Department of Electrical Engineering
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ELEG5693 Wireless Communications
Ch. 3 Modulation

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OUTLINE

- **Modulation**
- **Linear Modulation I**
- **Complex Signal Representation**
- **Pulse Shaping**
- **Linear Modulation II**
- **Non-linear Modulation**
- **FDMA**

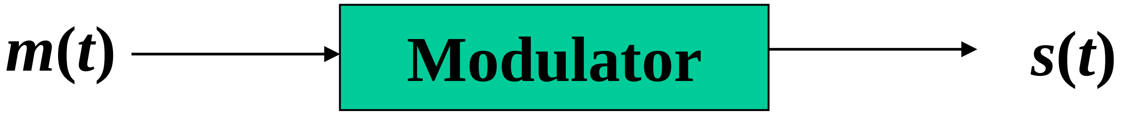
MODULATION: DEFINITION

- **What is modulation?**
  - The process of modifying a carrier signal (usually a sinusoid) in accordance with an information-bearing signal (modulating signal), such that the modified carrier signal (modulated signal) can be used to convey information.
  - Carrier signal:

$$c(t) = A_c \sin(2\pi f_c t + \theta)$$

    - Three parameters: amplitude, frequency, phase
    - One or all of them can be modified to carry information.
      - E.g. the amplitude is modified to carry information $m(t)$

$$s(t) = m(t) A_c \sin(2\pi f_c t + \theta)$$



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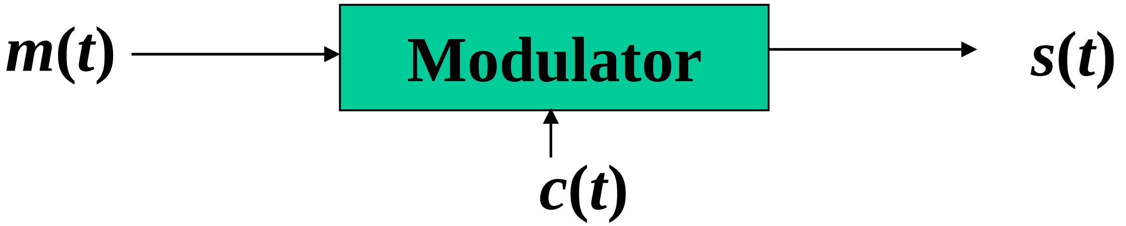
MODULATION: DEFINITION

- **What is modulation? (cont'd)**
  - Modulating signal $m(t)$
    - Original electrical information-bearing signal
    - E.g. electrical waveform representing voice, ASCII, digital video, etc.
    - Usually at low frequency
    - Also called baseband signal
  - Carrier signal $c(t)$
    - High frequency sinusoid to carry the information.
  - Modulated signal $s(t)$
    - Carrier signal modified by modulating signal
    - At the output of the modulator.
    - Also called bandpass signal, or RF signal



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MODULATION: WHY?

- **Why modulation?**
  - Shift the frequency of the message signal to the pre-allocated channel.
    - For example:
      - speech signal: 300 ~ 3100 Hz $m(t)$
      - Shift the signal to the allocated frequency range: 900MHz $c(t)$
  - Transfer the message into a form more suitable for wireless transmission.
    - Limited bandwidth
    - Make better use of the limited spectrum
    - High frequency signals are more suitable for wireless transmission.
  - Enables multiple access
    - Signals from different users can be shifted to different frequencies.
    - Modulation allows the simultaneous transmission of multiple users.

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MODULATION: TERMS

- **Demodulation**
  - Recover the original message signal $m(t)$ from the modulated signal $s(t)$.
- **Modulator**
  - Device used to perform modulation
- **Demodulator**
  - Device used to perform demodulation.

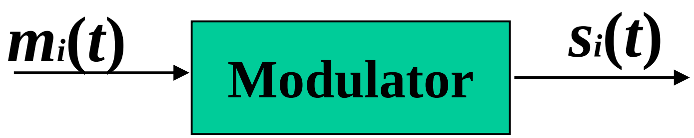
$r(t)$: modulated signal impaired by fading and noise.

MODULATION: CLASSIFICATIONS

- **Linear modulation v.s. non-linear modulation**
- **Analog modulation v.s. Digital modulation**
- **Amplitude modulation v.s. Angle modulation**

MODULATION: LINEAR V.S. NON-LINEAR

- **Principle of superposition**



  - If input is $m_1(t)+m_2(t)$, then output is $s_1(t)+s_2(t)$
  - If input is $a \cdot m_1(t)$, then output is $a \cdot s_1(t)$
- **If the input-output of the modulator satisfies principle of superposition, then the modulator is called linear modulator.**
  - E.g.

$$s(t) = m(t)A_c \sin(2\pi f_c t + \theta)$$

- **If the principle of superposition is not satisfied, the modulator is non-linear modulator.**

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MODULATION: ANALOG V.S. DIGITAL

- **Analog modulation**
  - $m(t)$ is an analog signal
    - $m(t)$ is a continuous function of time t.
    - $m(t)$ can take infinite number of values
  - Analog modulation is also called continuous-wave (CW) modulation.
  - AM radio, FM radio, first generation cell phone system
- **Digital modulation**
  - $m(t)$ is a digital signal
    - Take finite number of values
    - E.g. {-1, 1}, {-2, -1, 1, 2}, …
  - 2^{nd} generation cell phone system
  - Digital modulation systems have become more and more popular.

MODULATION: AMPLITUDE V.S. ANGLE

- **Amplitude modulation**
  - The amplitude of the carrier, A_c , is modified by the message signal $m(t)$.

- **Angle modulation**
  - The angle of the carrier is modified by the message signal $m(t)$.

$$\psi(t) = 2\pi f_c t + \theta$$

  - Frequency modulation: frequency is modified by $m(t)$.
  - Phase modulation: phase is modified by $m(t)$

OUTLINE

- Modulation
- **Linear Modulation Techniques I**
- Complex Signal Representation
- Pulse Shaping
- Linear Modulation Techniques II
- Non-linear modulation
- FDMA

LINEAR MODULATION: AM

- **Amplitude modulation (AM)**
  - The term, AM, is usually used when $m(t)$ is analog

$$s(t) = A_c m(t)\cos(2\pi f_c t)$$

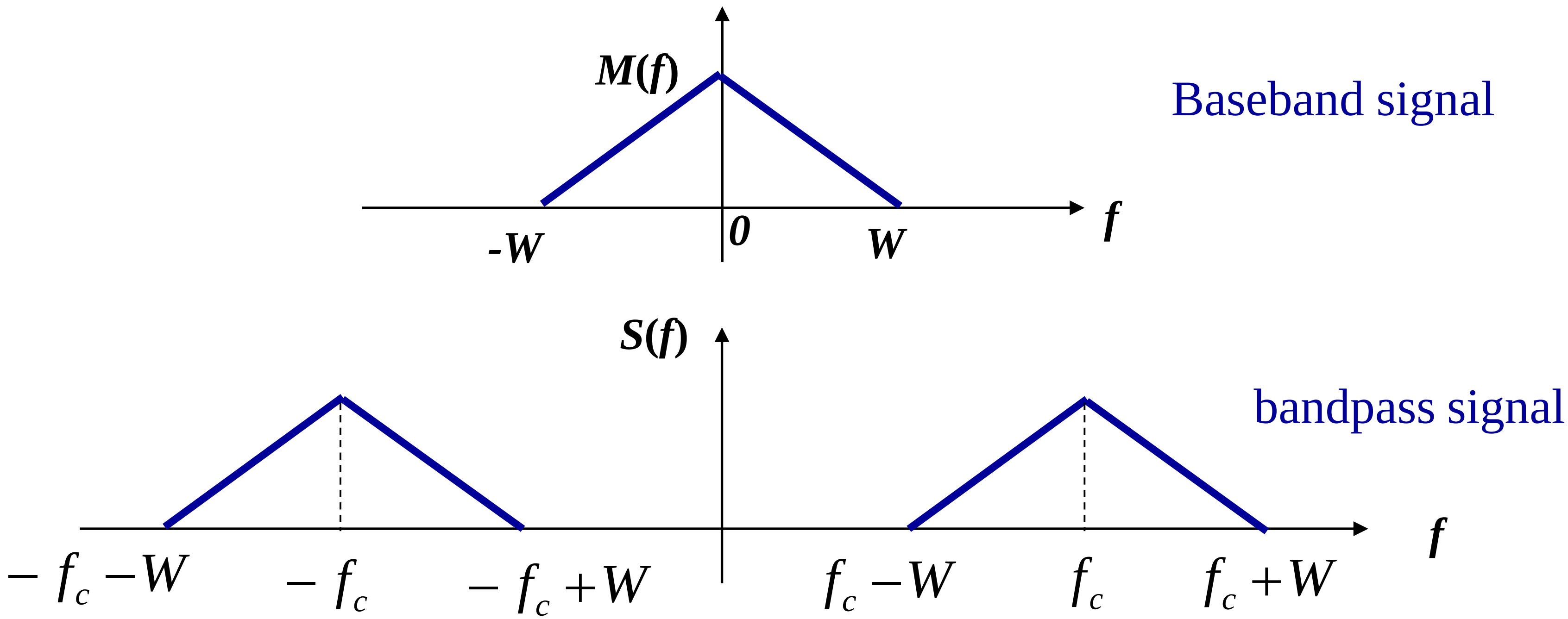
message signal

carrier frequency

- **Spectrum of AM**
  - $m(t) \rightarrow M(f)$
  - $A_c \cos(2\pi f_c t) \rightarrow \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$
  - $s(t) \rightarrow S(f) = ?$

LINEAR MODULATION: AM

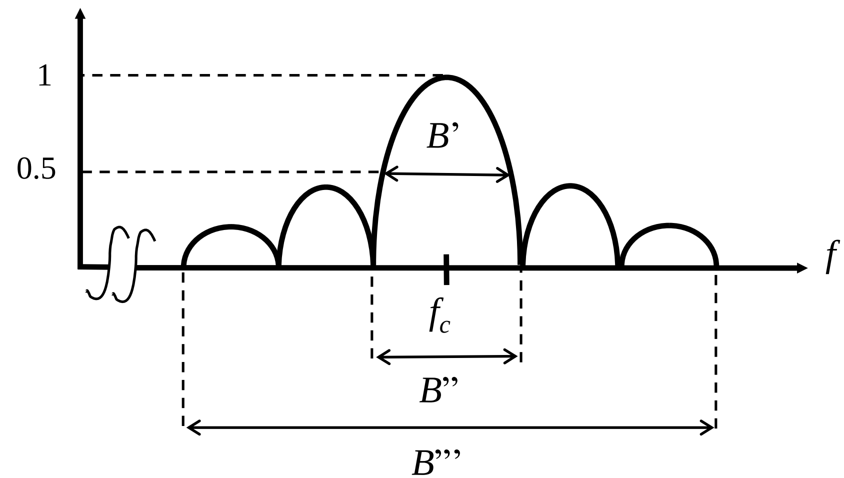
- **Spectrum of AM**



- **Center frequency shifted from 0 to** f_c
- **The bandwidth is doubled from *W* to 2*W***

