Department of Electrical Engineering
University of Arkansas

ELEG5663 Communication Theory
Ch. 4 Bandpass Modulation and Demodulation

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OUTLINE

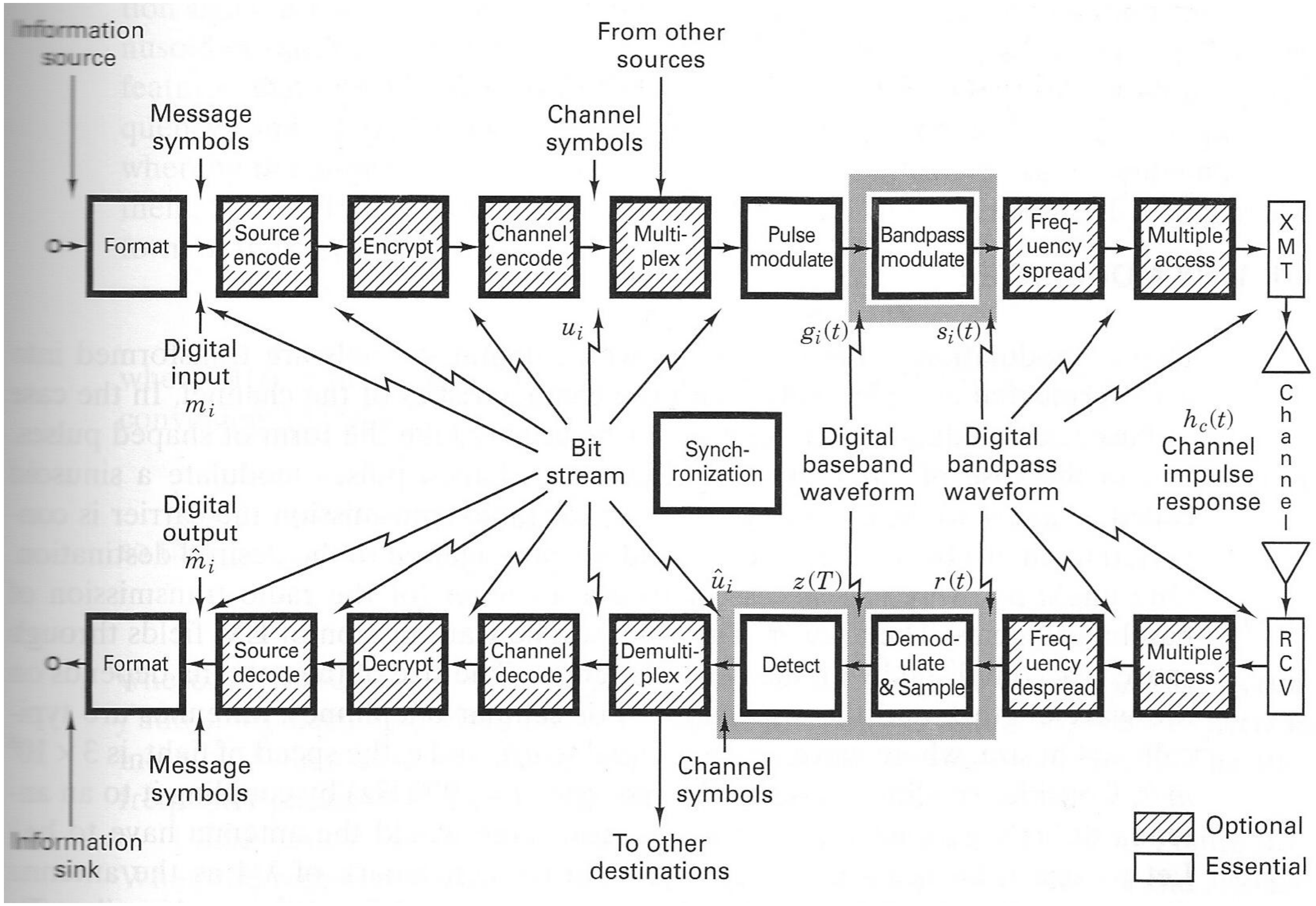
- **Introduction**
- **Bandpass Modulation**
- **Coherent Detection**
- **Noncoherent Detection**
- **Complex Envelope**

INTRODUCTION

- **Bandpass modulation**
  - Recall: baseband modulation
    - Mapping digital symbols onto baseband waveforms (pulse shapes) that are compatible with channel.
  - Bandpass modulation
    - The baseband pulse shapes are translated to a higher frequency by using a carrier wave (a high frequency sinusoid)
- **Why bandpass modulate?**
  - To transmit a signal, antennas are usually ¼ of wavelength λ
    - $\lambda = c / f$: higher frequency ➔ smaller wavelength ➔ smaller antenna.
      - E.g. 3KHz baseband signal, $\lambda =$
      - 1GHz bandpass signal, $\lambda =$
  - Translate the signal to a pre-allocated channel
    - E.g. frequency division multiple access (FDMA)

