Department of Electrical Engineering
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ELEG5693 Wireless Communications
Propagation and Noise Part II

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OUTLINE

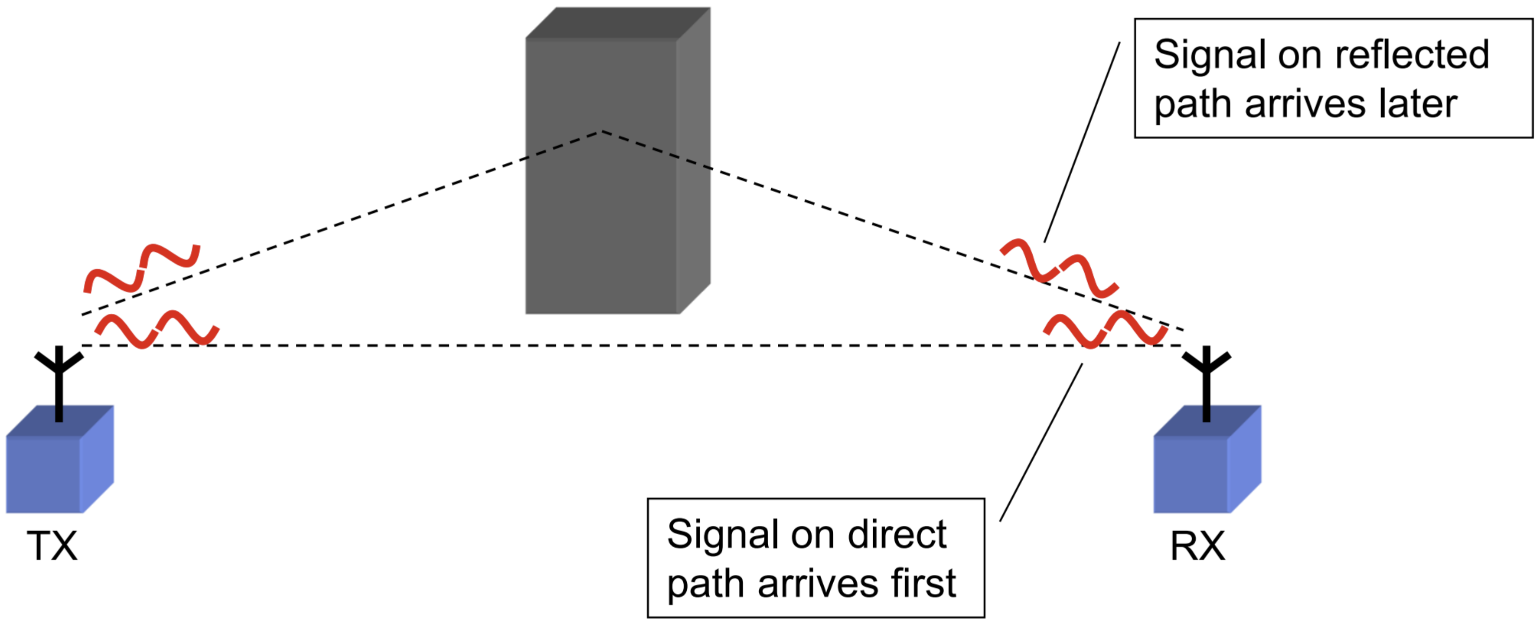
- Wireless channel
- Path loss
- Shadowing
- **Small scale fading**
- Simulation model
- Channel classifications
- Noise and interference

FADING: WHAT IS FADING?

- **Path loss and shadowing is caused by large objects that are distant from MS.**
  - Even the MS is moving, the change in the relative position between MS and those distant large objects is small.
  - Therefore, the impairments caused by those large distant objects change very slow with respect to (w.r.t.) time and position.
  - Shadowing is also referred to as large scale fading.
- **Small scale fading is caused by the effects of objects that are close to MS.**
  - The movement of MS w.r.t. nearby small objects will dramatically change the reflection or diffractions of propagated signals.
  - The signal at receiver (sum of the signals from all multiple paths) will change rapidly with the movement of MS.

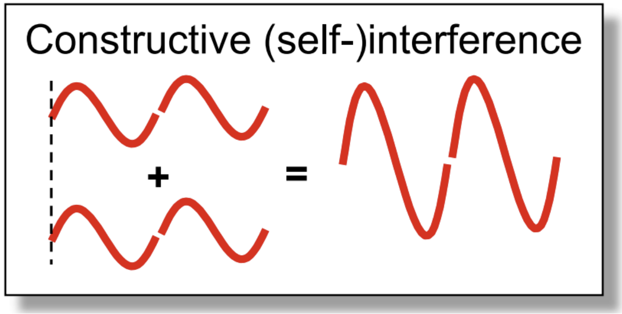
Small scale fading: rapid fluctuation of the received signals over short distance.