LINEAR MODULATION: AM

- **Bandwidth**
  - Baseband bandwidth (bandwidth of baseband signal): from 0 to W
  - Passband bandwidth (bandwidth of modulated signal): from Fc-W to $Fc+W$.
    - Passband bandwidth is twice of baseband bandwidth.
- **How do we define bandwidth?**
  - There are many different definitions of bandwidth
  - B': half power bandwidth (3dB bandwidth).
  - B'': null-to-null bandwidth
  - B''': absolute bandwidth
  - FCC definition: bandwidth contains 99% of signal power



power spectrum

LINEAR MODULATION: BPSK

- **Binary phase shift keying (BPSK) ➔ Digital modulation**

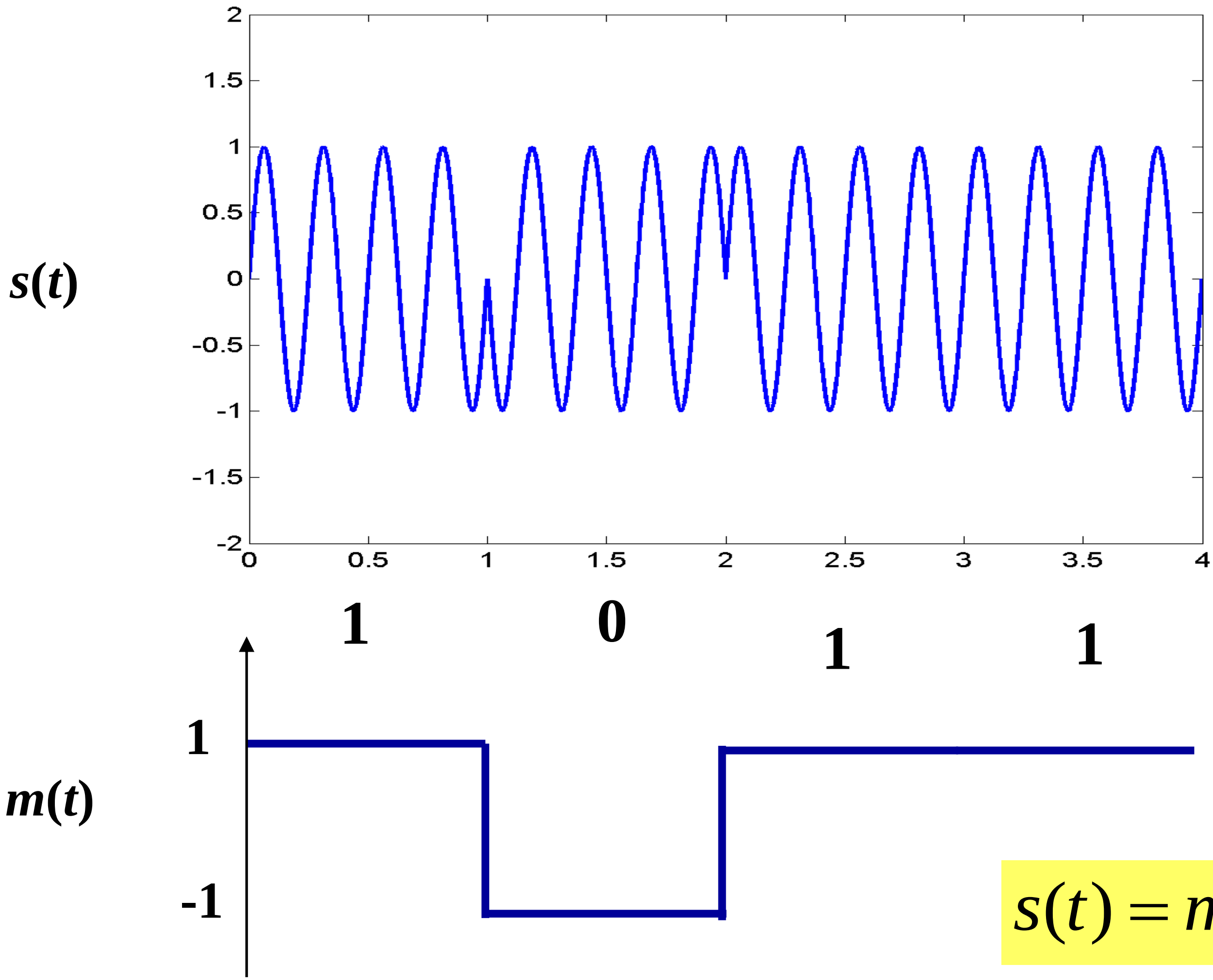
$$s(t) = \begin{cases} A_c \cos(2\pi f_c t) & \text{for binary 1} \\ A_c \cos(2\pi f_c t + \pi) & \text{for binary 0} \end{cases}$$

– Or alternatively

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t) & \text{for binary 1} \\ -A_c \cos(2\pi f_c t) & \text{for binary 0} \end{cases}$$

– Also called binary amplitude shift keying (BASK)

LINEAR MODULATION: BPSK

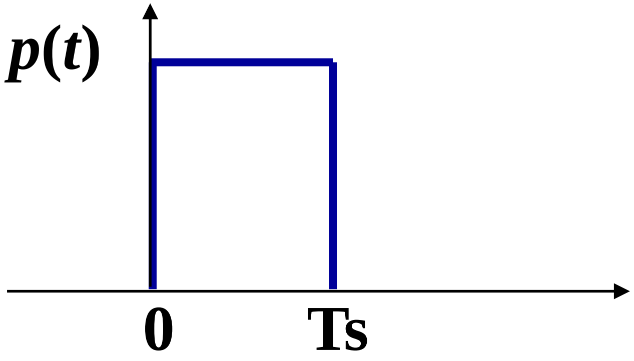


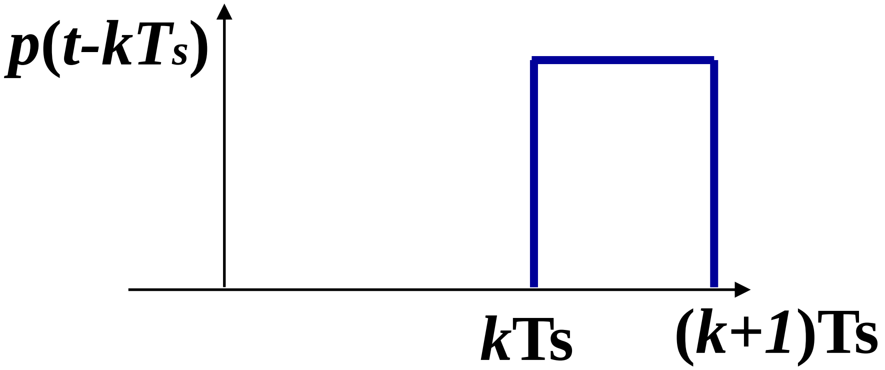
$$s(t) = m(t) \cdot \sin(2\pi f_c t)$$

LINEAR MODULATION: BPSK

$$m(t) = \sum_{k=1}^{\infty} b_k \, p(t - kT_s)$$

$$b_k = \begin{cases} 1 \\ -1 \end{cases} \qquad p(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & o.w. \end{cases}$$





***Ts*: symbol period. (The time duration to carry one symbol)**

- usually inverse proportional to signal BW

LINEAR MODULATION: MASK

- **M-ary amplitude shift keying (MASK)**
  - The information can take $M = 2^n$ values

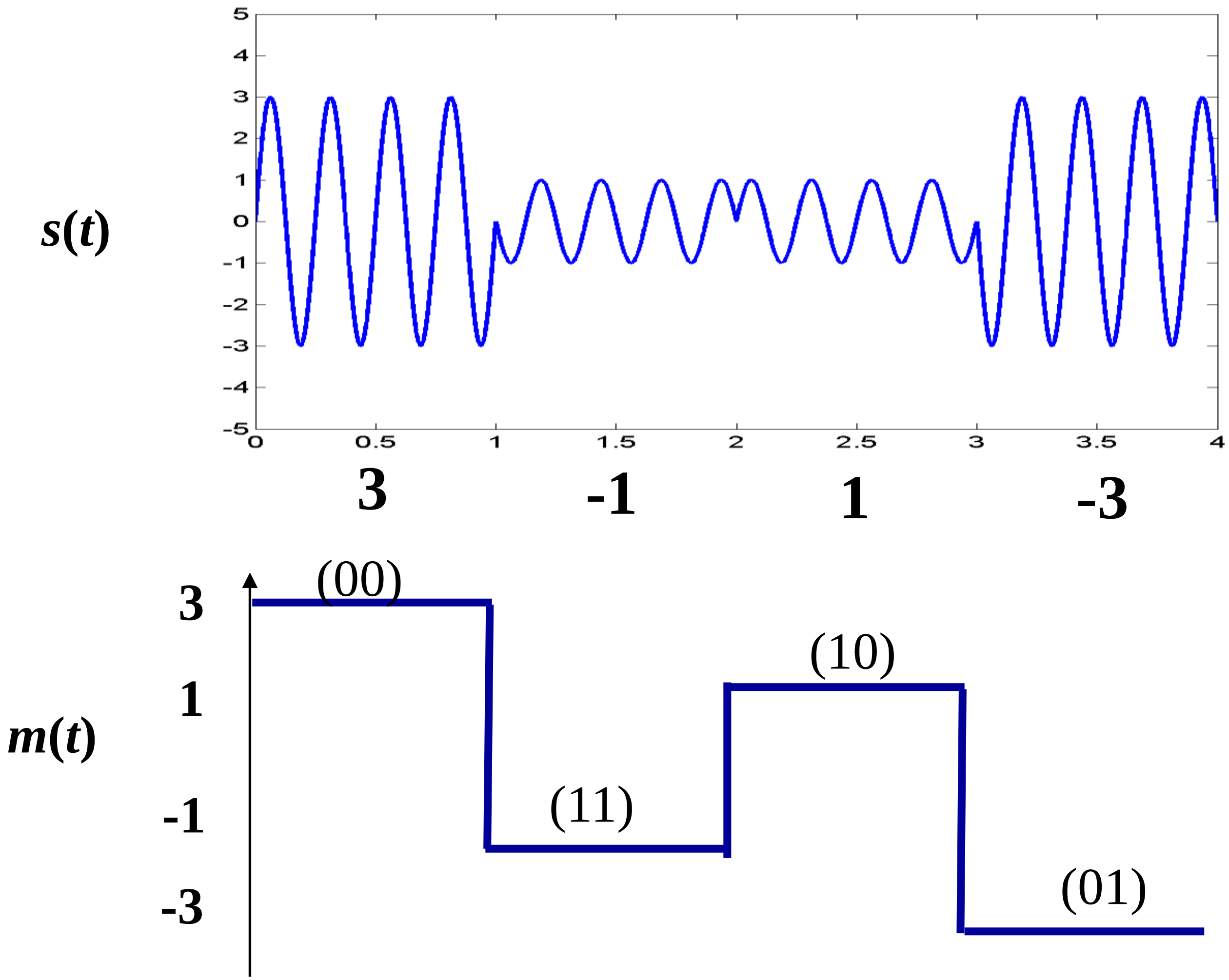
$$m(t) = \sum_{k=1}^{\infty} m_k \, p(t - kT_s)$$

$$m_k = \{s_1, s_2, \cdots, s_M\} \qquad p(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & o.w. \end{cases}$$

- **E.g. 4-ary amplitude shift keying (4ASK)**

$$m_k = \{-3, -1, 1, 3\}$$

LINEAR MODULATION: MASK



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LINEAR MODULATION: MASK

- **Each symbol can take *M* values**

  ➔Each symbol can represent $n = \log_2 M$ bits of information.