INTRODUCTION

- **Bandpass modulation and demodulation**



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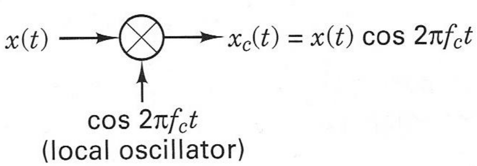
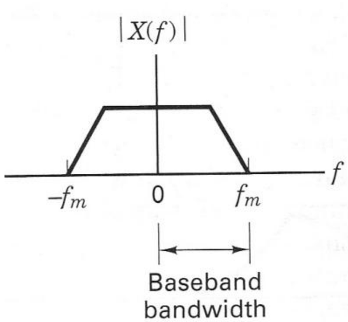
BANDPASS MODULATION

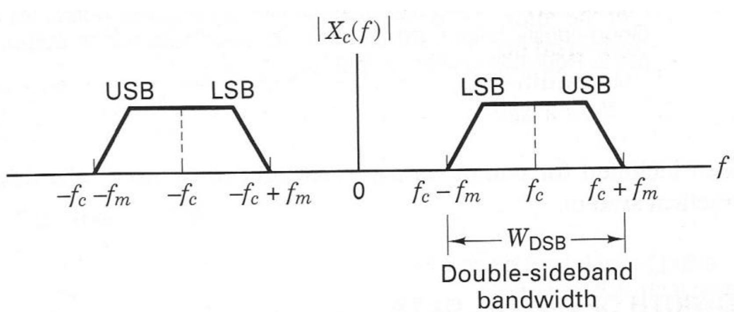
- **Bandpass modulation**
  - The amplitude, frequency, or phase of a radio frequency (RF) carrier, or a combination of them, is varied in accordance with the information to be transmitted.

$$s(t) = A(t)\cos[2\pi f_c t + \phi(t)]$$

  - Through bandpass modulation, a baseband signal is shifted to a higher frequency.
    - Example, amplitude shift keying.
    - Time domain: $x_c(t) = x(t)\cos 2\pi f_c t$
    - Frequency domain: $X_c(f) = \frac{1}{2}[X(f - f_c) + X(f + f_c)]$







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BANDPASS MODULATION

- **Types of bandpass modulation**
  - Coherent (the receivers exploits knowledge of the carrier's phase for detection)
    - Phase shift keying (PSK)
    - Frequency shift keying (FSK)
    - Amplitude shift keying (ASK)
    - Continuous phase modulation (CPM)
    - Hybrid: Quadrature amplitude modulation (QAM)
  - Noncoherent (the receivers operate without knowledge of the absolute value of the signal's phase)
    - Differential phase shift keying (DPSK)
    - Frequency shift keying (FSK)
    - Amplitude shift keying (ASK)
    - Continuous phase modulation (CPM)
    - Hybrid: Quadrature amplitude modulation (QAM)

BANDPASS MODULATION: VECTOR REPRESENTATION

- **Most bandpass modulated signals can be represented as**

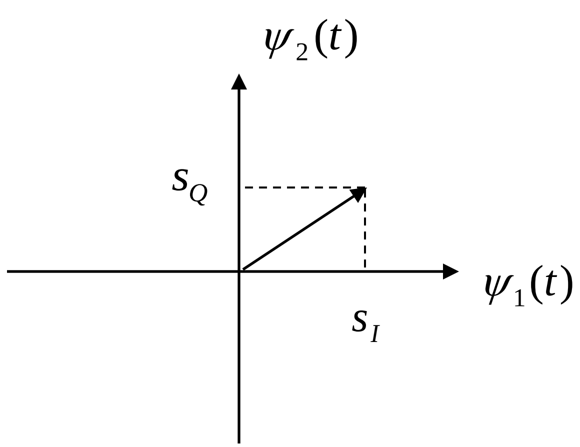
$$s(t) = s_I \sqrt{\frac{2}{T}} \cos 2\pi f_0 t + s_Q \sqrt{\frac{2}{T}} \sin 2\pi f_0 t \qquad 0 \le t \le T$$

  - Orthogonal representation of signal

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos 2\pi f_0 t \qquad \psi_2(t) = \sqrt{\frac{2}{T}} \sin 2\pi f_0 t \qquad 0 \le t \le T$$

    - $\psi_1(t)$ and $\psi_2(t)$ are orthonormal
      - Proof:

  - The bandpass modulated signal can be represented as $s(t) = s_I \psi_1(t) + s_Q \psi_2(t)$
    - $s(t) \Leftrightarrow [s_I, s_Q]$: there is a one-to-one relationship between $s(t)$ and the two dimensional vector $\lfloor s_I, s_Q \rfloor$