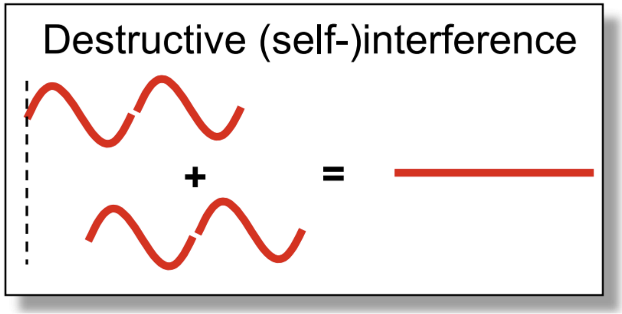
FADING: WHAT IS FADING



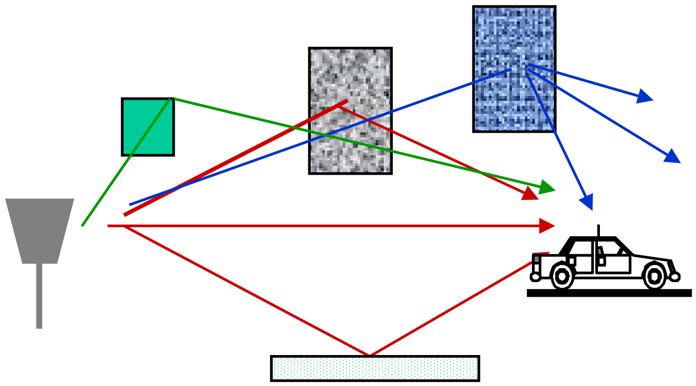
Signal on reflected path arrives later

Signal on direct path arrives first


Constructive (self-)interference

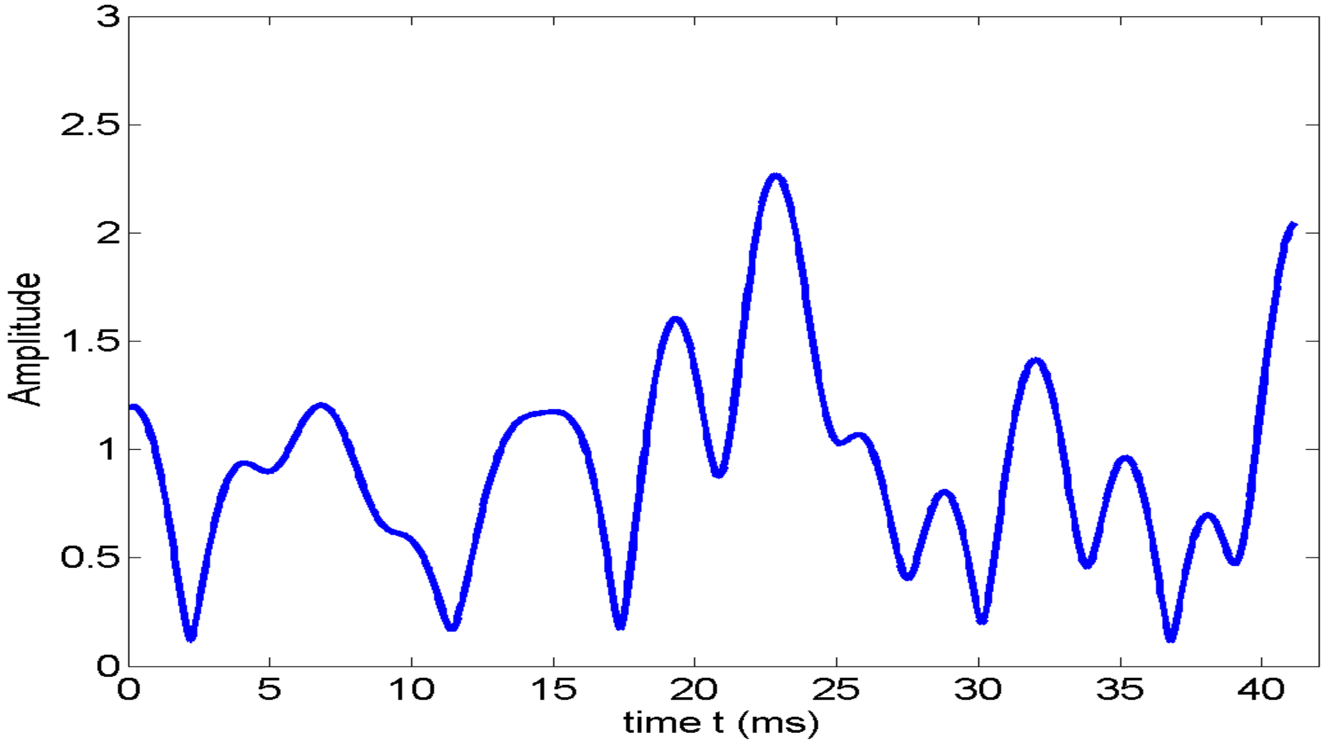

Destructive (self-)interference

FADING: WHAT IS FADING?



- **Random # of multipath components**
  - The amplitude, phase, and frequency of each component change w.r.t. the movement of MS.
  - The signal at the receiver is the summation of all the multipath components ➔ the amplitude, phase, and frequency of the received signal at receiver change w.r.t. the movement of MS.
  - The movement of surrounding objects (e.g. vehicles) will also cause the time variation of the signals.

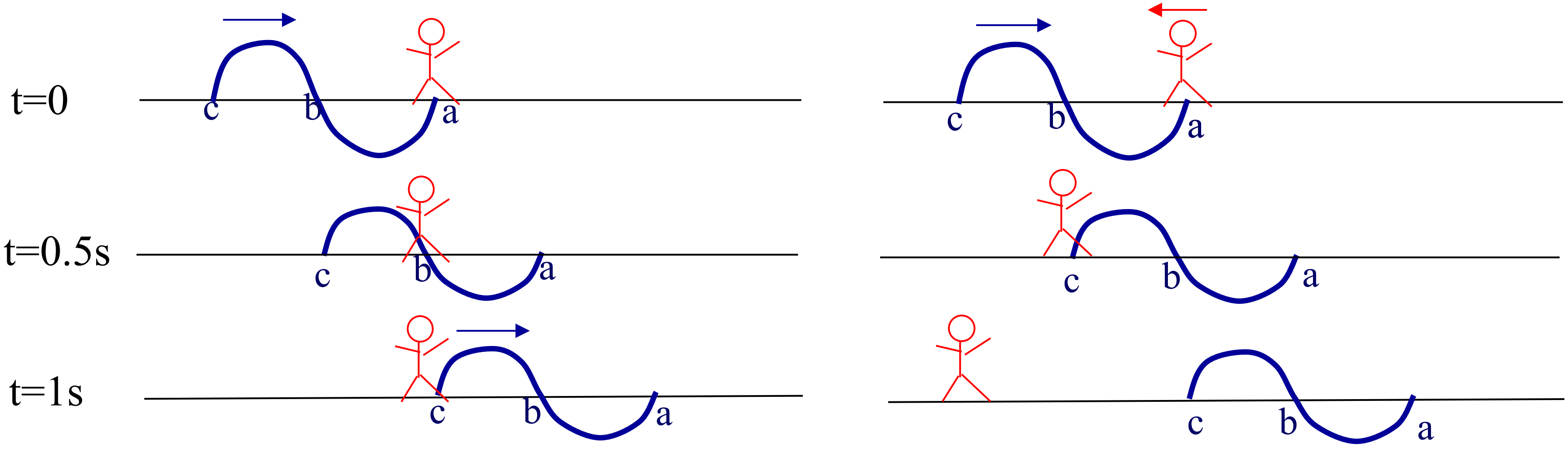
FADING: AN EXAMPLE



- **The rate of variation depends on two factors:**
  - Relative movement speed between Tx and Rx
  - Speed of surrounding objects

FADING: DOPPLER

- **What is Doppler?**
  - The whistle of the train coming from opposite direction sounds different with the train passing by.
    - The pitch of the sound (determined by sound frequency) is changing.
  - Rx signal frequency will change if the Rx is moving w.r.t. Tx.
  - Signal frequency change due to the relative movement between Tx and Rx is called Doppler effects.