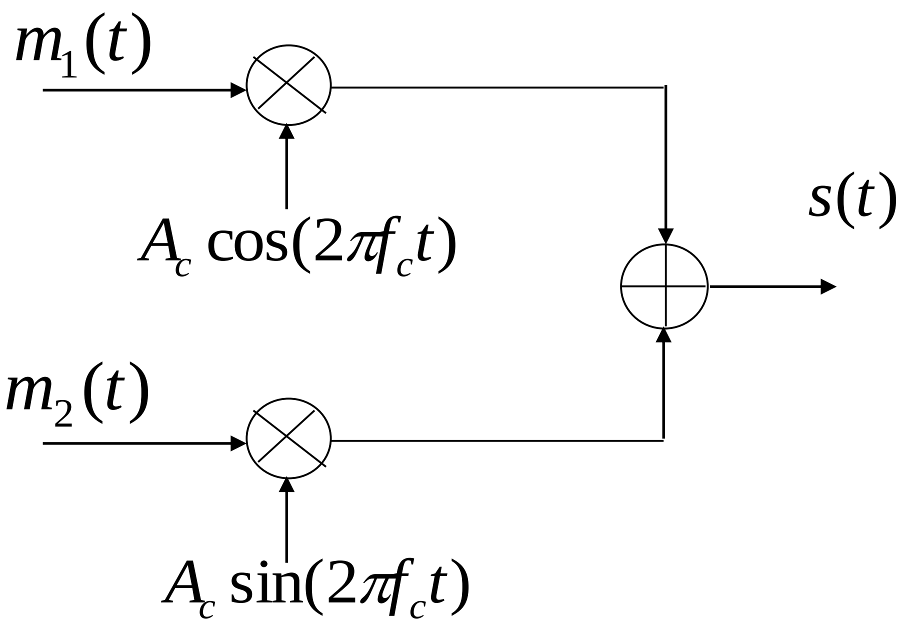
- **E.g.**
  - 4ASK ➔ M = 4 ➔ 2 bits/sym
  - 8ASK ➔ M = 8 ➔ 3 bits/sym
  - 16ASK ➔ M = 16 ➔ 4 bits/sym
- **Symbol rate (baud) R_s :**
  - # of modulation symbols/second
  - Generally, signal bandwidth is proportional to symbol rate!
- **Bit rate R_b:**
  - Bits/second
- **$R_b = R_s$ x (# of bits/sym)**

LINEAR MODULATION: MASK

- **M-ary modulation**
  - At the same symbol rate, (or fixed bandwidth)
    - Larger *M* ➔ more bits/symbol ➔ larger bit rate
  - M-ary modulation is good for band-limited system
    - In wireless systems, spectrum is precious
    - M-ary modulation is widely used in wireless systems!
    - Typical values used in wireless system: M = 2, 4, 8, 16
- **Why don't we use a very large M?**
  - If M ➔ infinity, then analog signal
  - At the same SNR
    - Larger M ➔ signals are more closed to each other ➔ it's harder to distinguish between all signals at receiver due to noise ➔ probability of error becomes larger!

LINEAR MODULATION: QPSK

- **Quadrature phase shift keying (QPSK)**
  - Combination of two BPSK streams
  - Quadrature: orthogonal (two carriers that are 90 degree apart)



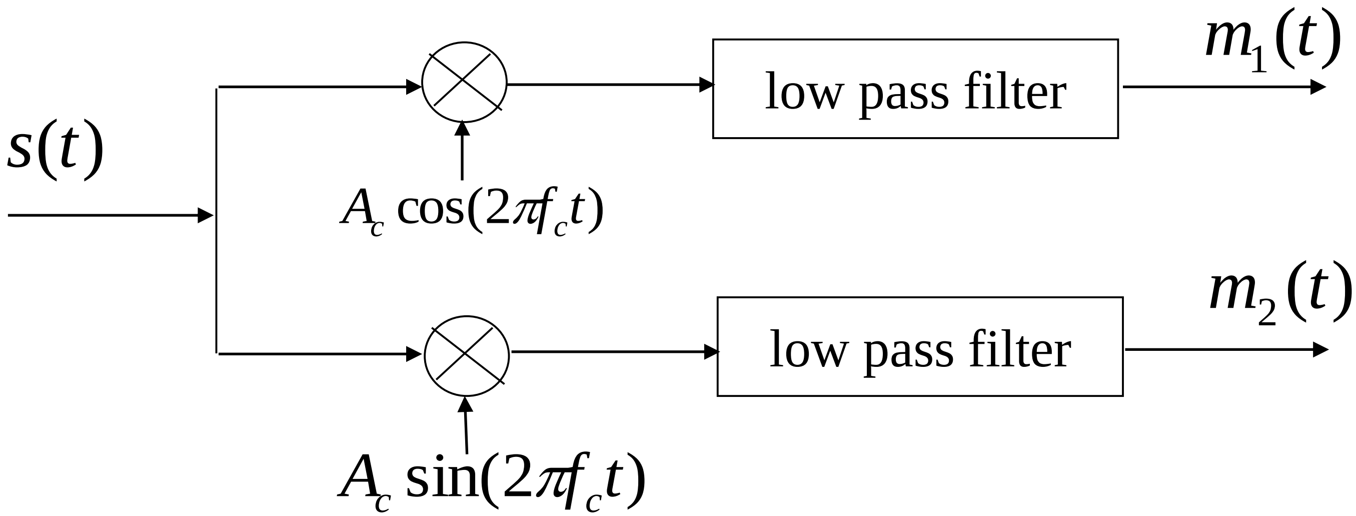
$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$$

Inphase signal

quadrature signal

LINEAR MODULATION: QPSK

- **Demodulator**
  - How could these two signals not interfere with each other?



$$s(t)\cos(2\pi f_c t) = A_c m_1(t)\cos^2(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)\cos(2\pi f_c t)$$

$$= \frac{A_c}{2} m_1(t) + \frac{A_c}{2} m_1(t)\cos(4\pi f_c t) + \frac{A_c}{2} m_2(t)\sin(4\pi f_c t)$$

After low pass filter: $\frac{A_c}{2} m_1(t)$

LINEAR MODULATION: QPSK

- **At each symbol period, two bits of information are transmitted**
  - 2 bits/symbol
  - Inphase signal $s_I(t) = A_c m_1(t)\cos(2\pi f_c t)$ has the same bandwidth as BPSK
  - Quadrature signal $s_Q(t) = A_c m_1(t)\sin(2\pi f_c t)$ has the same bandwidth as BPSK
  - The sum $s(t) = s_I(t) + s_Q(t)$ has the same bandwidth as BPSK
    - Sum in time domain ➔ sum in frequency domain ➔ bandwidth unchanged.
  - The same bandwidth as BPSK, but twice the bit rate of BPSK!

  - No interference between quadrature and inphase
    - Inphase has the same error performance as BPSK
    - Quadrature has the same error performance as BPSK
    - QPSK has the same error performance of BPSK!

One of the rare occasions that increase bit rate without sacrificing error performance!

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COMPLEX REPRESENTATION

- **Band-pass signal**

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

- **Complex base-band signal (also called the complex envelope of the signal)**

$$\tilde{s}(t) = s_I(t) + j s_Q(t)$$

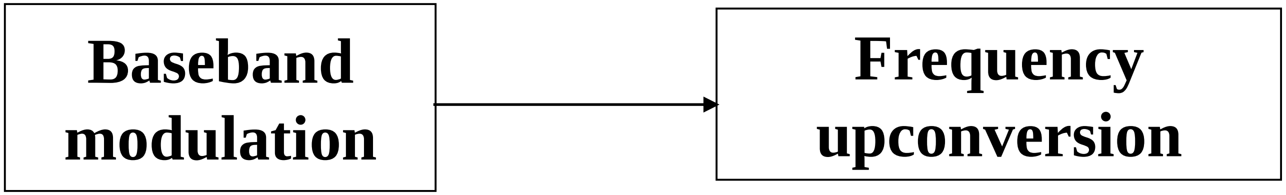
- **Relationship between complex baseband and band-pass**

$$s(t) = \mathrm{Re}\left\{\tilde{s}(t)\exp(j2\pi f_c t)\right\}$$

COMPLEX REPRESENTATION

- **Modulation can be decomposed into two steps:**
  - 1. complex baseband modulation, 2. frequency upconversion
    - The complex representation completely preserves the information of the modulating signal except the carrier frequency
  - It's suffice for us to examine baseband modulation only!

