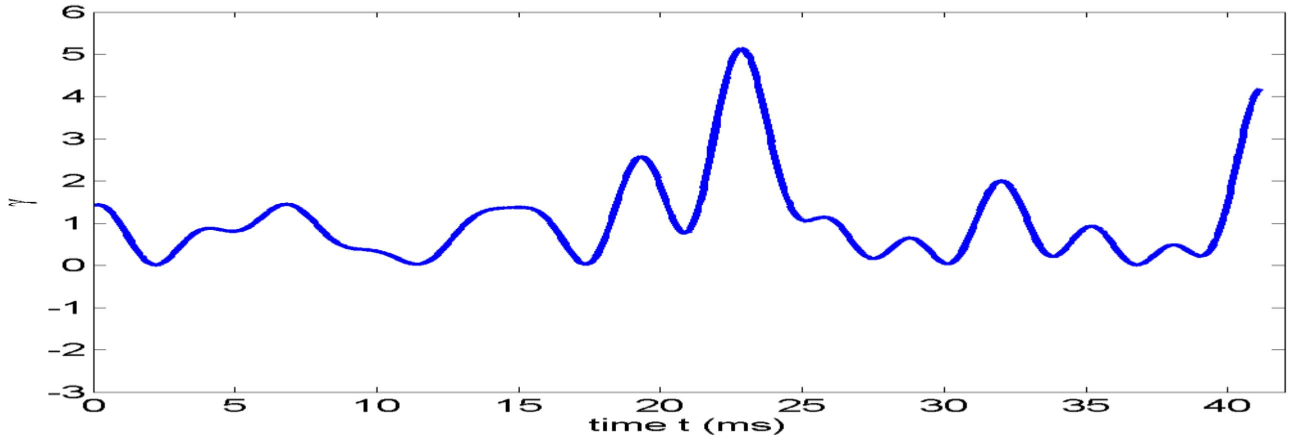
  - Instantaneous SNR: (signal power/noise power)

$$\gamma = |h|^2 \frac{E_s}{\sigma_n^2} = |h|^2 \gamma_0$$

SNR without fading

SPACE DIVERSITY

- **Instantaneous SNR**
  - Fading h varies with time ➔ SNR γ varies with time



  - The fluctuation of SNR degrades the performance of wireless communication system
  - If there is no fading (AWGN only), the SNR is constant
    - Space diversity will make the SNR more "flat"

SPACE DIVERSITY: COMMON TECHNIQUES

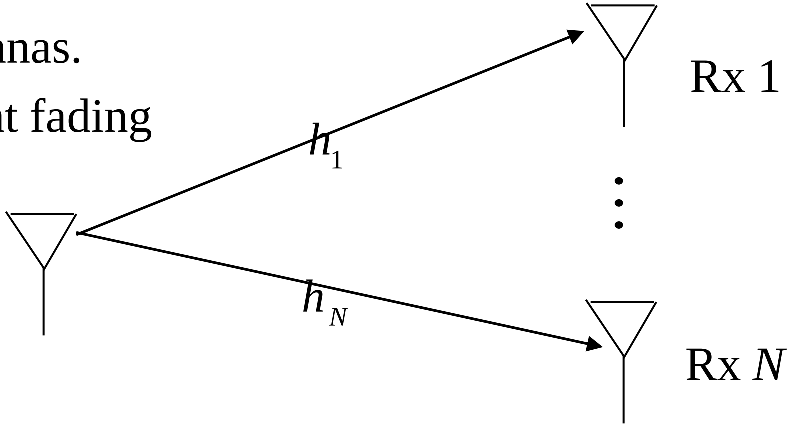
- **Common receive diversity techniques**
  - Selection diversity
  - Maximal Ratio Combining (MRC)
  - Equal Gain (EG) diversity
- **Common transmit diversity techniques**
  - Transmit diversity with feedback
  - Space time block code
  - Space time trellis code
- **MIMO**
  - Space time block code
  - Space time trellis code

SPACE DIVERSITY: RECEIVE DIVERSITY

- **System model for receive diversity**
  - 1Tx antennas, N Rx antennas.
  - The signal at the kth Rx antenna: $y_k = h_k \cdot x + n_k$
  - System equation in matrix format