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BANDPASS MODULATION: VECTOR REPRESENTATION

- **Vector representation of bandpass modulated signal**
  - Inphase component: s_I
  - Quadrature component: s_Q
  - The inphase carrier, $\psi_1(t) = \sqrt{\frac{2}{T}} \cos \omega_0 t$, and quadrature carrier, $\psi_1(t) = \sqrt{\frac{2}{T}} \sin \omega_0 t$ are orthonomral.
  - The bandpass modulated signal can be equivalently represented as vector

$$s(t) \Leftrightarrow \mathbf{s} = [s_I, s_Q]$$

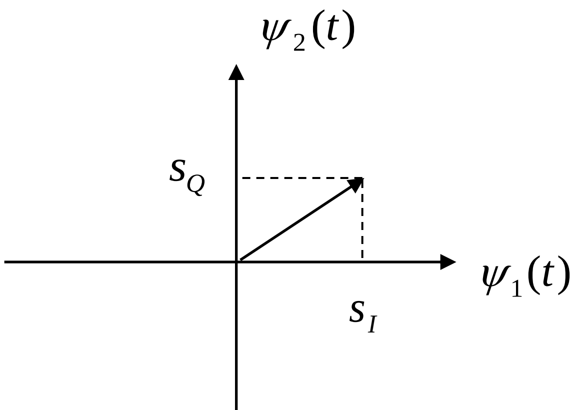
- **Complex number representation of bandpass modulated signal**
  - There is a one-to-one relationship between a 2D vector and complex number

$$s(t) \Leftrightarrow \mathbf{s} = [s_I, s_Q] \Leftrightarrow s_I + js_Q$$

  - The bandpass modulated signal can be equivalently represented as a complex number

$$s(t) \Leftrightarrow s_I + js_Q$$

  - The complex number representation is only used for mathematical convenience.



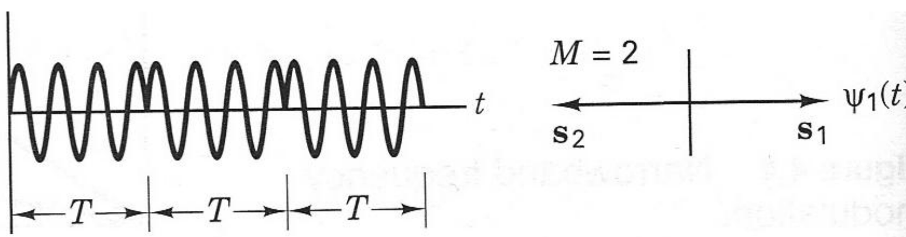
BANDPASS MODULATION: PSK

- **Phase shift keying (PSK)**

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos[\omega_0 t + \phi_i(t)] \qquad 0 \le t \le T, \quad i = 1, \cdots, M$$

  - $\omega_0 = 2\pi f_0$: carrier frequency
  - Each digital symbol is mapped to a different phase $\phi_i(t) = \frac{2\pi i}{M}, i = 1, \cdots, M$
  - Why $\sqrt{\frac{2E}{T}}$?
  - An example of BPSK

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + 2\pi i/M)$$
$$i = 1, 2, \ldots, M$$
$$0 \le t \le T$$



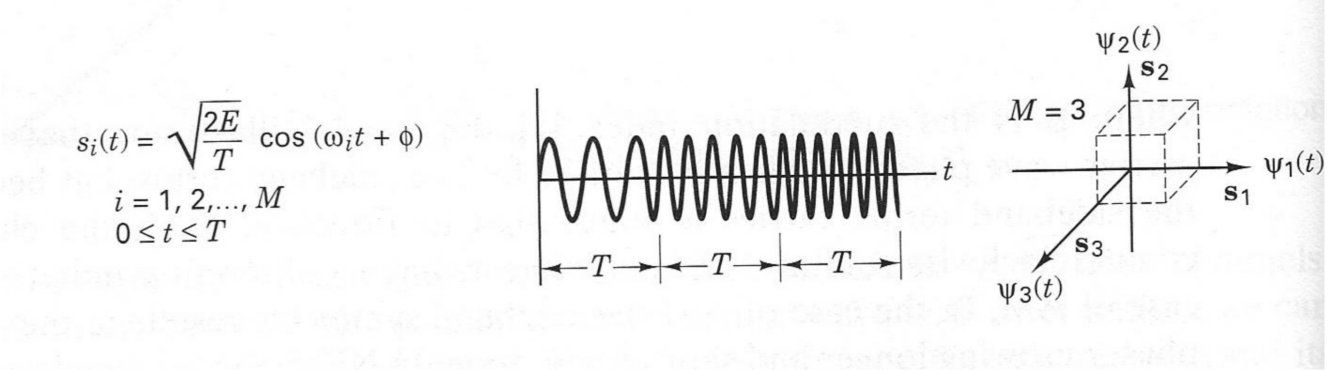
BANDPASS MODULATION: FSK

- **Frequency shift keying (FSK)**

$$s_i(t) = \sqrt{\frac{2E}{T}}\cos[\omega_i t + \phi]$$

$$0 \le t \le T$$
$$i = 1, \cdots, M$$

  - $\omega_i = 2\pi f_i$
  - Each digital symbol is mapped to a different frequency.
  - The set of signals, $\{s_i(t)\}_{i=1}^{M}$, could be orthogonal or non-orthogonal.
  - An example of orthogonal FSK.