T = 1s

T = 0.5s

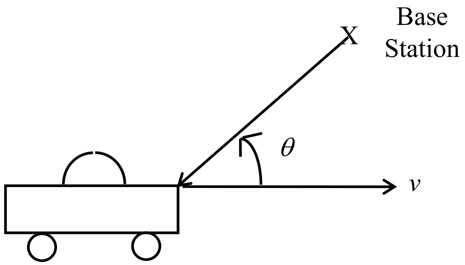
FADING: DOPPLER

- **Consider Tx sends out a sinusoid with frequency 1Hz**
  - If Rx moves toward Tx, the signal observed by Rx will have a shorter period → frequency increased
  - If Rx moves away from Tx, the signal observed by Rx will have a longer period → frequency decreased

- **The amount of frequency change is called Doppler shift**
  - Doppler shift depends on
    - Relative speed between Tx and Rx
    - The frequency of the original signal

FADING: DOPPLER

- **Relationship between speed and Doppler shift**



$$\Delta f = \frac{v}{\lambda}\cos\theta$$

v : relative speed λ : wavelength

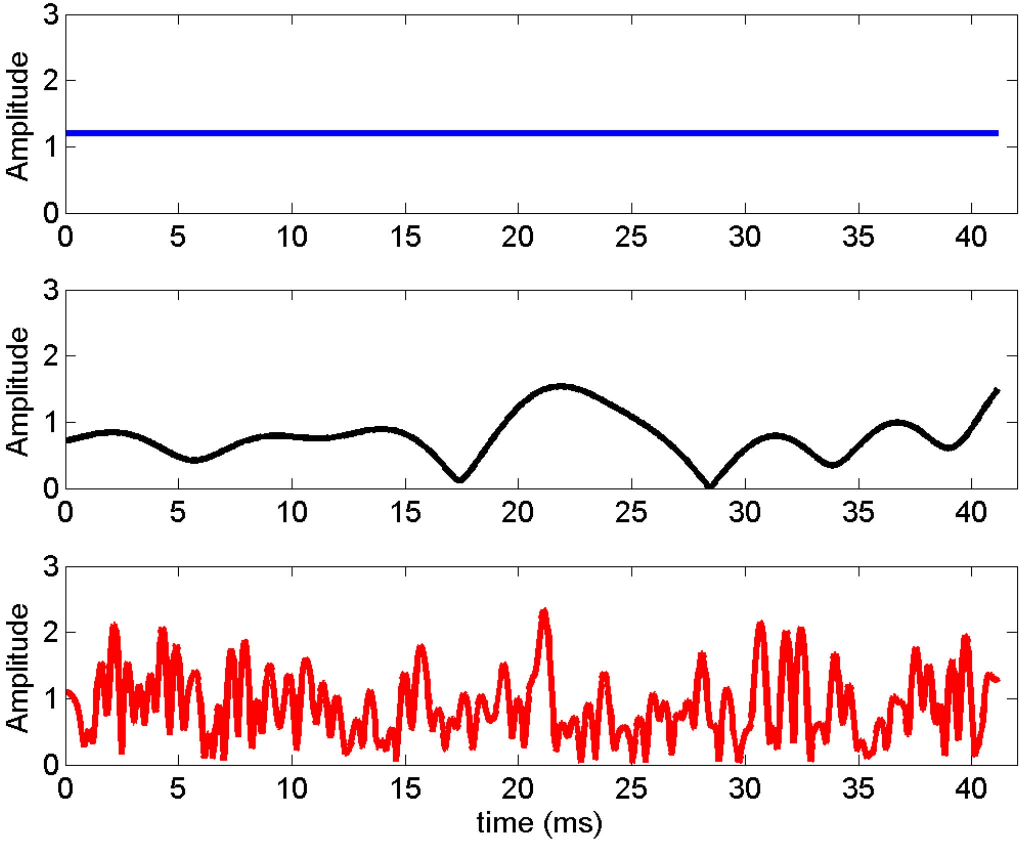
- **Maximum Doppler shift:**

$$f_D = \frac{v}{\lambda}$$

- **Example:** find the maximum Doppler shift of 900MHz system with mobile speed 120km/Hr

FADING: DOPPLER

- **At given frequency**
  - v ↑➔ f_D ↑➔ channel changes more rapidly



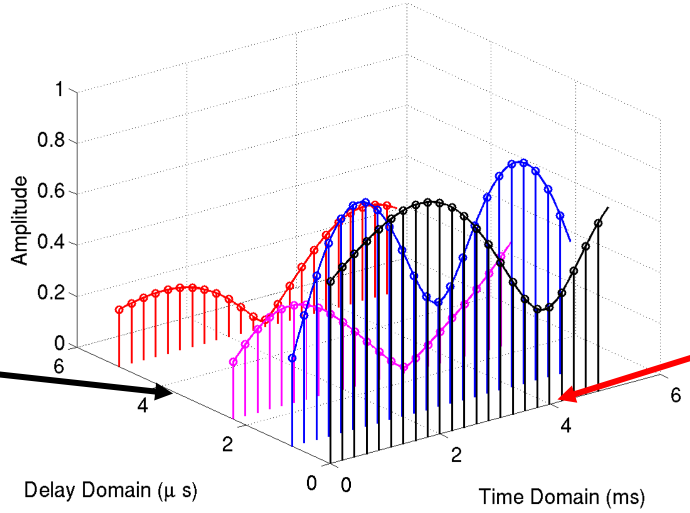
$f_D = 0\text{Hz}$

$f_D = 100\text{ Hz}$

$f_D = 1000\text{ Hz}$

FADING: IMPULSE RESPONSE

- **The impulse response of fading is time-varying!**



Relative multipath delay τ

Time variation t

N multipath components

$$c(t,\tau)=\sum_{n=1}^{N}\alpha_n(t)\cos[2\pi f_c t+\varphi_n(t)]\times\delta(\tau-\tau_n)$$

- f_c : system operating frequency (e.g. 900MHz, 1.8GHz)
- t: the time variation (both amplitude and phase changes with respect to time)
- τ : relative delay between multipath components
- $\varphi_n(t)$: depends on path distance and Doppler shift $(2\pi f_D t)$

FADING: IMPULSE RESPONSE

- **Complex baseband representation**

$$c(t,\tau) = \sum_{n=1}^{N} \alpha_n(t)\,\mathrm{Re}\left[e^{j2\pi f_c t + \varphi_n(t)}\right] \times \delta(\tau - \tau_n)$$

$$= \mathrm{Re}\left\{ e^{j2\pi f_c t} \left[\sum_{n=1}^{N} \alpha_n(t) e^{j\varphi_n(t)} \times \delta(\tau - \tau_n) \right] \right\}$$

$$h(t,\tau) = \sum_{n=1}^{N} \alpha_n(t) e^{j\varphi_n(t)} \times \delta(\tau - \tau_n)$$

- **Maximum delay spread**
  - The time interval between the first multipath and the last multipath

$$\tau_{\max} = \tau_N - \tau_1$$

FADING: FLAT FADING

- **Flat fading**
  - Maximum delay spread $\tau_{\max}$ << system symbol period Ts
  - Relative to the symbol period, all the multipath components arrive at almost the same time ➔ Doesn't need to consider the delay variable τ

$$h(t) = \sum_{n=1}^{N} \alpha_n(t) e^{j\varphi_n(t)} = \sum_{n=1}^{N} \alpha_n(t)\cos[\varphi_n(t)] + j \cdot \sum_{n=1}^{N} \alpha_n(t)\sin[\varphi_n(t)]$$