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_N \end{bmatrix} \cdot x + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_N \end{bmatrix}$$

  - The same signal is received by N antennas.
    - Each signal undergoes independent fading



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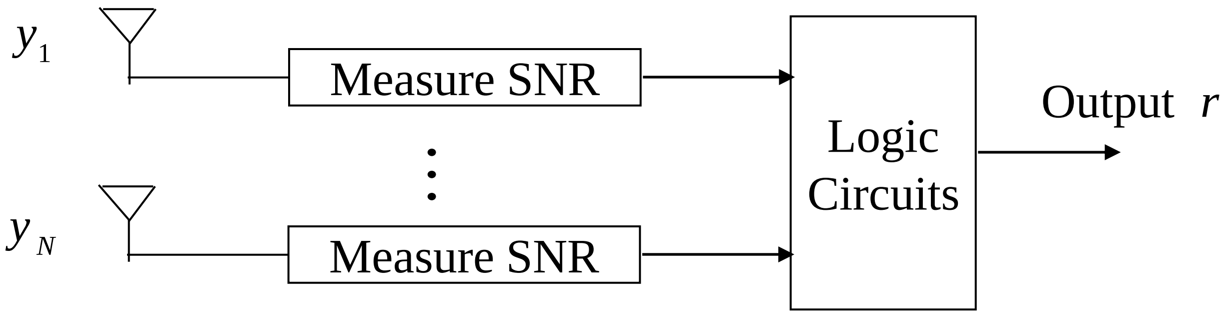
SPACE DIVERSITY: SELECTION DIVERSITY

- **Selection diversity**
  - Out of the N received signals, at each symbol period, choose the one with the maximum instantaneous SNR.
    - Demodulation and decoding is performed for the signals with the maximal SNR.
    - Signals on other antennas are discarded.