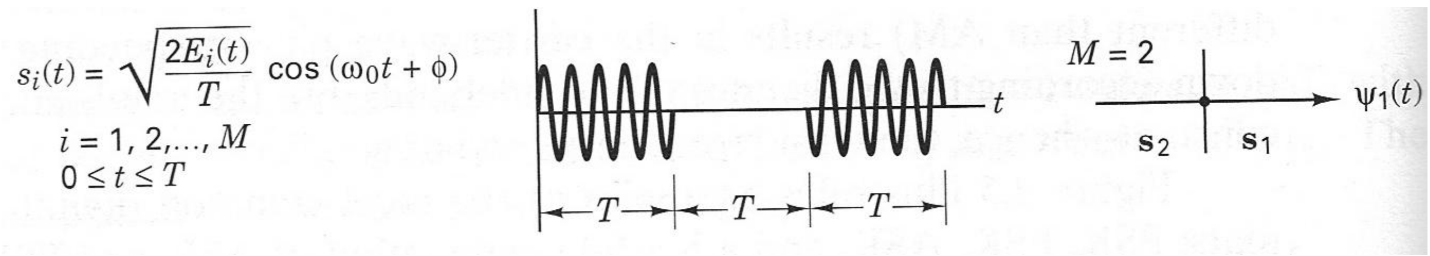
$$\int_0^T s_i(t)s_j(t)dt = \frac{E}{T}\delta_{ij}$$

BANDPASS MODULATION: ASK

- **Amplitude shift keying (ASK)**

$$s_i(t) = \sqrt{\frac{2E_i(t)}{T}} \cos[\omega_0 t + \phi] \qquad \begin{array}{l} 0 \le t \le T \\ i = 1, \cdots, M \end{array}$$

  - Each digital symbol is mapped to a different amplitude
  - An example of BASK (on-off keying)

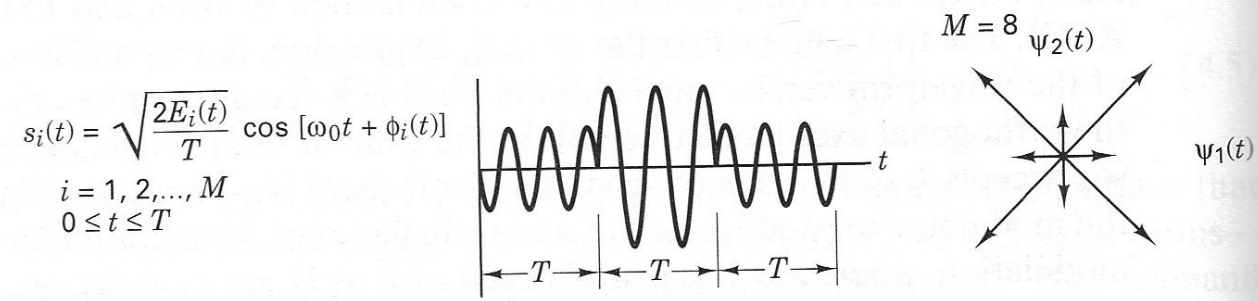


BANDPASS MODULATION: ASK

- **Amplitude phase keying (APK)**

$$s_i(t) = \sqrt{\frac{2E_i(t)}{T}} \cos[\omega_0 t + \phi] \qquad 0 \le t \le T, \quad i = 1, \cdots, M$$

  - Both amplitude and phase are altered by the digital symbol.
  - An example of a 8-ary APK



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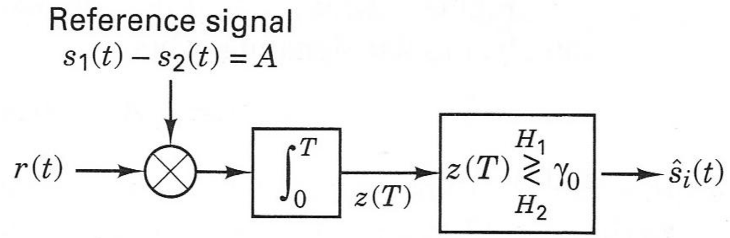
COHERENT DETECTION: BPSK

- **Recall: correlation receiver of binary baseband signal**



$$\gamma_0 = \frac{a_1 + a_2}{2}$$

  – the output of correlation receiver is the same as the output of matched filter and sampler

$$a_1 = \int_0^T s_1(t)\left[s_1(t) - s_2(t)\right]dt \qquad a_2 = \int_0^T s_2(t)\left[s_1(t) - s_2(t)\right]dt$$

  – Bit error probability

$$P(E) = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) \qquad E_d = \int_0^T [s_1(t) - s_2(t)]^2\,dt$$