  - Inphase component

$$h_I(t) = \sum_{n=1}^{N} \alpha_n(t)\cos[\varphi_n(t)]$$

  - Quadrature component

$$h_Q(t) = \sum_{n=1}^{N} \alpha_n(t)\sin[\varphi_n(t)]$$

FADING: FLAT FADING

$$h(t) = h_I(t) + j \cdot h_Q(t)$$

- **Both $h_I(t)$ and $h_Q(t)$ are the sum of many multipath components**
  - Each multipath component is a random process
  - $h_I(t)$ and $h_Q(t)$ are random processes

- **Central limit theorem**
  - The sum of N independent and identically distributed (i.i.d.) random variables tends to Gaussian distribution when N is large enough.

- **Based on central limit theorem, at time time t, both $h_I(t)$ and $h_Q(t)$ are Gaussian distributed!**

FLAT FADING: RAYLEIGH FADING

- **If there is no LOS between Tx and Rx**
  - $h_I(t)$ and $h_Q(t)$ are zero-mean Gaussian distributed $\sim N(0,\sigma^2)$
  - The amplitude (or envelope) of $h(t)$

$$|h(t)| = \sqrt{h_I^2(t) + h_Q^2(t)}$$

  - The fading envelope $|h(t)|$ follows Rayleigh distribution

$$f_{|h(t)|}(z) = \frac{z}{\sigma^2}\exp\left(-\frac{z^2}{2\sigma^2}\right)$$

  - Average power of fading

$$E\left[|h(t)|^2\right] = E\left[h_I^2(t)\right] + E\left[h_Q^2(t)\right] = 2\sigma^2$$

FLAT FADING: RICIAN FADING

- **If there is LOS component**
  - $h_I(t)$ and $h_Q(t)$ are non-zero-mean Gaussian distributed $\sim N(s,\sigma^2)$
  - The fading envelope

$$|h(t)| = \sqrt{h_I^2(t) + h_Q^2(t)}$$

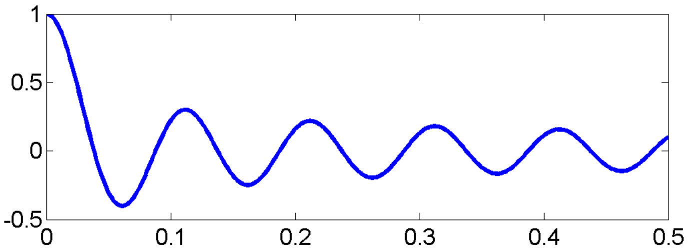
follows Rician distribution

FLAT FADING: TIME DOMAIN CORRELATION

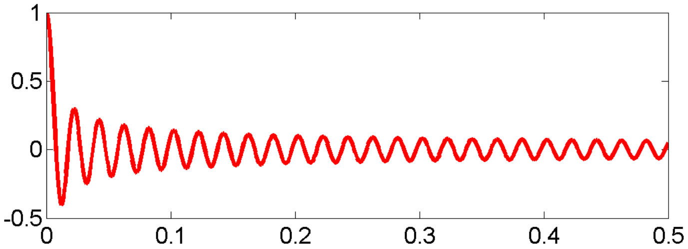
- **The time domain correlation of *h*(t) is**

$$R_h(\alpha) = E\left[h(t+\alpha)h^*(t)\right] = P_h \cdot J_0(2\pi f_D \alpha)$$

  - $J_0(x)$: zero-order Bessel function of the first kind



Fd = 10Hz

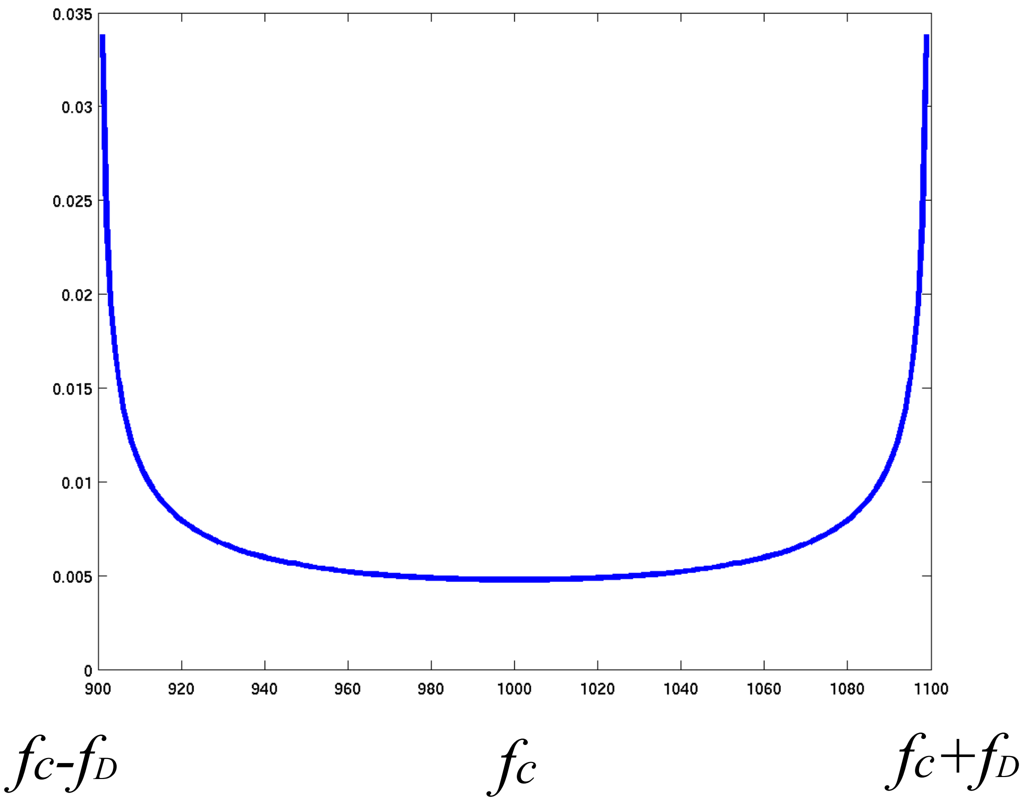


Fd = 50Hz

  - Generally speaking, for given time interval α
    - Larger speed v ➔ larger f_D ➔ smaller $|R_h(\alpha)|$.