$$s(t) = \mathrm{Re}\left\{\tilde{s}(t)\exp(j2\pi f_c t)\right\}$$



Modulation

COMPLEX REPRESENTATION: BASEBAND MODULATION

- **BPSK**

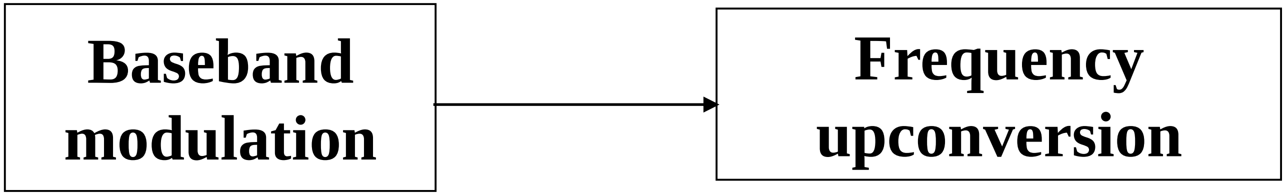
$$s_I(t) \in \{-1,1\} \qquad s_Q(t) = 0$$

$$\tilde{s}(t) \in \{-1,1\} \qquad (M = 2)$$

- **QPSK**

$$s_I(t) \in \{-1,1\} \qquad s_Q(t) \in \{-1,1\}$$

$$\tilde{s}(t) \in \{-1-j, -1+j, 1-j, 1+j\} \qquad (M = 4)$$



Modulation

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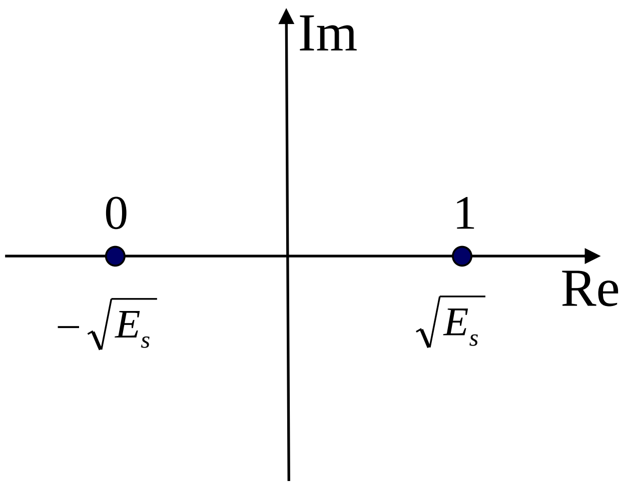
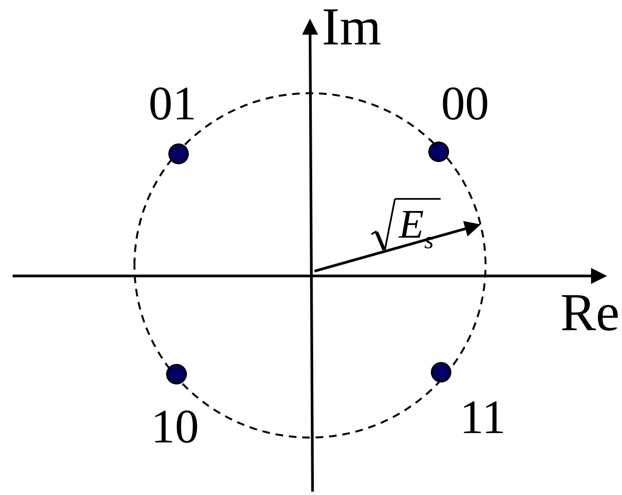
COMPLEX REPRESENTATION: CONSTELLATION

- **Modulation Constellation**
  - The collection of all modulation symbols in their complex representation.
  - E_s : the energy of one symbol



BPSK



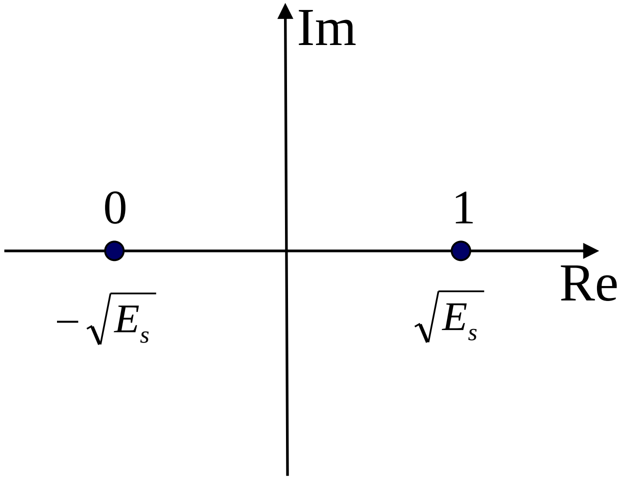
QPSK

$$\tilde{s}(t) \in \left\{ \sqrt{E_s}\exp(j0), \sqrt{E_s}\exp(j\pi) \right\}$$

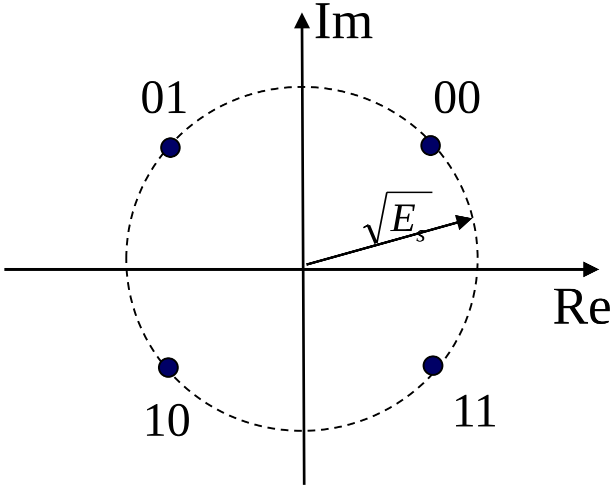
$$\tilde{s}(t) \in \left\{ \begin{matrix} \sqrt{E_s}\exp(j\frac{\pi}{4}), \sqrt{E_s}\exp\left(j\frac{3\pi}{4}\right), \\ \sqrt{E_s}\exp(j\frac{5\pi}{4}), \sqrt{E_s}\exp\left(j\frac{7\pi}{4}\right) \end{matrix} \right\}$$

COMPLEX REPRESENTATION

- **Baseband modulation examples**



BPSK



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BPSK:

QPSK:

COMPLEX REPRESENTATION: SYMBOL ENERGY

- **Symbol energy *Es***
  - If modulation symbol is

$$\tilde{s}(t) = \sqrt{E_s} \exp\left[j\theta(t) \right]$$

  - The energy of one symbol is E_s .
    - Proof:

- **Bit energy *Eb***
  - The energy of one bit
  - # of bits per symbol: $\log_2 M$

$$E_b = E_s / \log_2 M$$

COMPLEX REPRESENTATION

- **Signals pass through channel with flat fading and AWGN**

$$\tilde{y}(t) = \tilde{h}(t) \times \tilde{s}(t) + \tilde{n}(t)$$

$\tilde{h}(t) = h_I(t) + jh_Q(t)$ **: time-varying flat fading**

$\tilde{n}(t) = n_I(t) + jn_Q(t)$ **: AWGN**

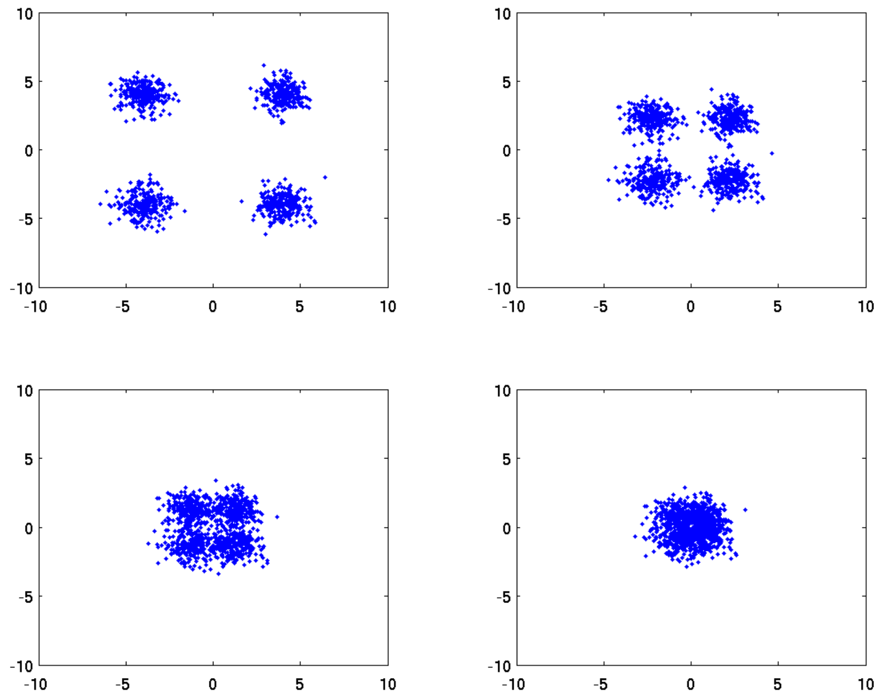
Both the inphase component and quadrature component are going to be distorted.

COMPLEX REPRESENTATION: SNR

- **Signal to noise ratio (SNR)**
  - The ratio between signal power and noise power
  - E.g. same level of noise, signal power becomes smaller

SNR=15dB

SNR=10dB



SNR=5dB

SNR=0dB

QPSK: $\tilde{y}(t) = \tilde{s}(t) + \tilde{n}(t)$

COMPLEX REPRESENTATION: SNR

- **Baseband demodulation**
  - Find the modulation symbol that has the smallest Euclidean distance with the received signal.
  - E.g. QPSK: r = x + n

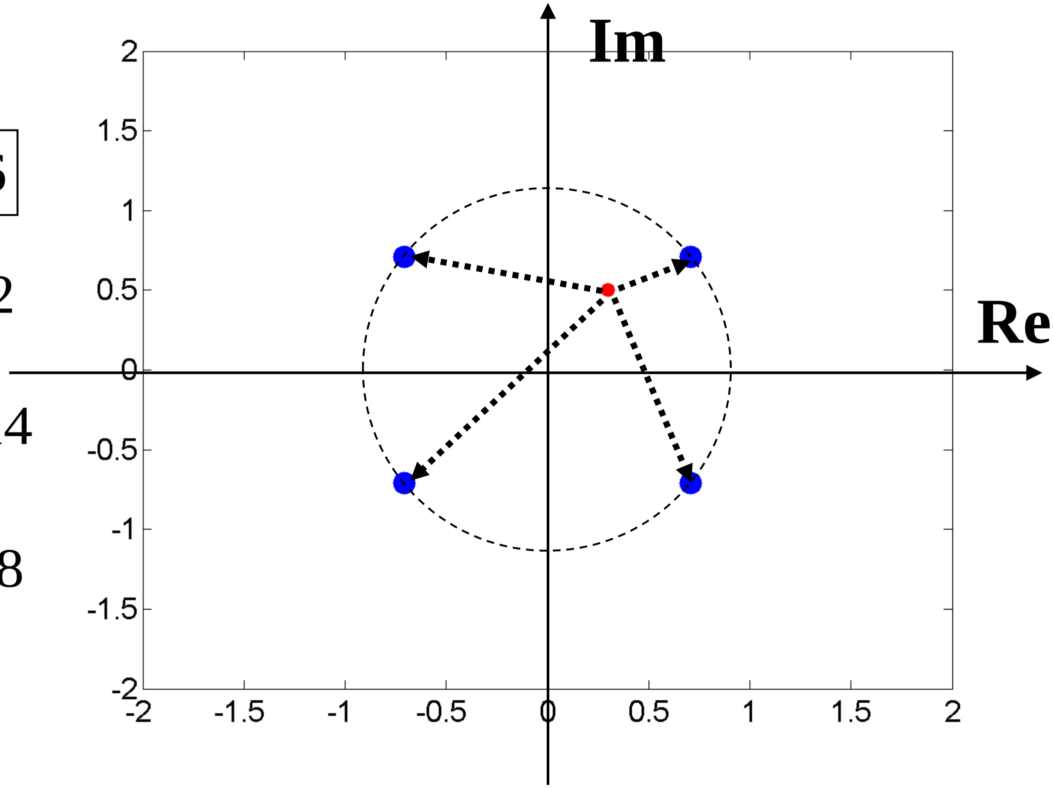
    r = 0.3 + j 0.5

$$|0.3 + j0.5 - \exp(j\pi/4)|^2 = 0.2086$$

$$|0.3 + j0.5 - \exp(j3\pi/4)|^2 = 1.0572$$

$$|0.3 + j0.5 - \exp(j5\pi/4)|^2 = 2.4714$$

$$|0.3 + j0.5 - \exp(j7\pi/4)|^2 = 1.6228$$



COMPLEX REPRESENTATION: SNR

- **SNR**
  - The ratio between signal power and noise power

$$SNR = \frac{P_s}{P_n} = \frac{E_s / T_s}{N_0 B}$$

Symbol period

Power spectral density of noise

Signal bandwidth

Symbol rate ***Rs = 1/Ts***

Bit rate ***Rb***

$$SNR = \frac{E_s}{N_0} \cdot \frac{R_s}{B} = \frac{E_b}{N_0} \cdot \frac{R_s \log_2 M}{B} = \frac{E_b}{N_0} \cdot \frac{R_b}{B}$$