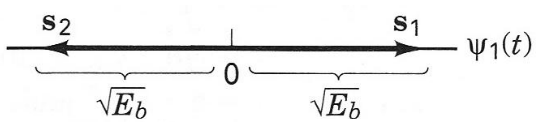
COHERENT DETECTION: BPSK

- **Recall: correlation receiver of binary baseband signal (Cont'd)**
  - Bit error probability with vector signal representation

$$s_1(t) = a_{11}\psi_1(t) + a_{12}\psi_2(t) \qquad s_2(t) = a_{21}\psi_1(t) + a_{22}\psi_2(t)$$

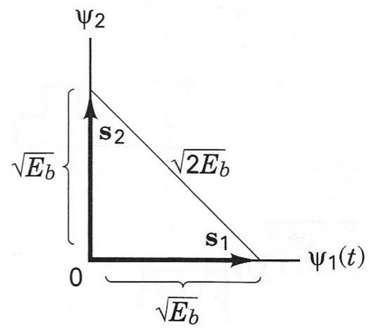
$$E_d = \int_0^T [s_1(t) - s_2(t)]^2 dt =$$

  - Bipolar signal



$$P(E) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

  - Orthogonal signal



$$P(E) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

- E_d is the squared distance between the two modulation points.

COHERENT DETECTION: BPSK

- **Binary Phase Shift Keying (BPSK)**

$$s_1(t) = \sqrt{\frac{2E}{T}}\cos[\omega_0 t] \qquad s_2(t) = \sqrt{\frac{2E}{T}}\cos[\omega_0 t + \pi] = -\sqrt{\frac{2E}{T}}\cos[\omega_0 t]$$

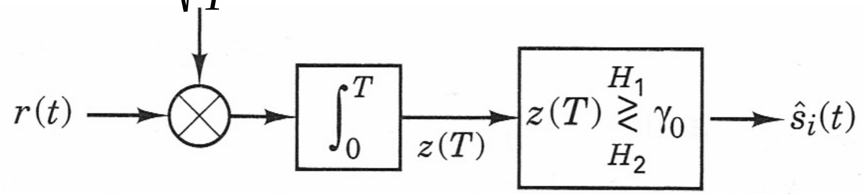
  – Orthogonal representation

$$\psi_1(t) = \sqrt{\frac{2}{T}}\cos\omega_0 t \qquad \psi_2(t) = \sqrt{\frac{2}{T}}\sin\,\omega_0 t \qquad 0 \le t \le T$$

$$s_1(t) = \sqrt{E}\psi_1(t) \qquad s_2(t) = -\sqrt{E}\psi_1(t)$$

  – Correlation receiver

$$\psi_1(t) = \sqrt{\frac{2}{T}}\cos\omega_0 t$$



$$\gamma_0 = \frac{a_1 + a_2}{2}$$

$$a_1 = \int_0^T s_1(t)\psi_1(t)dt =$$

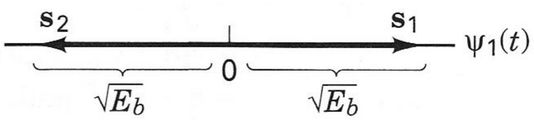
$$a_2 = \int_0^T s_2(t)\psi_1(t)dt =$$

COHERENT DETECTION: BPSK

- **Bit error probability of BPSK**

$$P(E) = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) \qquad E_d = \int_0^T [s_1(t) - s_2(t)]^2 \, dt$$

$$E_d = \int_0^T [s_1(t) - s_2(t)]^2 \, dt =$$



$$P(E) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

COHERENT DETECTION: BPSK

- **BPSK: revisit maximum likelihood detector**

$$r(t) = s_i(t) + n(t)$$

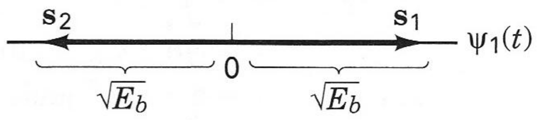
$$z(T) = \int_0^T r(t)\psi_1(t)dt = \int_0^T [s_i(t) + n(t)]\psi_1(t)dt = a_i + n \qquad a_1 = \sqrt{E_b} \qquad a_2 = -\sqrt{E_b}$$

– Likelihood function

$$p(z \mid s_i) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(z - a_i)^2}{2\sigma^2} \right]$$

– Maximum likelihood detection rule:

- If $|z - a_1| < |z - a_2|$, detect $s_1(t)$
- If $|z - a_1| > |z - a_2|$, detect $s_2(t)$
- Equivalently:
  – In the orthogonal representation of signal, choose the signal that has the smallest Euclidean distance with the received sample