FLAT FADING: POWER SPECTRAL DENSITY

- **Power spectral density is the Fourier transform of auto-correlation function.**



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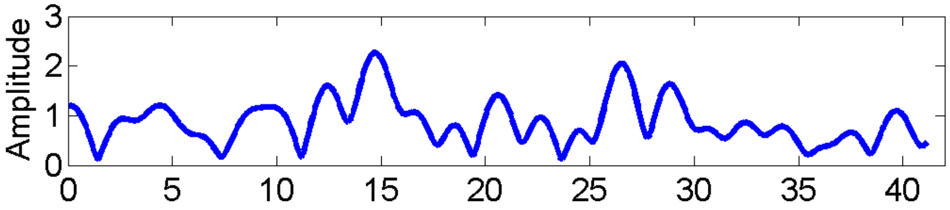
- **Simulation model**
- Channel classifications
- Noise and interference

SIMULATOR



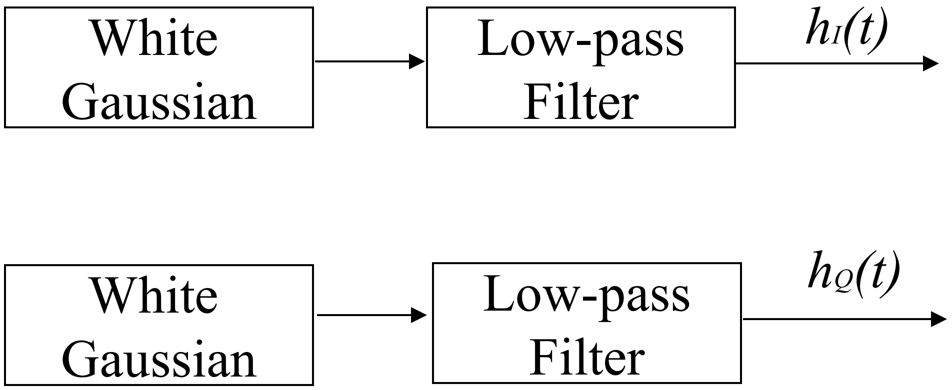
$$h(t) = h_I(t) + j \cdot h_Q(t)$$

- **Flat Rayleigh fading is a random process**
  - At any time instant *t*, $|h(t)| = \sqrt{h_I^2(t) + h_Q^2(t)}$ is Rayleigh distributed.
    - Both the real part $h_I(t)$ and the imaginary part $h_Q(t)$ are zero mean Gaussian distributed.
  - The auto-correlation function must satisfy

$$R(\tau) = E\left[h(t+\tau)h^*(t)\right] = J_0(2\pi f_D \tau)$$

SIMULATOR

- **How to generate flat Rayleigh fading with computer program?**
  - Method 1: Filtered Gaussian noise
    - Rely on low-pass filter to introduce the time-domain correlation among symbols


White Gaussian

Low-pass Filter

$h_I(t)$

White Gaussian

Low-pass Filter

$h_Q(t)$

  - The low-pass filter is hard to design.

SIMULATOR

- **Method 2: Sum-of-sinusoid**

$$h_I(nT_s)=\frac{1}{\sqrt{M}}\sum_{m=1}^{M}\cos\left\{2\pi f_D\cos\left[\frac{(2m-1)\pi+\theta}{4M}\right]\cdot nT_s+\alpha_m\right\}$$

$$h_Q(nT_s)=\frac{1}{\sqrt{M}}\sum_{m=1}^{M}\sin\left\{2\pi f_D\cos\left[\frac{(2m-1)\pi+\theta}{4M}\right]\cdot nT_s+\beta_m\right\}$$

$$h(nT_s)=h_I(nT_s)+j\cdot h_Q(nT_s)$$

θ,α_m,β_m : uniformly distributed in $[0,2\pi]$

M : a constant. The larger, the more accurate. Usually 8 or 16.

T_s : time duration between samples.

h = Rayleigh(N, fd, Ts)

$$\mathbf{h}=\left[h(0T_s),h(1Ts),h(2Ts),\cdots,h((N-1)Ts)\right]$$

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CLASSIFICATION

- **Fading**
  - Amplitude and phase distortions of transmitted signal

- **Classification criterions**
  - Scale
    - Large scale fading, small scale fading
  - Small scale fading
    - Flat fading v.s. frequency selective fading
    - Fast fading v.s. slow fading
    - Rayleigh fading v.s. Rician fading

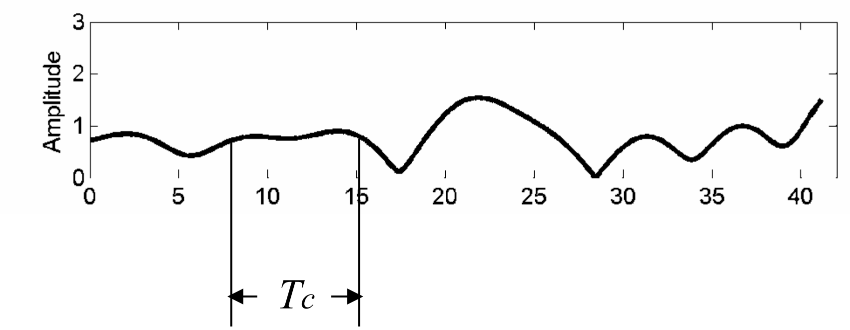
CLASSIFICATION: SCALE

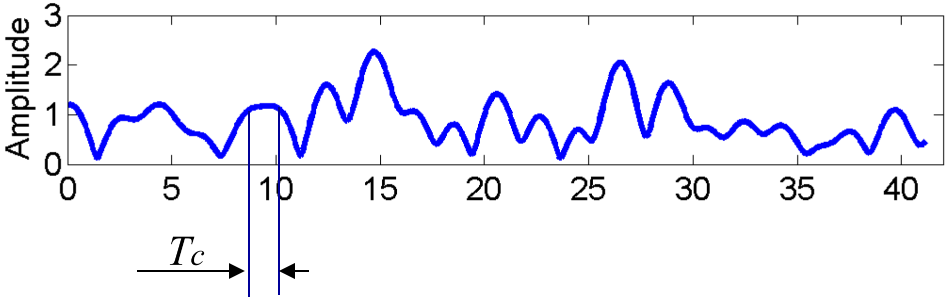
- **Large scale fading**
  - Path loss (signal power loss as a function of distance)
    - Due to distance between Tx and Rx, reflection of large objects
  - Shadowing
    - Obstruction from large objects
- **Small scale fading**
  - Amplitude and phase distortions from local objects ➔ highly sensitive to locations of MS
  - Due to the superposition of multiple electromagnetic waveforms
  - Caused by two independent propagation mechanisms
    - (1) time dispersion (delay spread)
      - Determines frequency selective or flat
    - (2) frequency dispersion (Doppler spread)
      - Determines fast or slow