COMPLEX: BANDWIDTH EFFICIENCY

- **Bandwidth efficiency**
  - How many bits can be accommodated in unit bandwidth (bps/Hz)

$$\eta_B = \frac{R_b}{B}$$

bit rate (R_b), bandwidth (B)

  - Measures the ability of the modulation technique to accommodate data in a limited bandwidth
    - Larger bandwidth efficiency ➔ more data can be transferred in limited bandwidth
- **Generally, the bandwidth is proportional to symbol rate**

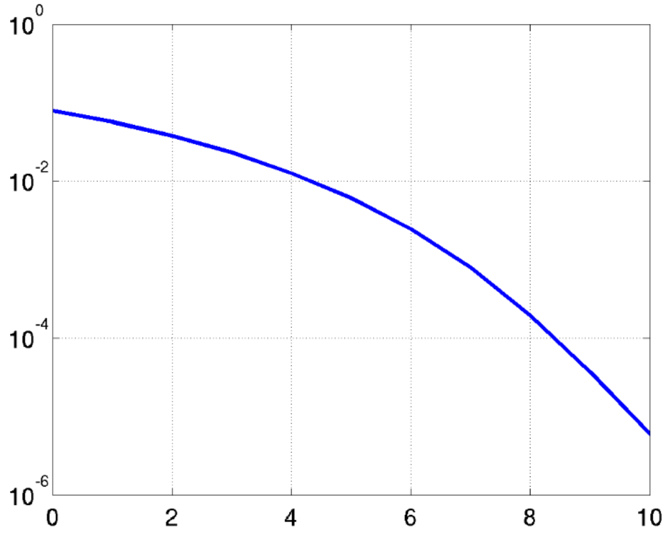
$$B \propto R_s$$

  - For M-ary modulation, fixed bandwidth B ➔ fixed symbol rate R_s
    - Larger M ➔ larger $R_b = R_s \log_2 M$ ➔ larger bandwidth efficiency

COMPLEX: ENERGY EFFICIENCY

- **Energy efficiency**
  - At given noise level, how much energy is required to achieve a certain bit error rate.
  - Measured as the value of Eb/N0 required for certain bit error rate (BER).
    - Eb: energy per bit. N0: noise power spectral density.

BER



Eb/N0 (dB)

BER v.s. Eb/N0 is the most important measure of digital communication systems!

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- **Pulse Shaping**
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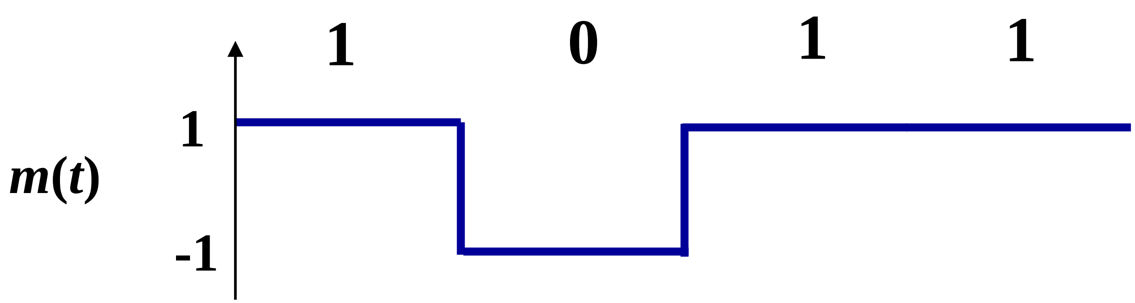
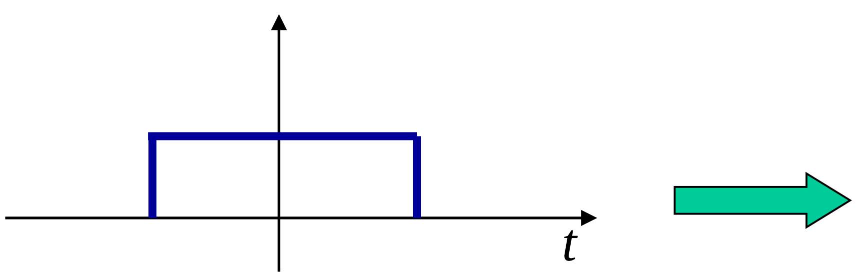
PULSE SHAPING

- **Why pulse shaping?**
  - In previous discussions, the message $m(t)$ is represented as a series rectangular pulses.

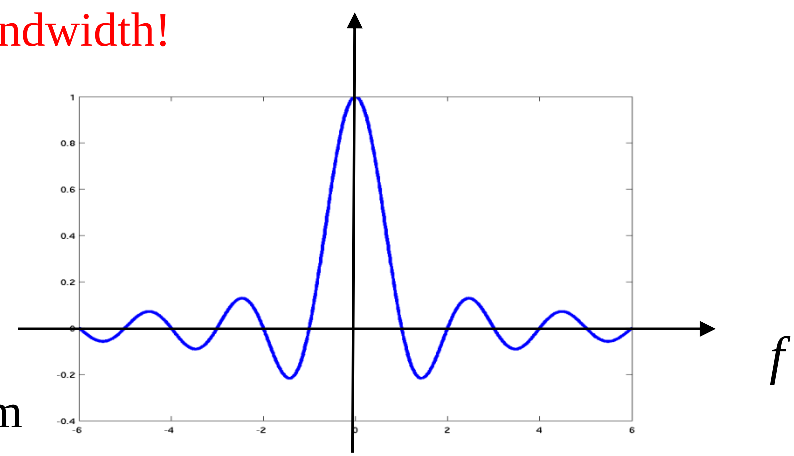
$$m(t) = \sum_{k=1}^{\infty} m_k \, p(t - kT_s) \qquad p(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & o.w. \end{cases}$$



  - Rectangular pulse has unlimited bandwidth!

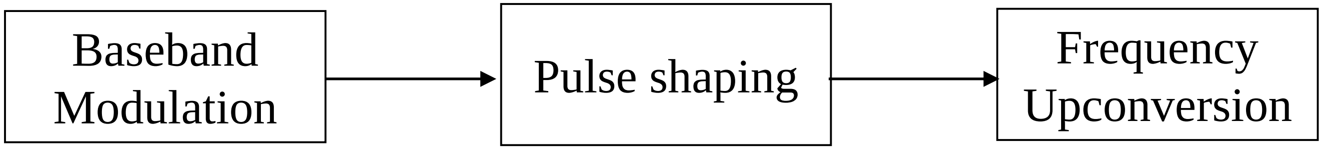


Fourier Transform

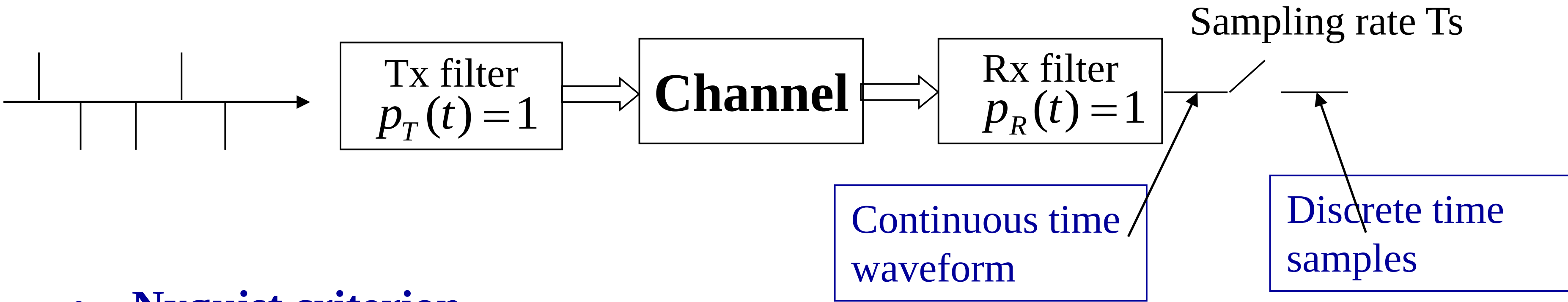


PULSE SHAPING

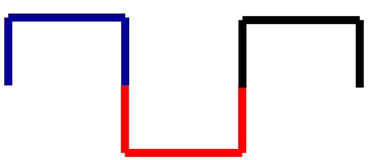
- **Why pulse shaping? (Cont'd)**
  - Baseband signal has unlimited bandwidth.
  - Spectrum of pass-band signal is shifted from baseband signal.
  - Thus, pass-band signal has unlimited bandwidth.
  - In wireless channel, only limited bandwidth resource is available!
  - We want to choose a non-rectangular pulse shape that is limited in bandwidth!