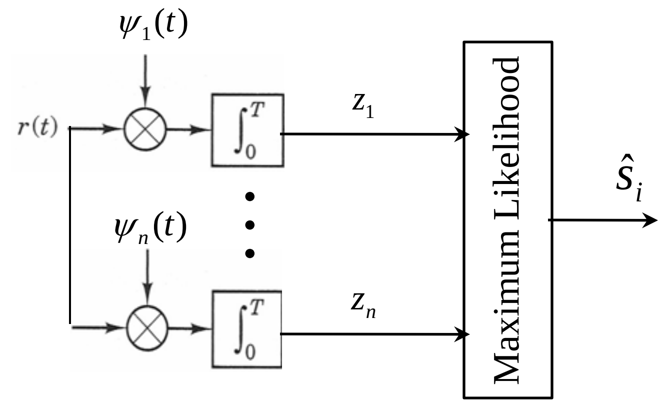


COHERENT DETECTION: BPSK

- **Example**
  - Find the expected number of bit errors made in one day by the following continuously operating coherent BPSK receiver. The data rate is 1 kbps. The input waveforms are $s_1(t) = A\cos\omega_0 t$ and $s_2(t) = -A\cos\omega_0 t$ where A = 2mV and the double-sided noise PSD is $10^{-10} W / Hz$.

COHERENT DETECTION: MAXIMUM LIKELIHOOD

- **Maximum Likelihood Detection**
  - Signal: $s_i(t) = \sum_{j=1}^{n} a_{ij}\psi_j(t)$
  - Rx Signal: $r(t) = s_i(t) + n(t)$
  - Correlation receiver:
  - Likelihood function



COHERENT DETECTION: MAXIMUM LIKELIHOOD

- **Maximum Likelihood Detection**
  - Choose $s_i(t)$ that minimizes

$$\sum_{k=1}^{n}(z_k - a_{ik})^2$$

- **Graphical Interpretation**
  - M-ary modulation, there are M constellation points:

$$\mathbf{s}_m = (a_{m1}, \cdots, a_{mn})$$

  - After correlation detector, there is an *n*-dimension point:

$$\mathbf{z} = (z_1, \cdots, z_n)$$

  - Detection: choose $\mathbf{s}_i$ such that the Euclidean distance is minimized

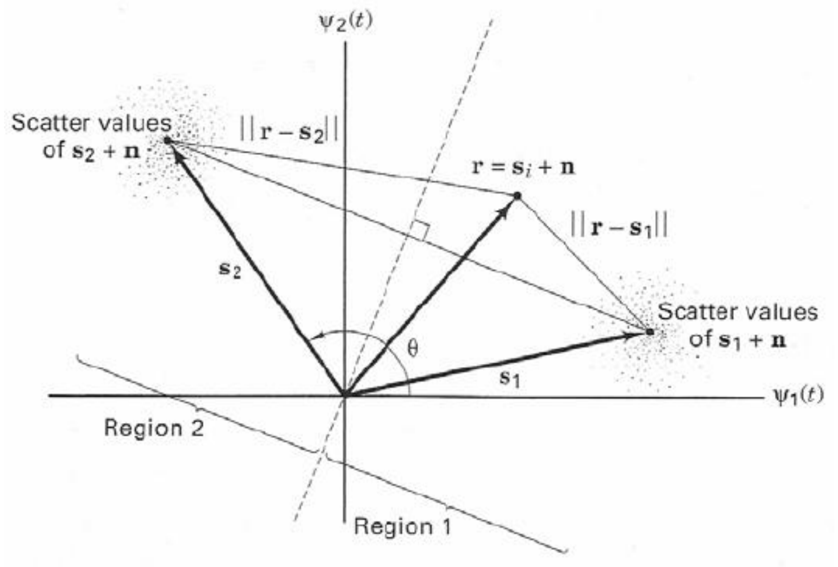
$$\left\|\mathbf{s}_i - \mathbf{z}\right\|$$

COHERENT DETECTION

- **Maximum Likelihood detection: 2-Dimension**
  - Choose $\mathbf{s}_i$ such that the Euclidean distance between the vector $\mathbf{r}$ and $\mathbf{s}_i$

$$\|\mathbf{r}-\mathbf{s}_i\| = \sqrt{(r_I - s_{iI})^2 + (r_Q - s_{iQ})^2}$$

    are minimized.
  - Decision region example
    - Whenever the received signal $\mathbf{r}$ is located in region 1, choose signal $\mathbf{s}_1$
    - Whenever the received signal $\mathbf{r}$ is located in region 2, choose signal $\mathbf{s}_2$



COHERENT DETECTION: MPSK

- **Multiple Phase Shift Keying (MPSK)**

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[\omega_0 t - \frac{2\pi i}{M}\right] \qquad 0 \le t \le T \qquad i = 1, \cdots, M$$