- SNR of the kth Rx antenna

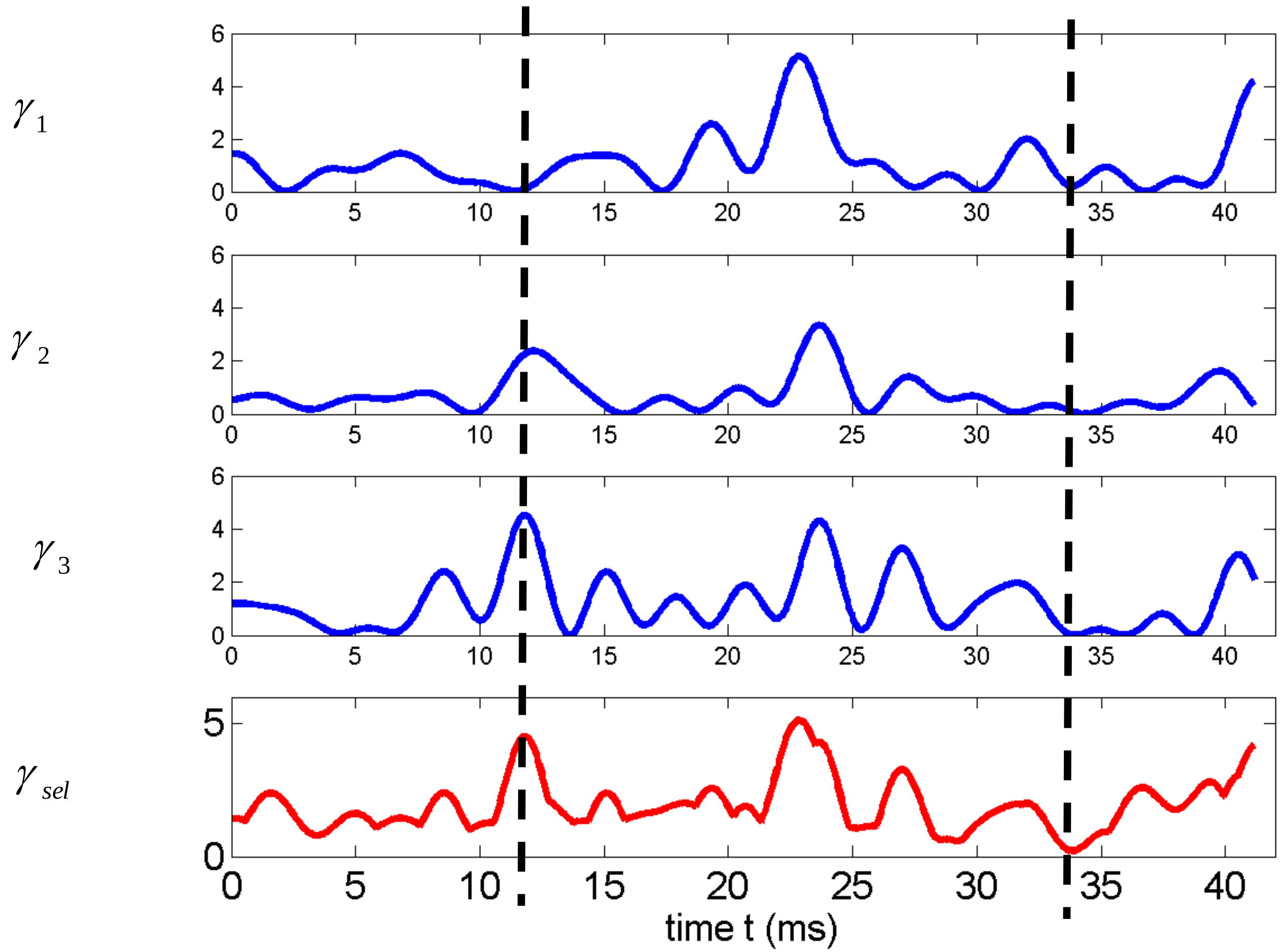
$$\gamma_k = |h_k|^2 \cdot \gamma_0$$

- SNR after selection diversity

$$\gamma_{sel} = \max \{\gamma_1, \gamma_2, \cdots, \gamma_N\}$$

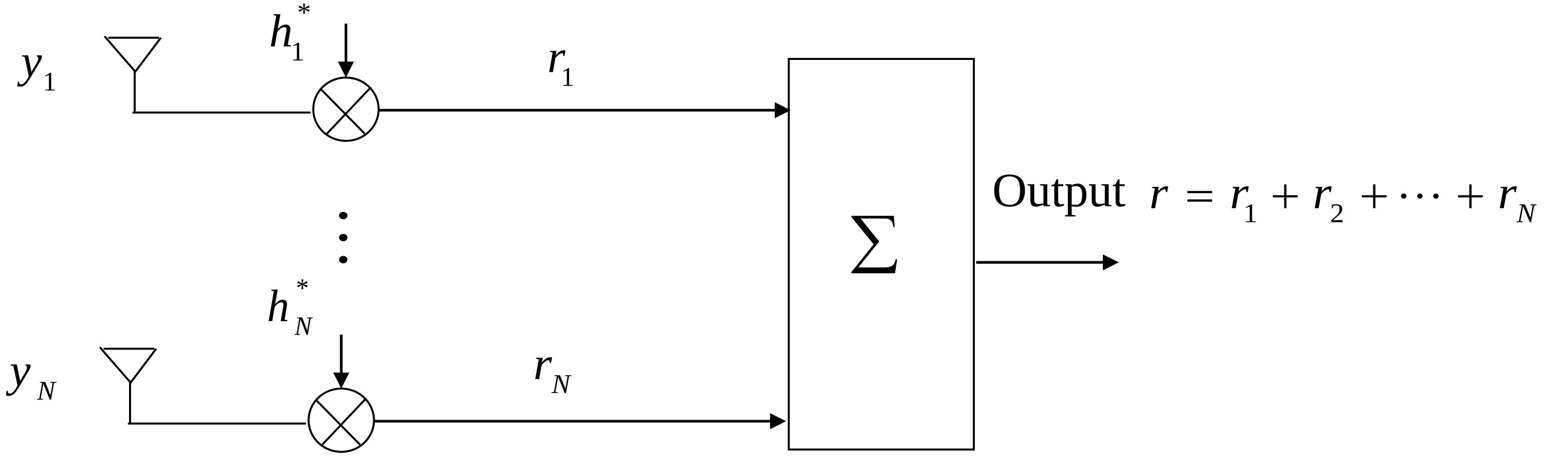
SPACE DIVERSITY: SELECTION DIVERSITY

- **SNR after selection diversity (N = 3)**



SPACE DIVERSITY: MAXIMAL RATIO COMBINGING

- **Maximal Ratio Combining (MRC)**



  - The signals from all Rx antennas are processed and combined together
  - The output on the *k*th branch

$$r_k = |h_k|^2 \cdot x + h_k^* \cdot n_k$$

  - The output of the MRC receiver

$$r = \sum_{k=1}^{N} |h_k|^2 \cdot x + \sum_{k=1}^{N} \left(h_k^* \cdot n_k\right)$$