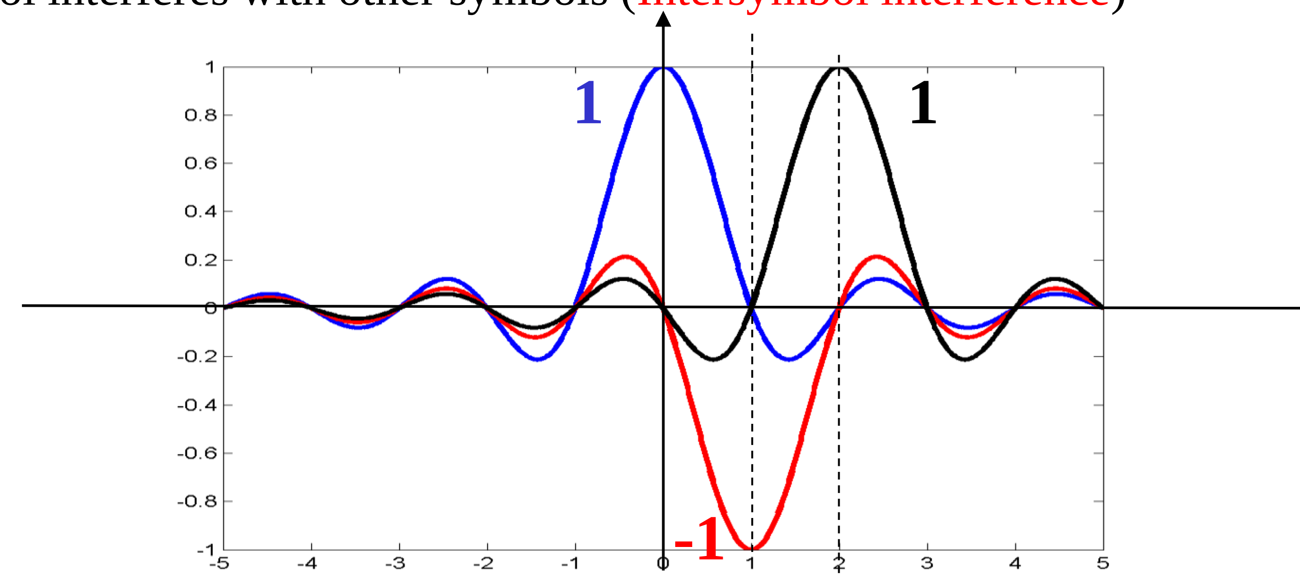


PULSE SHAPING

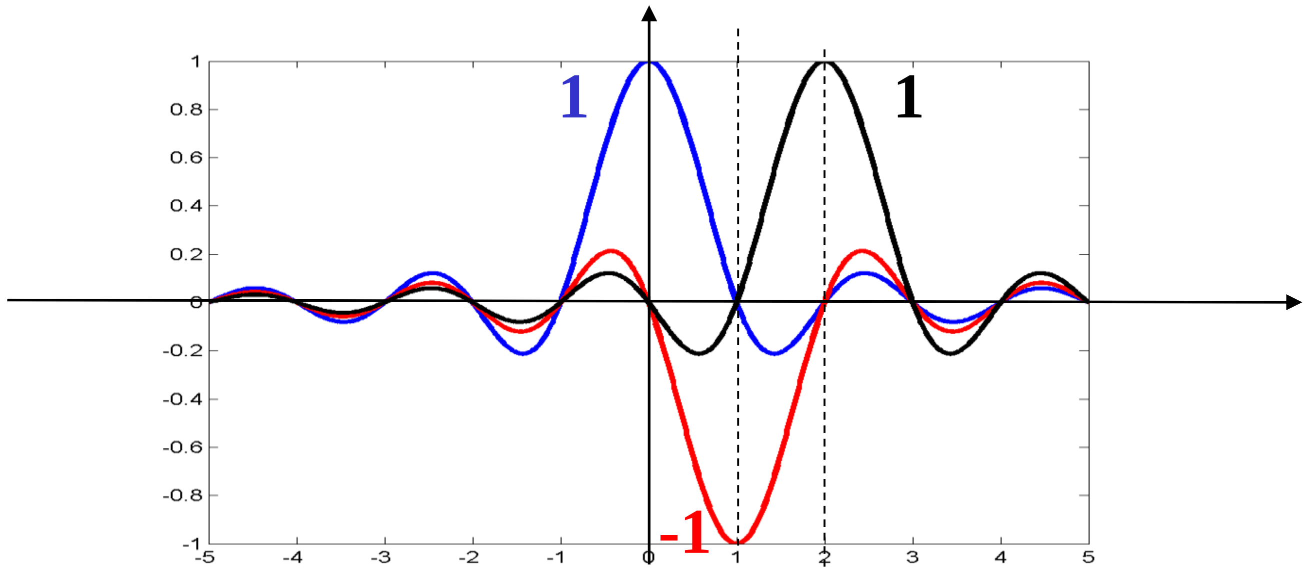


- **Nyquist criterion**
  - Design the overall response of Tx filter and Rx filter, $p(t) = p_T(t) \otimes p_R(t)$ such that $p(0) = 1$, and $p(nT_s) = 1 \quad n \neq 0$
  - Avoid one symbol interferes with other symbols (Intersymbol interference)

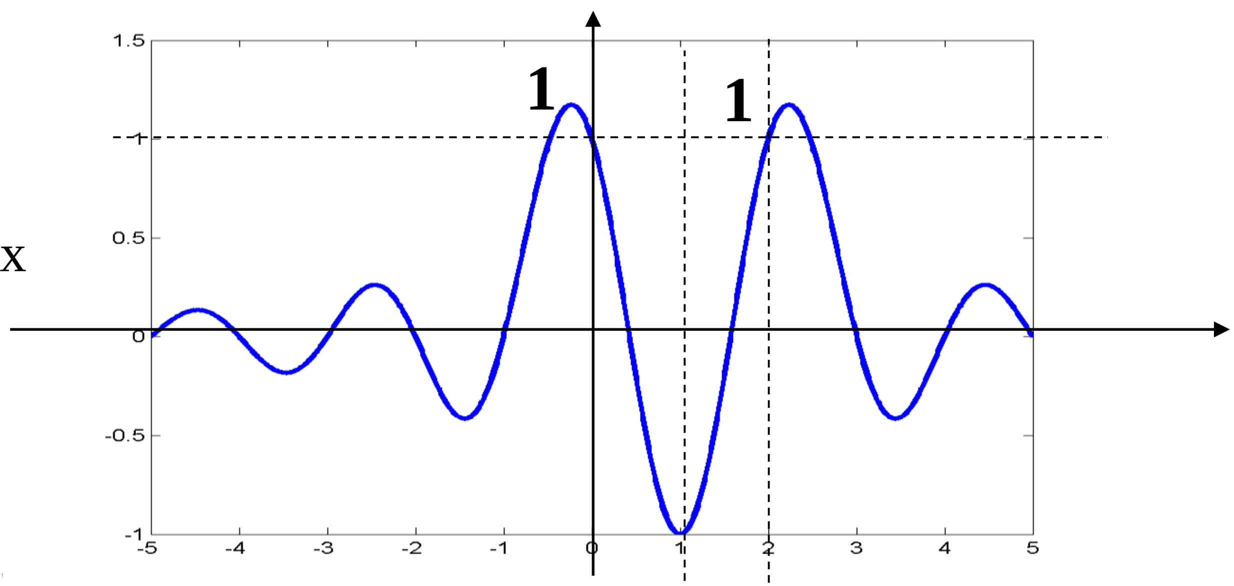




PULSE SHAPING



Waveform at Rx

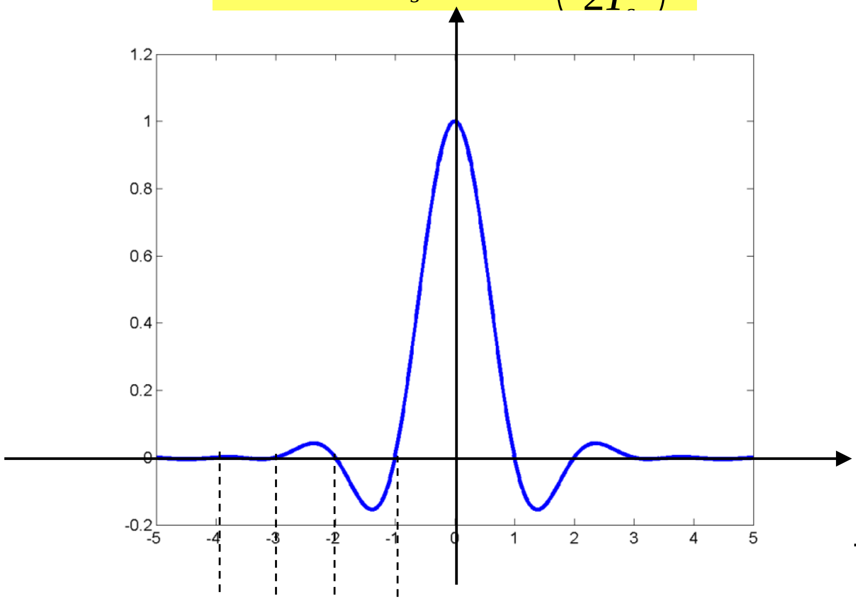


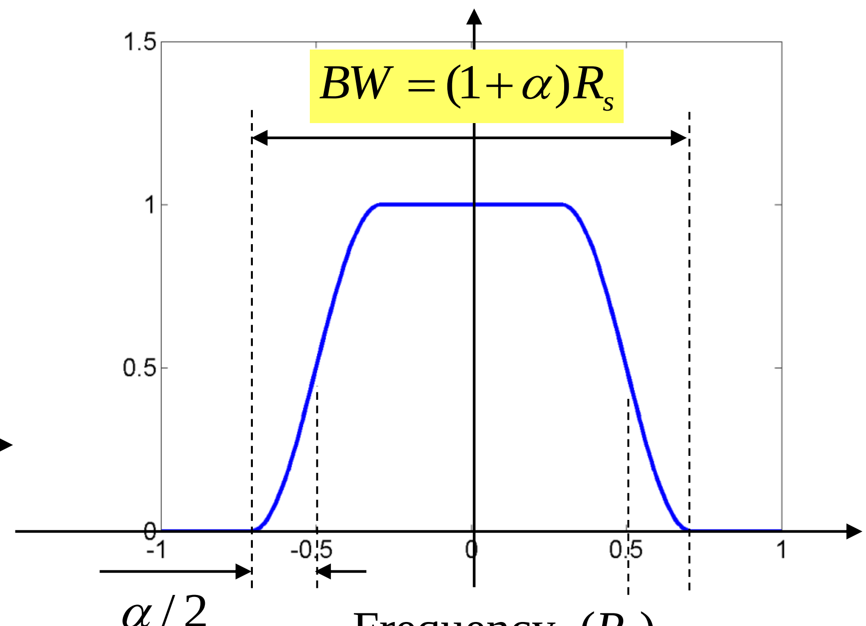
PULSE SHAPING

- **Raised cosine (RC) pulse**

$$p(t)=\frac{\sin\left(\frac{\pi t}{T_s}\right)}{\frac{\pi t}{T_s}}\cdot\frac{\cos\left(\frac{\pi\alpha t}{T_s}\right)}{1-\left(\frac{4\alpha t}{2T_s}\right)^2}$$

$$\Rightarrow \quad P_{RC}(f)$$





Satisfy Nyquist criterion

α: Roll-off factor

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PULSE SHAPING

- **Root raised cosine (RRC) pulse**
  - The frequency domain response of RRC filter is the square root of the frequency response of RC filter

$$P_{RRC}(f) = \sqrt{P_{RC}(f)}$$

  - RRC filter has the same bandwidth as RC filter
- **The overall response is RC filter**
  - Satisfy Nyquist criterion

$$P_{overall}(f) = \sqrt{P_{RC}(f)} \times \sqrt{P_{RC}(f)} = P_{RC}(f)$$

PULSE SHAPING

- **Root Raised Cosine (RRC)**

$$p(t) = 4\alpha \frac{\cos\left[(1+\alpha)\pi t R_s\right] + \sin\left[(1-\alpha)\pi t R_s\right](4\alpha t R_s)^{-1}}{\pi\sqrt{T_s}\left[1 - 16\alpha^2 t^2 R_s^2\right]}$$