  - Orthogonal representation of MPSK signals

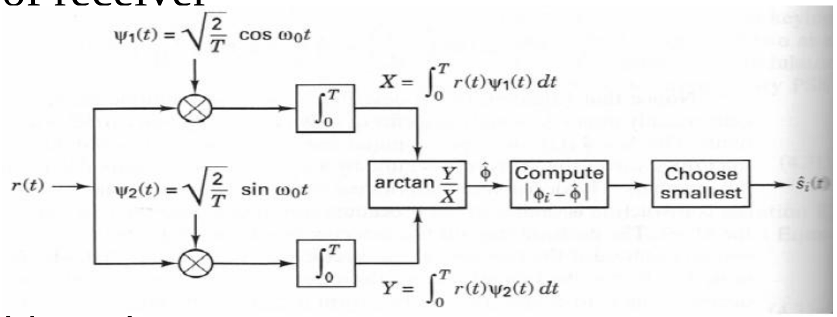
$$s_i(t) = \sqrt{E} \cos\left(\frac{2\pi i}{M}\right)\sqrt{\frac{2}{T}} \cos(\omega_0 t) + \sqrt{E} \sin \cos\left(\frac{2\pi i}{M}\right)\sqrt{\frac{2}{T}} \sin(\omega_0 t)$$

$$s_i(t) = \sqrt{E} \cos\left(\frac{2\pi i}{M}\right)\psi_1(t) + \sqrt{E} \sin \cos\left(\frac{2\pi i}{M}\right)\psi_2(t)$$

  - Inphase component $s_I = \sqrt{E} \cos\left(\frac{2\pi i}{M}\right)$
  - Quadrature component $s_Q = \sqrt{E} \sin\left(\frac{2\pi i}{M}\right)$

COHERENT DETECTION: MPSK

- **MPSK coherent detection**
  - Structure of receiver



  - Likelihood functions

$$X = \int_0^T (s_i(t) + n(t))\psi_1(t)dt = s_{iI} + n_I \qquad Y = \int_0^T (s_i(t) + n(t))\psi_2(t)dt = s_{iQ} + n_Q$$

$$p(X \mid s_i) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(X - s_{iI})^2}{2\sigma^2}\right] \qquad p(Y \mid s_i) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(Y - s_{QI})^2}{2\sigma^2}\right]$$

$$p(X, Y \mid s_i) = \frac{1}{\sigma^2 2\pi} \exp\left[-\frac{(X - s_{iI})^2 + (Y - s_{iQ})^2}{2\sigma^2}\right]$$

  - Maximum likelihood detection
    - Choose $s_i(t)$ that minimizes $(X - s_{iI})^2 + (Y - s_{iQ})^2$

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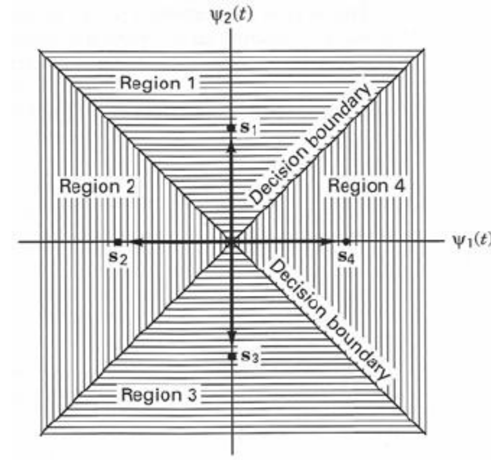
COHERENT DETECTION: MPSK

- **MPSK coherent detection (Cont'd)**
  - The distance between $\mathbf{r} = (X, Y)$ and $\mathbf{s}_i = (s_{iI}, s_{iQ})$

$$d_i = \sqrt{(X - s_{iI})^2 + (Y - s_{iQ})^2}$$

  - Maximum likelihood decision rule

$$X = \int_0^T r(t)\psi_1(t)dt \qquad Y = \int_0^T r(t)\psi_2(t)dt$$



    - Choose $s_i(t)$ that minimizes the distance between $\mathbf{r} = (X, Y)$ and $\mathbf{s}_i = (s_{iI}, s_{iQ})$
    - Equivalently, choose $s_i(t)$ with phase ϕ_i that is closest to the phase of the signal at the output of the correlator:
      - Find $s_i(t)$ that minimize $|\phi - \hat{\phi}|$ $\qquad \hat{\phi} = \arctan \frac{Y}{X}$

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COHERENT DETECTION: MPSK

- **Example:**
  - For a system with 8PSK, if the received symbols are:

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos[\omega_0 t + \frac{2\pi i}{8}] \qquad m = 0,1,\cdots,7$$

  - Find the vector representation is the signal
  - Plot the constellation diagram
  - If the samples after coherent receiver are (X, Y) = [(1, 3), (-1, 2), (4, -1)], find the detected symbols.