CLASSIFICATION: FAST FADING V.S. SLOW FADING

- **The time domain variation of fading is determined by maximum Doppler spread f_D**
  - Doppler shift: signal frequency change due to relative movement between Tx and Rx.
  - Larger speed v ➔ larger f_D ➔ channel varies faster.
- **Coherence time T_c**
  - The time period over which the channel is strongly correlated (didn't change too much)
  - Inverse proportional to f_D

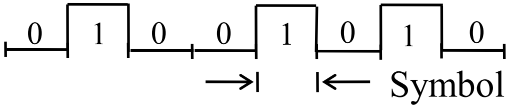
$$T_c \approx \frac{1}{2 f_D}$$





CLASSIFICATION: FAST FADING V.S. SLOW FADING

- **System symbol period v.s. signal bandwidth**



Symbol Period = T_s

Signal BW = $B_s \approx 1 / T_s$

- **Fast fading**
  - If $Ts > Tc$, or $Bs < f_D$
    - $Ts > Tc$: channel changes within one symbol period→ fast fluctuation

- **Slow fading**
  - If $Ts << Tc$, or $Bs >> f_D$
    - $Ts << Tc$: channel keeps constant during several symbol periods → slow amplitude fluctutation.

(Coherence time Tc, Doppler spread f_D) is related to fast fading or slow fading

CLASSIFICATION: FAST FADING V.S. SLOW FADING

- **Example: A cell phone user is in a vehicle moves at a speed of 120km/hr. The carrier frequency is 1800MHz.**
  - (a) What is the maximum Doppler spread?
  - (b) What is the coherence time of the channel?
  - (c) The symbol period of a system is 3ms. Is the system experiencing fast fading or slow fading?
  - (d) The symbol rate of IS-136 system is 24.3ksym/s. Is the system experiencing fast fading or slow fading?

CLASSIFICATION: FLAT V.S. FREQUENCY SELECTIVE

- **Maximum delay spread**
  - The time interval between the first multipath and the last multipath

$$\tau_{\max} = \tau_N - \tau_1$$

- **Mean delay spread**

$$\bar{\tau} = \sum_{n=1}^{N} \frac{P_n}{P_{total}} \cdot \tau_n$$

P_n : the average power of the nth multipath

$P_{total} = \sum_{n=1}^{N} P_n$: the total power of the all multipath

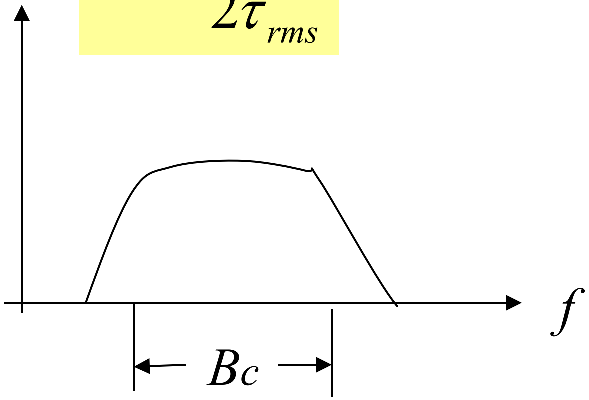
- **Root mean square (rms) delay spread**

$$\tau_{rms} = \sqrt{\sum_{n=1}^{N} \frac{P_n}{P_{total}} \cdot (\tau_n - \bar{\tau})^2} = \sqrt{\sum_{n=1}^{N} \frac{P_n}{P_{total}} \cdot \tau_n^2 - \bar{\tau}^2}$$

CLASSIFICATION: FLAT V.S. FREQUENCY SELECTIVE

- **Coherence bandwidth *Bc***
  - The bandwidth over which the channel is strongly correlated (didn’t change too much)
    - The spectrum over coherence bandwidth is almost “flat”
  - Inverse proportional to rms delay spread

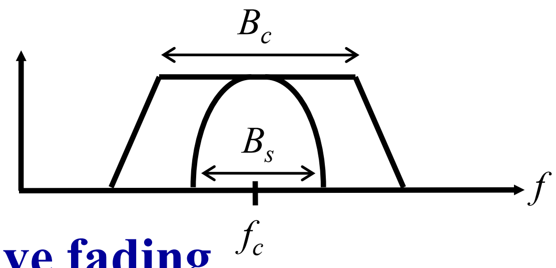
$$B_c \approx \frac{1}{2\tau_{rms}}$$



(Coherence bandwidth *Bc,* rms delay spread τ_{rms}) is related to fast fading or slow fading

CLASSIFICATION: FLAT V.S. FREQUENCY SELECTIVE

- **Flat fading**
  - If $Bs << Bc$, or $Ts >> \tau_{rms}$
  - $Bs << Bc$: signal bandwidth << channel bandwidth
  - $Ts >> \tau_{rms}$: relative arrival time between multipath components is negligible
    - Doesn't need to consider delay variable $h(t,\tau) \Rightarrow h(t)$



- **Frequency selective fading**
  - If $Bs >> Bc$, or $Ts << \tau_{rms}$
  - $Bs >> Bc$: signal bandwidth >> channel bandwidth
    - Signal spectrum will be seriously distorted by channel!
  - $Ts << \tau_{rms}$: symbol period smaller than rms delay spread
    - The relative arrival time between the multipath components is no longer negligible!

CLASSIFICATION: FREQUENCY SELECTIVE FADING

- **Frequency selective fading**
  - rms delay spread τ_{rms} $>>$ system symbol period Ts
    - The relative arrival time between the multipath components is no longer negligible!
  - The N multipath components are divided into L clusters
    - Within each cluster, there are still many multipath components
    - Multipath components belonging to the lth cluster arrives at approximately the same time τ_l .