SPACE DIVERSITY: MRC

- **SNR of MRC**

$$r = \sum_{k=1}^{N} |h_k|^2 \cdot x + \sum_{k=1}^{N} \left(h_k^* \cdot n_k\right)$$

  – Power of signal component

$$\left[\sum_{k=1}^{N} |h_k|^2\right]^2 \cdot E_s$$

  – Power of noise

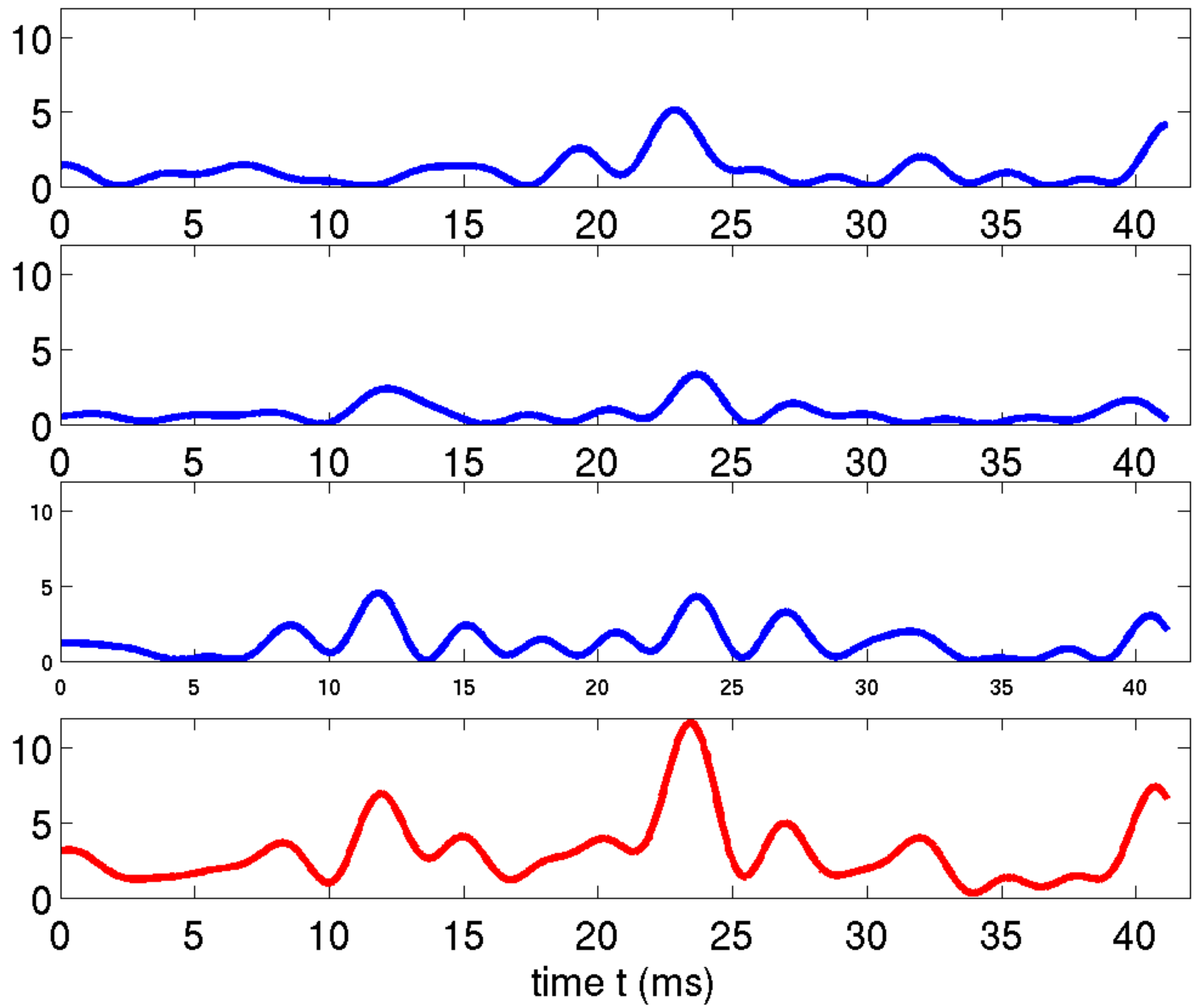
$$E\left|\sum_{k=1}^{N} \left(h_k^* \cdot n_k\right)\right|^2 = \sum_{k=1}^{N} |h_k|^2 \cdot \sigma_n^2$$