- **RRC filter is used at both Tx and Rx**
  - Overall response is Raised Cosine
  - Signal bandwidth is $(1+\alpha)R_s$
    - Symbol rate: $R_s = 1/T_s$
    - Signal bandwidth is proportional to symbol rate

OUTLINE

- Modulation
- Linear Modulation Techniques I
- Complex Signal Representation
- Pulse Shaping
- **Linear Modulation Techniques II**
- Non-linear modulation
- FDMA

LINEAR MODULATION II: 16-QAM

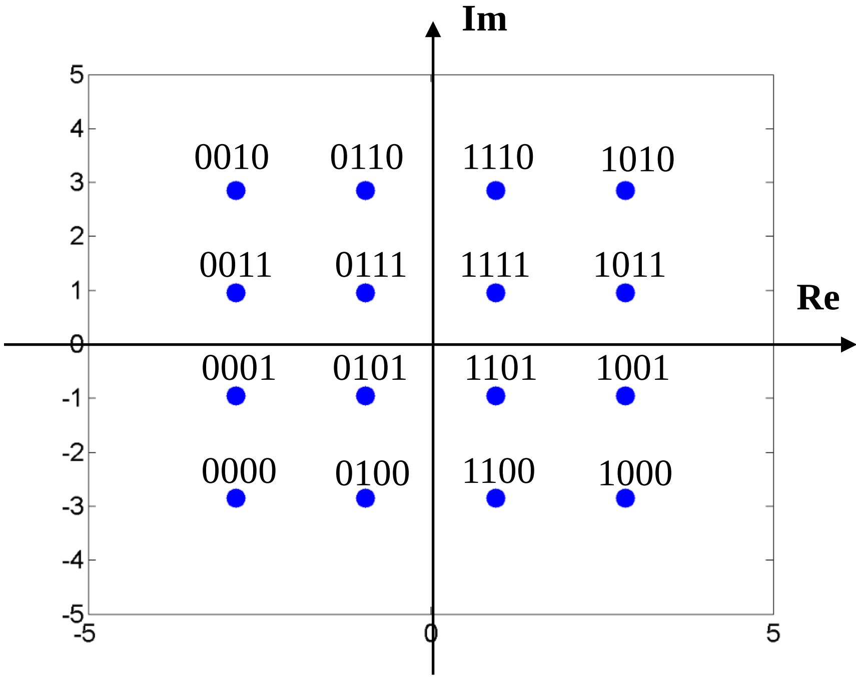
- **16 Quadrature amplitude modulation (16QAM)**
  - Inphase: 4 amplitude shift keying
  - Quadrature: 4 amplitude shift keying

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

$$\tilde{s}(t) = s_I(t) + j \cdot s_Q(t)$$

$$s_I(t) \in \{-3,-1,1,3\} \qquad s_Q(t) \in \{-3,-1,1,3\}$$

- **16-QAM symbols**
  - Total 4 x 4 = 16 possible combinations.
  - M = 16 ➔ $\log_2 M = 4$ bits/sym
- **Gray mapping**
  - Two adjacent symbols differ only in 1 bit
  - The most common error is that the receiver decide in favor of the neighbor of the correct symbol.
    - With Gray mapping, only 1 bit error.



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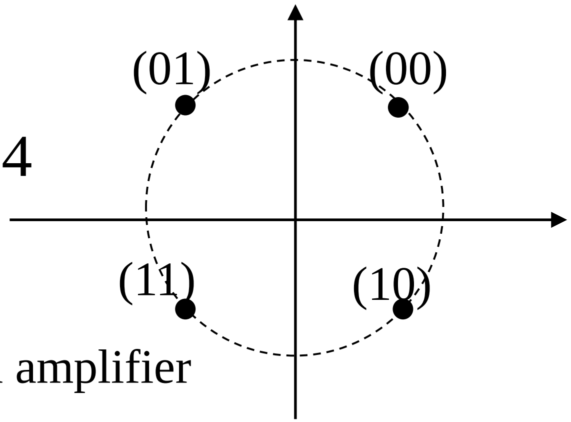
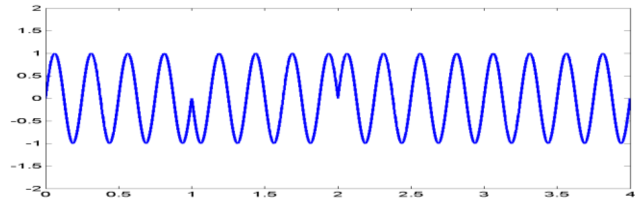
LINEAR MODULATION II: OQPSK

- **Offset quadrature phase shift keying (OQPSK)**
  - Motivation:
    - QPSK ($\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$)
    - 00 10 01 11 ➔ $\pi/4$, $7\pi/4$, $3\pi/4$, $5\pi/4$
    - Rapid phase change (e.g. $7\pi/4$ ➔ $3\pi/4$) could result in amplitude fluctuation ➔ carries information in amplitude ➔ high requirement in amplifier ➔ increase error in wireless communication systems.
    - We want to avoid phase change of π
      - Avoid two bits (inphase, quadrature) change at the same time !





  - OQPSK: Delay the quadrature component by half symbol period such that the inphase bit and quadrature bit don’t change at the same time.

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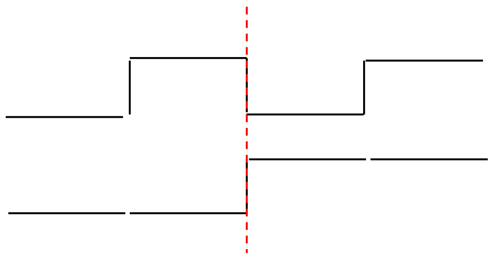
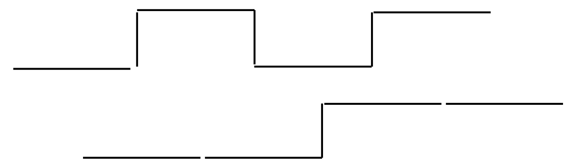
LINEAR MODULATOIN II: OQPSK



QPSK

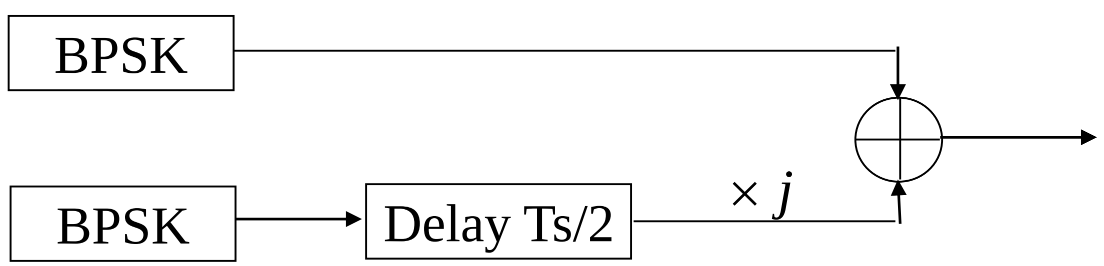


OQPSK

- **OQPSK**
  - Delay the quadrature component by half symbol period
    - It's guaranteed at any moment, only one bit is changing
    - The phase shift of π is completely avoided!

$$\tilde{s}(t) = s_I(t) + j \cdot s_Q(t - T_s/2)$$

  - Disadvantage: phase change every Ts/2 ➔ larger bandwidth



LINEAR MODULATION II: MPSK

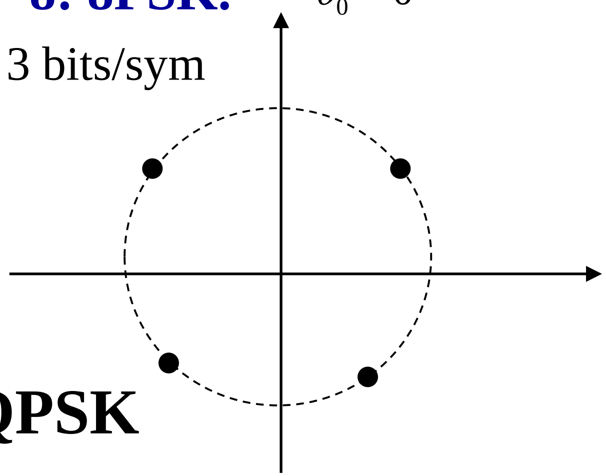
- **Multi-ary Phase Shift Keying (MPSK)**
  - Phase modulation: use phase to carry information.

$$\tilde{s}(t) = \sqrt{E_s}\exp\left(-j2\pi\frac{m(t)}{M}+\theta_0\right) \qquad m(t)\in\{0,1,\cdots,M-1\}$$

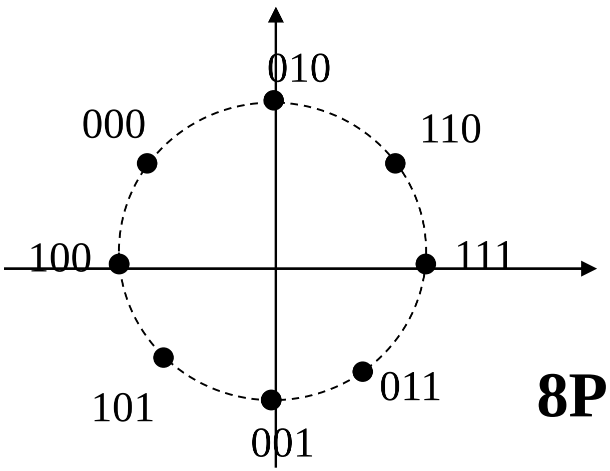
- **M = 4: QPSK.** $\theta_0 = \frac{\pi}{4}$ $\qquad m(t)\in\{0,1,2,3\}$

$$\tilde{s}(t) = \left\{\sqrt{E_s}\exp\left(-j\frac{\pi}{4}\right),\sqrt{E_s}\exp\left(-j\frac{3\pi}{4}\right),\sqrt{E_s}\exp\left(-j\frac{5\pi}{4}\right),\sqrt{E_s}\exp\left(-j\frac{7\pi}{4}\right),\right\}$$