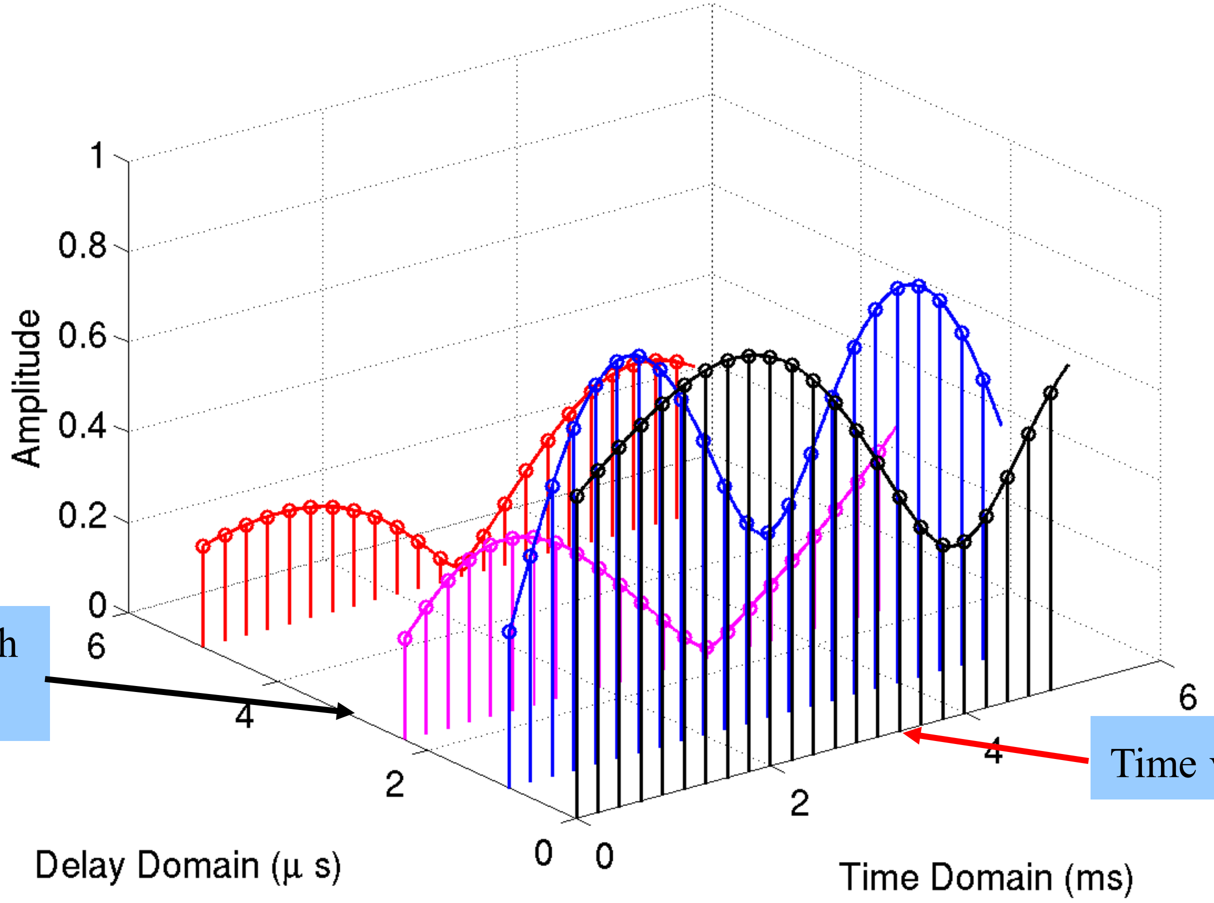
$$h(t,\tau) = \sum_{l=1}^{L} h_l(t) \times \delta(\tau - \tau_l) = \sum_{l=1}^{L} \left[h_{Il}(t) + j h_{Ql}(t) \right] \times \delta(\tau - \tau_l)$$

  - h_l(t) is the sum of all the multipath components within the same cluster
    - Resolvable multipath component
  - The inphase and quadrature components of h_l(t) are Gaussian distributed.
  - The frequency selective fading can be viewed as the combination of multiple flat fading
    - Each branch (cluster) h_l(t) can be viewed as flat fading

CLASSIFICATION: FREQUENCY SELECTIVE FADING

- **Each branch of frequency selective fading can be viewed as a flat fading**
  - All the properties discussed for flat fading can be directly applied to each branch of frequency selective fading
    - E.g. Inphase and quadrature components are Gaussian distributed.
    - fading envelope: Rayleigh v.s. Rician

CLASSIFICATION: FREQUENCY SELECTIVE FADING

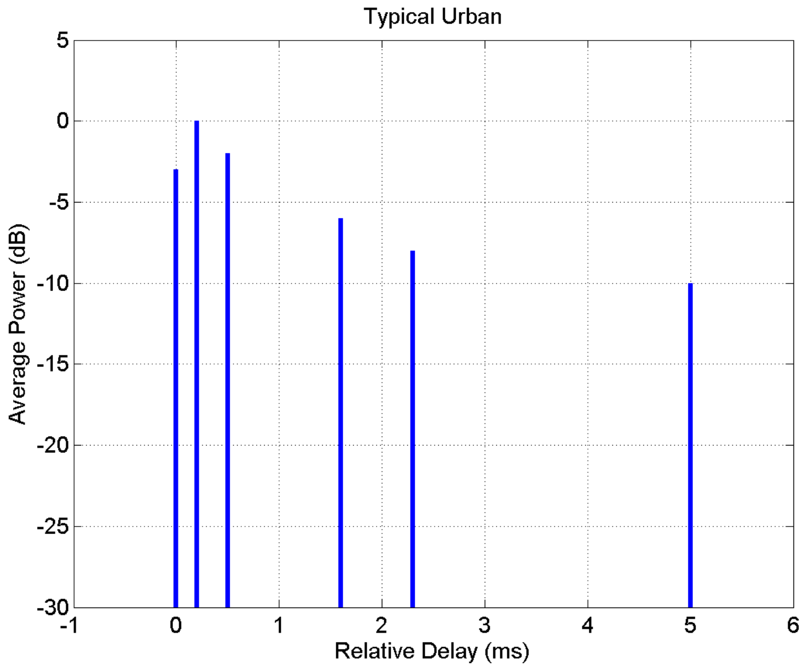


Relative multipath delay τ

Time variation t

CLASSIFICATION: FREQUENCY SELECTIVE FADING

- **Power delay profile**
  - The average power of each resolvable multipath component, w.r.t. the relative delay



Relative delay (ms)	0	0.2	0.5	1.6	2.3	5.0
Average power	0.1897	0.3785	0.2388	0.0951	0.0600	0.0379

CLASSIFICATION: FREQUENCY SELECTIVE FADING

- **Example:**
  - (a) Find the maximum delay spread, mean delay spread, and rms delay spread of the following power delay profile.
  - (b) What is the coherence bandwidth of the channel?
  - (c) For a system with symbol rate 0.5KHz, is this a flat fading or frequency selective fading?
  - (d) For a system with symbol rate 1000KHz, is this a flat fading or frequency selective fading?