  – SNR

$$\gamma = \frac{\left[\sum_{k=1}^{N} |h_k|^2\right]^2 \cdot E_s}{\sum_{k=1}^{N} |h_k|^2 \cdot \sigma_n^2} \Rightarrow \gamma = \sum_{k=1}^{N} |h_k|^2 \cdot \gamma_0 = \sum_{k=1}^{N} \gamma_k$$

- **The SNR of MRC equals to the sum of SNRs of individual branches!**
  – MRC is the optimum diversity receiver (highest SNR among all diversity receivers.)

SPACE DIVERSITY: MRC



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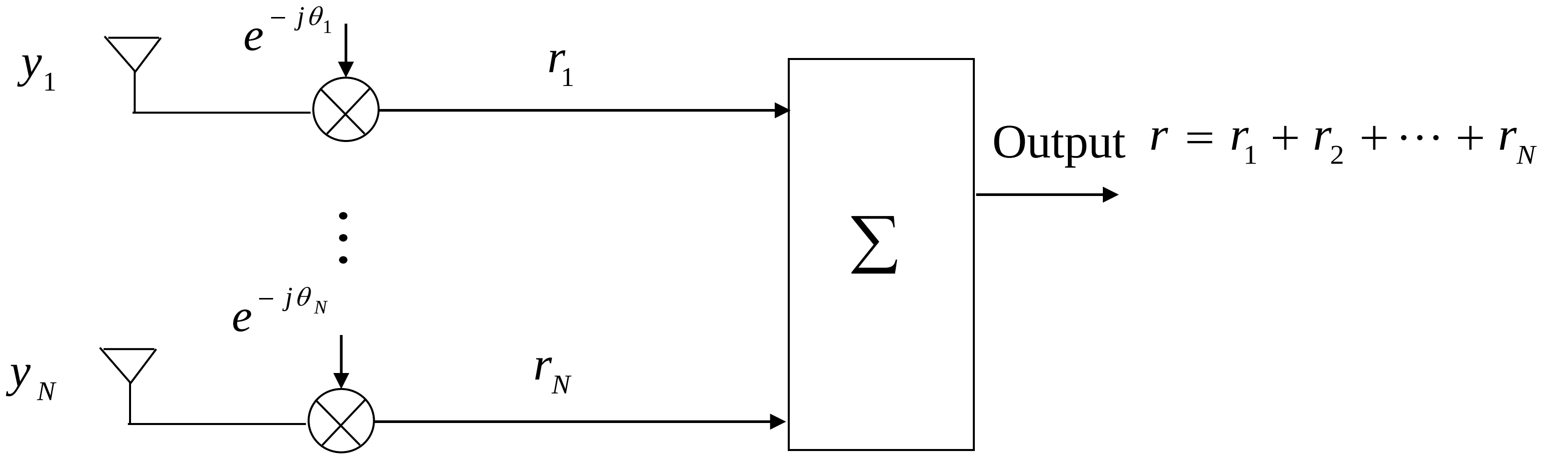
SPACE DIVERSITY: EQUAL GAIN COMBINING

- **Equal gain combining (EGC)**



  - θ_k : the phase of h_k , i.e., $h_k = |h_k| e^{j\theta_k}$
  - The output of the *k*th branch:

$$r_k = |h_k| \cdot x + e^{-j\theta_k} \cdot n_k$$

  - The output of the Equal Gain receiver

$$r = \sum_{k=1}^{N} |h_k| \cdot x + \sum_{k=1}^{N} \left(e^{-j\theta_k} \cdot n_k \right)$$