- **M = 8: 8PSK.** $\theta_0 = 0$
  - 3 bits/sym



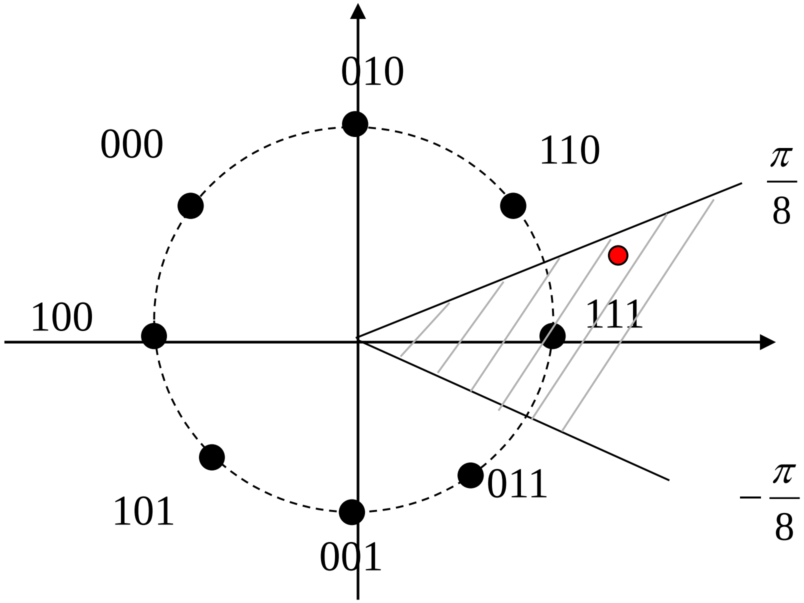
QPSK



8PSK

LINEAR MODULATION II: MPSK

- **MPSK demodulation**
  - Find the modulation symbol that has the smallest Euclidean distance with the received signal.



LINEAR MODULATION II: MPSK

- **Tradeoff between spectral efficiency and energy efficiency**
  - Spectral efficiency: data rate that is accommodated in unit bandwidth (bps/Hz) = data rate/bandwidth
    - Given fixed bandwidth ➔ fixed symbol rate
    - Large M ➔ large # bits/sym ➔ large data rate ➔ large spectral efficiency
  - Energy efficiency: at given noise level, how much signal power is required to achieve a certain bit error rate.
    - Measured as the value of Eb/N0 required for certain bit error rate (BER).
    - Given fixed bandwidth ➔ fixed symbol rate
    - Large M ➔ constellation symbols are more "dense" ➔ more sensitive to noise ➔ low energy efficiency

Increase modulation level M ➔ Increase spectral efficiency

Increase modulation level M ➔ Decrease energy efficiency

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NON-LINEAR MODULATION: FREQUENCY MODULATION

- **Frequency modulation (FM)**
  - The term, FM, is usually used when *m*(*t*) is analog
    - E.g. FM radio
  - Use frequency to carry information

$$f(t) = f_c + k_f m(t)$$

  - Relationship between phase and frequency
    - Fixed frequency

$$\psi(t) = 2\pi f_c t + \theta$$

    - Variable frequency

$$f(t) = \frac{1}{2\pi}\frac{d\psi(t)}{dt} \qquad \psi(t) = 2\pi\int_0^t f(t)dt$$

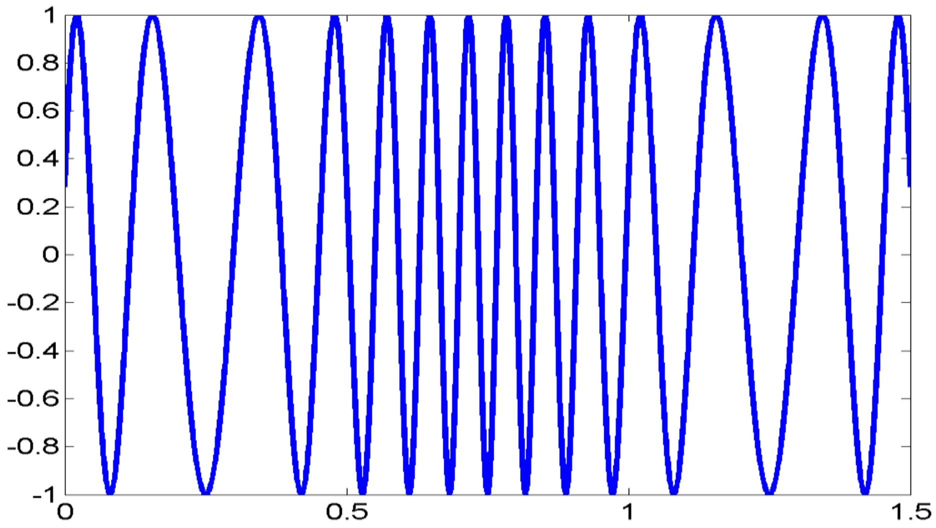
  - FM

$$\psi(t) = 2\pi f_c t + k_f \int_0^t m(t)dt$$

NON-LINEAR MODULATION: FREQUENCY MODULATION

- **FM**

$$s(t) = A_c \cos\left[2\pi f_c t + k_f \int_0^t m(t)dt\right]$$



NON-LINEAR MODULATION: FREQUENCY MODULATION

- **FM bandwidth**
  - The spectrum of FM is much more complicated than the spectrum of AM
  - Pass-band Bandwidth of FM signal can be approximated by Carson's rule.

$$B_T \approx 2(\beta + 1)B_s$$

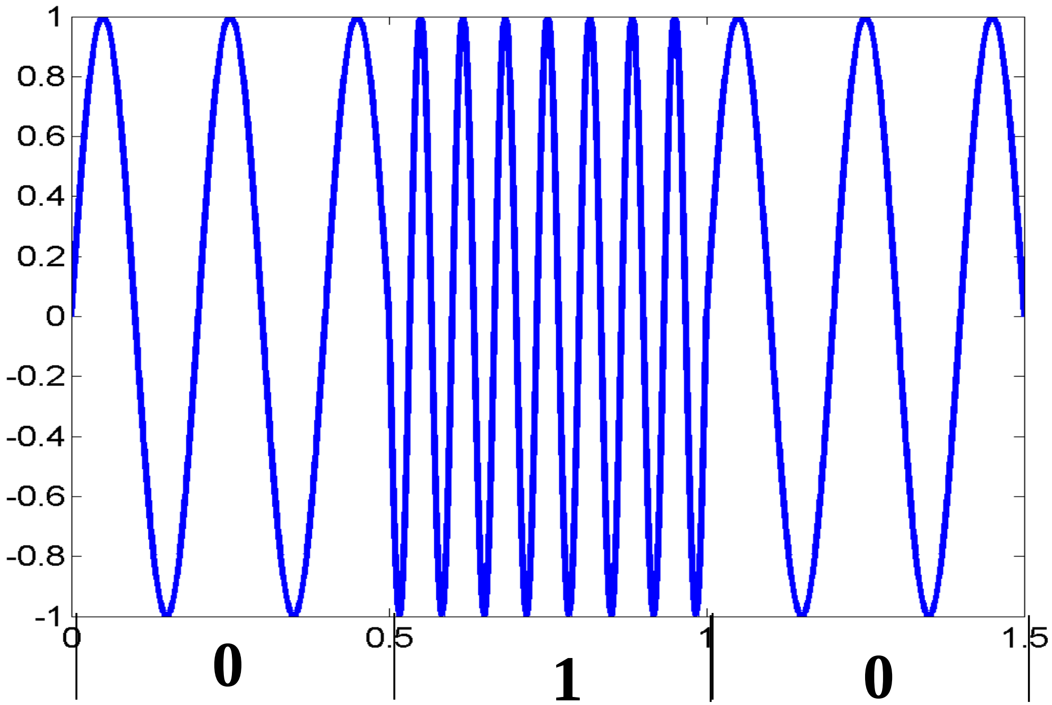
  - β :modulation index.
  - B_s :Bandwidth of baseband signal

- **E.g.**
  - AMPS, $\beta = 3, B_s = 4kHz$
  - Bandwidth: 32 KHz

NON-LINEAR MODULATION: BFSK

- **Binary frequency shift keying (BFSK)**
  - Constant envelope

$$s(t) = \begin{cases} A_c \sin\left[2\pi(f_c - \Delta f)t\right], & 0 \\ A_c \sin\left[2\pi(f_c + \Delta f)t\right], & 1 \end{cases}$$



If continuous phase (CP) ➔ CP FSK ➔ No abrupt phase change!

NON-LINEAR MODULATION: MSK

- **Minimum shift keying (MSK)**
  - A special case of CP BFSK

$$s(t) = \begin{cases} A_c \sin\left[2\pi\left(f_c - \frac{R_s}{4}\right)t\right], & 0 \\ A_c \sin\left[2\pi\left(f_c + \frac{R_s}{4}\right)t\right], & 1 \end{cases}$$

  - CP BFSK with $\Delta f = \frac{R_s}{4}$
- **Advantages:**
  - bandwidth much smaller than BPSK/QPSK
  - Constant envelope
  - Very popular modulation scheme for mobile radio
- **Usually used with Gaussian pulse shaping filter**
  - GMSK (used by GSM)

NON-LINEAR MODULATION: PRACTICAL ISSUE

- **Power amplifier non-linearity**
  - Power amplifier: amplify signal power before transmission
    - To meet the SNR requirement at receiver.
  - Amplifier categories: Class A, Class B, Class AB, Class C, Class D
    - Class A: linear amplifier
      - Signals amplitude at the operational range is amplified linearly.
      - The information in amplitude is preserved.
      - Should be used for amplitude modulation.
      - Requires more transmission power ➔ short battery life.
    - Class C: non-linear amplifier
      - Different amplitude range are amplified by different factors
      - The amplitude of the signals is distorted.
      - CANNOT be used for amplitude modulated systems. CAN be used for frequency modulated systems.
      - Requires less transmission power ➔ longer battery life
    - From A to B to C to D, linearity becomes worse.

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FDMA

- **Frequency division multiple access (FDMA)**
  - Available spectrum is divided into a set of frequency bands, and each frequency band is assigned to a user
- **FDMA is not FDD**
  - FDD: frequency division duplex
  - The downlink and uplink of one user is using different frequencies.
- **Example: AMPS**
  - FDMA
  - Analog voice: FM
  - Digital control: BFSK (one frequency for ‘0’, one frequency for ‘1’).

FDMA: ACI

- **Adjacent Channel Interference (ACI)**
  - Interference from adjacent channels.
  - For some modulations (FM, FSK), the modulated signal has unlimited bandwidth.
    - The signal in one channel will leak into adjacent channels.
    - To reduce ACI, introduce some guard bands between adjacent channels.
  - To improve overall bandwidth efficiency, we want to have as many channel as possible in a limited spectrum.
    - Tradeoff between bandwidth efficiency and power efficiency.