Relative delay (ms)	0	1
Average power	0.4	0.6

CLASSIFICATION: RAYLEIGH V.S. RICIAN

- **Fading envelope**

$$|h(t)| = \sqrt{h_I^2(t) + h_Q^2(t)}$$

- **Rayleigh fading**
  - Fading envelope |h(t)| follows Rayleigh distribution
  - Non LOS (no dominant multipath components)

- **Rician fading**
  - Fading envelope |h(t)| follows Rician distribution
  - One dominant component (LOS) along with weaker multipath signals

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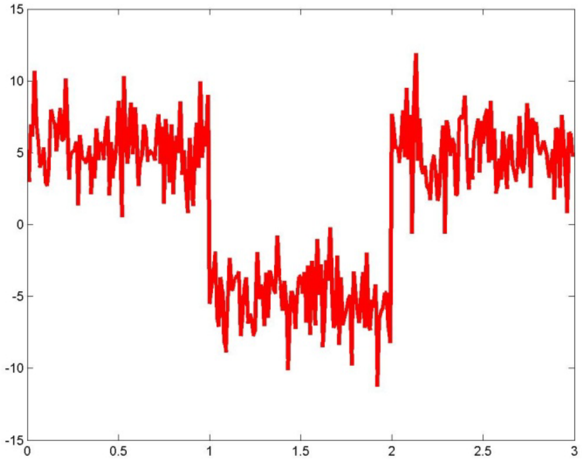
NOISE

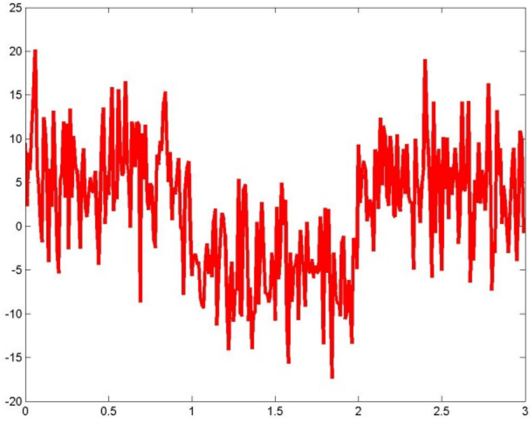
- **Noise and interference**
  - Unwanted electrical signals interfering with the desired signal
  - Arises from outside natural or artificial sources
    - Artificial source: noise from automobile ignition, signal from other communication system, etc.
    - Natural source: thermal noise, atmospheric disturbances.
- **Noise v.s. fading**
  - Noise arises from outside sources
    - fading arises from the signal propagation itself
  - Noise is added to the desired signal ➔ the desired signal is buried by noise (noise only has negative effects on signal).
    - Fading results in signal power fluctuation ➔ signal power may become larger or become smaller (fading might benefit system performance).
  - Noise is present in all communication systems.
    - Fading is unique to wireless communications.

NOISE: SIGNAL TO NOISE RATIO

- **Signal to noise ratio (SNR):**
  - the ratio of the signal power to the noise power at the receiver.
  - SNR = S/N, with S being the signal power, and N being the noise power observed by the receiver.
  - High SNR ➔ Signal is strong, and noise is weak ➔ Better communication quality.
  - Improve SNR ➔ Improve Tx power ➔ More power consumption



High SNR



Low SNR

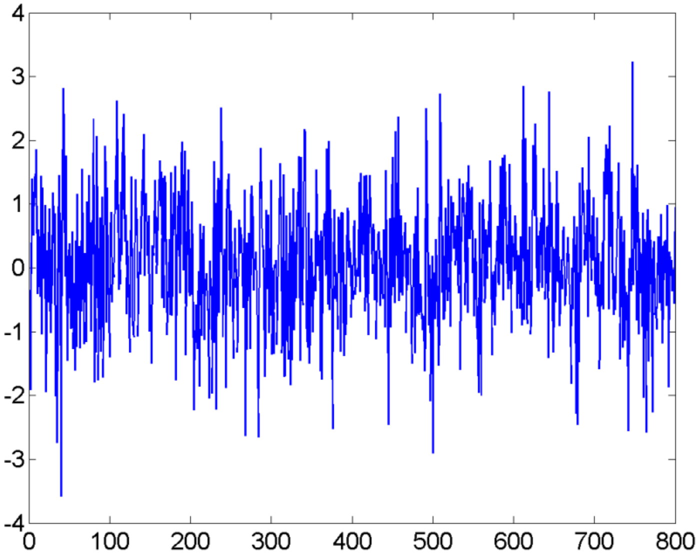
NOISE: THERMAL NOISE

- **Thermal noise**
  - At the temperature above 0K (absolute temperature, =-273 centigrade), the electrons inside the conductor will move randomly.
  - This random movement of electrons will cause random voltage fluctuations of the transmitted signals.
  - Temperature increase ➔ electrons movement becomes stronger ➔ noise power increase

NOISE: THERMAL NOISE

- **Thermal noise is a random process**
  - At any time instant, the random voltage due to thermal noise follows Gaussian distribution with zero mean.
    - The random voltage is caused by the sum of the motions of a large number of electrons (central limit theorem).
  - The noise samples at any two different time instants are uncorrelated. Auto-correlation function

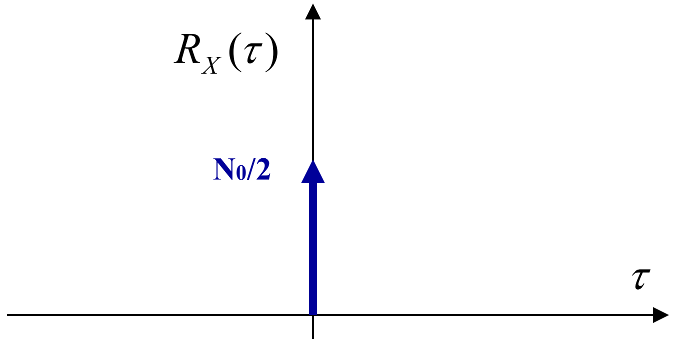
$$R_x(\tau) = E\left[n(t)n^*(t-\tau)\right] = \frac{N_0}{2}\delta(\tau)$$

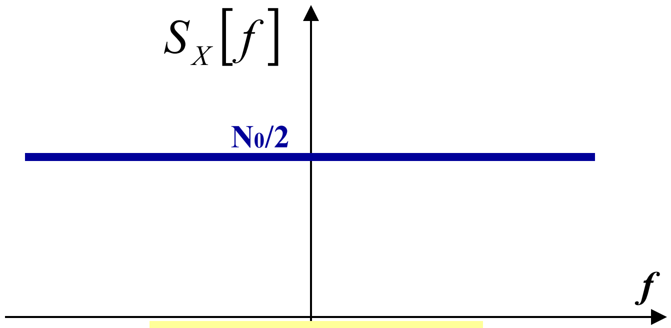


NOISE: THERMAL NOISE

- **Power spectral density**
  - Fourier transform of autocorrelation function

$$S_x(f) = \int_{-\infty}^{+\infty} \frac{N_0}{2}\delta(\tau)e^{-j2\pi f\tau}\,d\tau = \frac{N_0}{2}$$





white spectrum

Additive White Gaussian Noise (AWGN)

NOISE: COCHANNEL INTERFERENCE

- **Cochannel interference (CCI)**
  - Due to frequency reuse, cells using the same frequency range will generate interference with each other.
  - Unique to cellular system.