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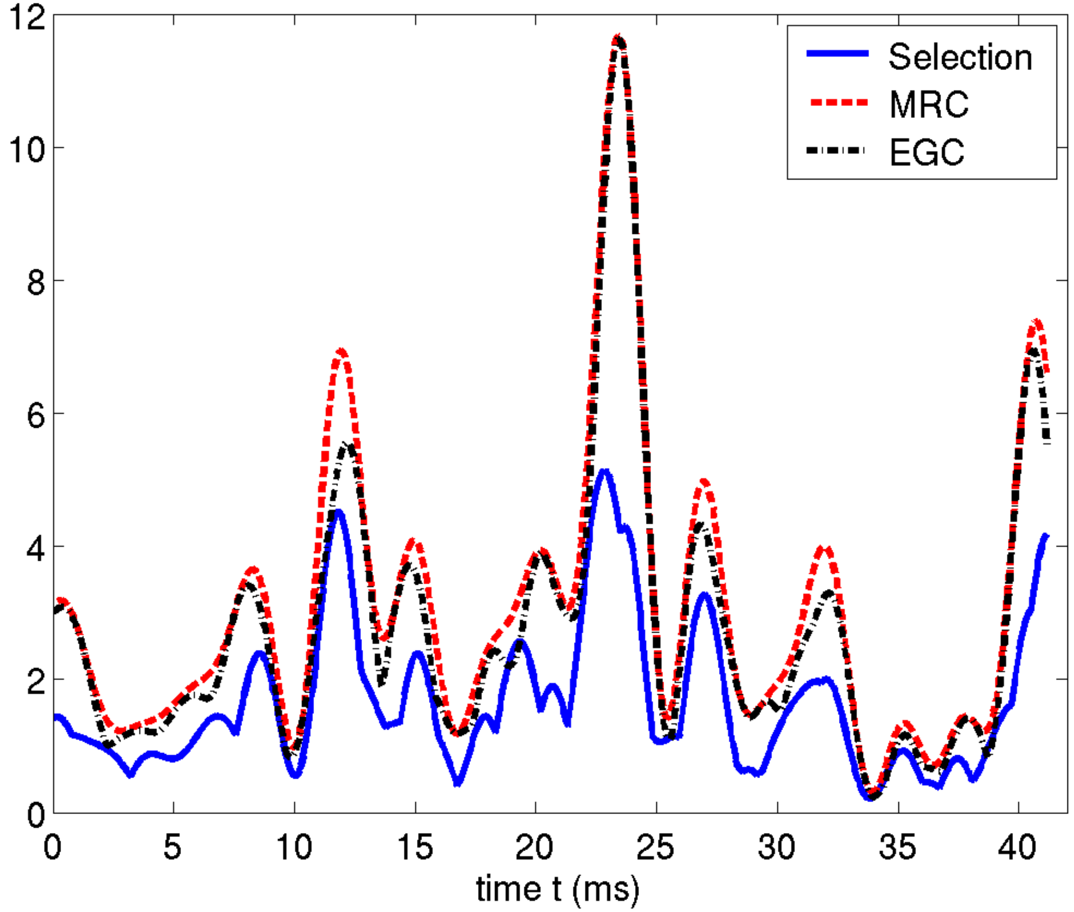
SPACE DIVERSITY: EQUAL GAIN COMBINING

- **SNR of equal gain combining**

$$\gamma = \frac{1}{N}\left[\sum_{k=1}^{N} |h_k|\right]^2 \cdot \gamma_0$$

- **Comparison of Selection diversity, MRC, and EGC**



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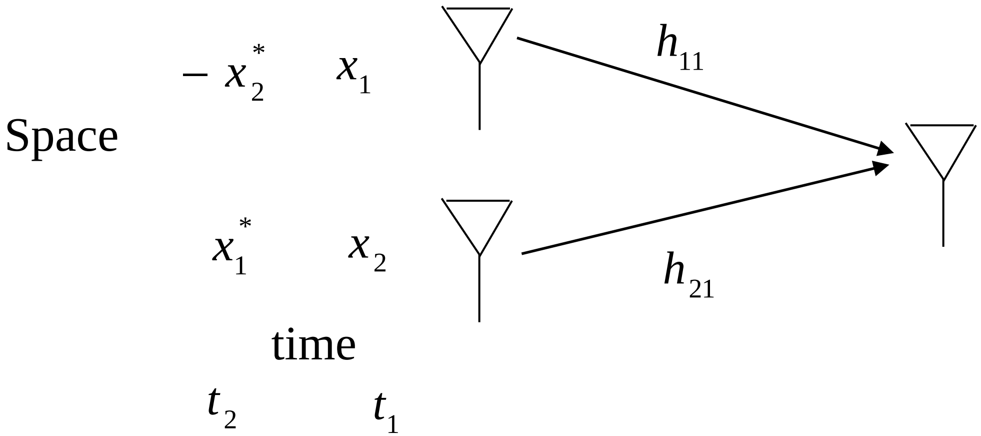
- **Space Time Block Code**

STBC

- **Orthogonal Space Time Block Code**
  - Proposed by Alamouti in 1998
    - "A simple transmit diversity technique for wireless communications", IEEE Journal on Selected Areas of Communications, vol. 16, 1998.
  - Suitable for system with 2 Tx antennas and arbitrary number of receive antennas
  - A simple, yet powerful space diversity technique
    - It doesn't require any knowledge of the channel at transmitter.
      - ALL transmit diversity techniques before STBC require some knowledge of the channel at the transmitter
      - The transmitter can learn the channel state through feedback from receiver ➔ extra bandwidth is used for feedback
  - It has been adopted by 3GPP HSDPA (High Speed Downlink Packet Access)

STBC

- **Structure (2 Tx, 1 Rx)**



  - Assumption: the channel keeps constant during two consecutive symbol periods (t_1 and t_2) .
  - At time t_1 , transmit x_1 on antenna 1, transmit x_2 on antenna 2.

$$y_1 = h_{11}x_1 + h_{21}x_2 + n_1$$

  - At time t_2 , transmit $-x_2^*$ on antenna 2, transmit x_1^* on antenna 1.

$$y_2 = -h_{11}x_2^* + h_{21}x_1^* + n_2$$

STBC

- **System equation**

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \\ h_{21}^* & -h_{11}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}$$

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}$$

- **Channel matrix is orthogonal**

$$\mathbf{H}^H \mathbf{H} =$$

- **Receiver**

$$\mathbf{r} = \mathbf{H}^H \mathbf{y} =$$

$$r_1 = \left[|h_{11}|^2 + |h_{21}|^2 \right] x_1 + h_{11}^* n_1 + h_{21} n_2^*$$

$$r_2 = \left[|h_{11}|^2 + |h_{21}|^2 \right] x_2^* + h_{21}^* n_1 - h_{11} n_2^*$$

STBC

- **2 Tx, *M* Rx**
  - At time t_1 , transmit x_1 on antenna 1, transmit x_2 on antenna 2.
    - At the m-th Rx antenna

$$y_{1m} = h_{1m} x_1 + h_{2m} x_2 + n_{1m}$$

  - At time t_2 , transmit $-x_2^*$ on antenna 1, transmit x_1^* on antenna 2.
    - At the m-th Rx antenna

$$y_{2m} = -h_{1m} x_2^* + h_{2m} x_1^* + n_{2m}$$

  - System equation at the m-th Rx antenna

$$\begin{bmatrix} y_{1m} \\ y_{2m}^* \end{bmatrix} = \begin{bmatrix} h_{1m} & h_{2m} \\ h_{2m}^* & -h_{1m}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_{1m} \\ n_{2m}^* \end{bmatrix}$$

$$\mathbf{y}_m = \mathbf{H}_m \mathbf{x} + \mathbf{n}_m$$

STBC

- **2 Tx, *M* Rx (Cont'd)**
  - System equation for all the Rx antennas

$$\begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_M \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_M \end{bmatrix} \mathbf{x} + \begin{bmatrix} \mathbf{n}_1 \\ \vdots \\ \mathbf{n}_M \end{bmatrix}$$

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

  - Receiver

$$\mathbf{r} = \mathbf{H}^H \mathbf{y} =$$