COHERENT DETECTION: MFSK

- **Multiple frequency shift keying (MFSK)**

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos \omega_i t \qquad 0 \le t \le T \qquad i = 1, \cdots, M$$

- The value of ω_i can be chosen such that $\{\cos \omega_i t\}_{i=1}^{M}$ are mutually orthogoanal ➔ orthogonal MFSK
  - Orthogonal MFSK is a special case of MFSK
  - We are only going to examine orthogonal MFSK
- Orthogonal MFSK

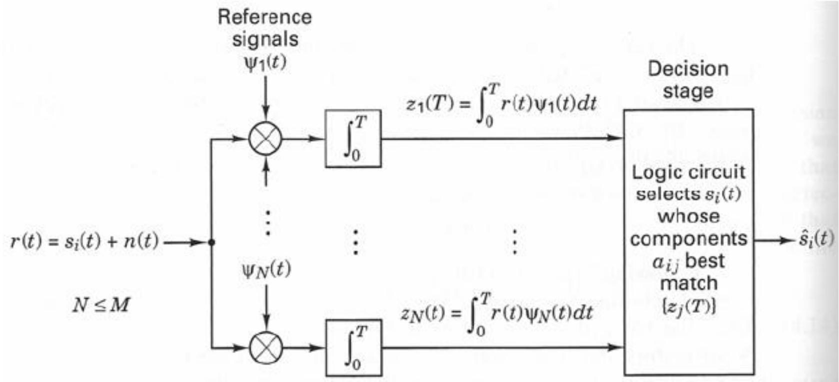
$$\psi_i(t) = \sqrt{\frac{2}{T}} \cos \omega_i t \qquad \int_0^T \psi_i(t)\psi_j(t)dt = \delta_{ij}$$

- Example: Show the following system is orthogonal BFSK if $f_c >> \frac{1}{T}$

$$f_1 = f_c - \frac{1}{2T} \qquad f_2 = f_c + \frac{1}{2T}$$

COHERENT DETECTION: MFSK

- **Coherent receiver structure of MFSK**
  - Structure of a receiver $s_i(t) = s_{i1}\psi_1(t) + s_{i2}\psi_2(t) + \cdots + s_{iM}\psi_M(t)$



  - Output of the correlation detector for MFSK

$$r_{im} = \int_0^T (s_i(t) + n(t))\psi_m(t)dt = s_{im} + n_m$$

- Output of the correlation detector of MFSK is coordinate in orthogonal signal representation

$$\mathbf{s}_i = (s_{i1}, s_{i2}, \cdots, s_{iM})$$

$$\mathbf{r} = (r_{i1}, r_{i2}, \cdots, r_{iM})$$

COHERENT DETECTION: MFSK

- **Maximum Likelihood detection**
  - Choose $s_i(t)$ that minimizes the Euclidean distance between

$$\mathbf{s}_i = (s_{i1}, s_{i2}, \cdots, s_{iM}) \qquad \mathbf{r} = (r_{i1}, r_{i2}, \cdots, r_{iM})$$

- **Bit error probability of BFSK**

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COHERENT DETECTION

- **Vector representation of bandpass communication system**
  - Recall vector representation of bandpass modulated signal

$$\psi_1(t) = \sqrt{\frac{2}{T}}\cos 2\pi f_0 t \qquad \psi_2(t) = \sqrt{\frac{2}{T}}\sin 2\pi f_0 t \qquad 0 \le t \le T$$

$$s_i(t) = s_{iI}\psi_1(t) + s_{iQ}\psi_2(t) \qquad n(t) = n_I\psi_1(t) + n_Q\psi_2(t)$$

$$r(t) = s_i(t) + n(t) = (s_I + n_I)\psi_I(t) + (s_Q + n_Q)\psi_Q(t) \qquad r(t) = r_I\psi_I(t) + r_Q\psi_Q(t)$$

$$r_I = s_{iI} + n_I$$

$$r_Q = s_{iQ} + n_Q$$

  - The bandpass communication system is equivalently represented as the summation of signal vector and noise vector

$$\mathbf{r} = \mathbf{s}_i + \mathbf{n}$$

$$\mathbf{r} = [r_I, r_Q] \qquad \mathbf{s}_i = [s_{iI}, s_{iQ}] \qquad \mathbf{n} = [n_I, n_Q]$$

COHERENT DETECTION

- **Maximum likelihood detection**
  - For AWGN with two-sided PSD $\frac{N_0}{2}$
    - The noise variance per dimension is $\sigma^2 = \frac{N_0}{2}$
  - Likelihood functions

$$p(r_I \mid s_{iI}) = \frac{1}{\sigma^2\sqrt{2\pi}} \exp\left[-\frac{(r_I - S_{iI})^2}{2\sigma^2}\right]$$

$$p(r_Q \mid s_{iQ}) = \frac{1}{\sigma^2\sqrt{2\pi}} \exp\left[-\frac{(r_Q - S_{iQ})^2}{2\sigma^2}\right]$$

  - n_I and n_Q are independent

$$p(r_I, r_Q \mid s_{iI}, s_{iQ}) = \frac{1}{2\pi\sigma^4} \exp\left[-\frac{(r_I - s_{iI})^2 + (r_Q - s_{iQ})^2}{2\sigma^2}\right]$$

  - Maximize $p(r_I, r_Q \mid s_{iI}, s_{iQ})$ ➔ Minimize $(r_I - s_{iI})^2 + (r_Q - s_{iQ})^2$
  - Minimize the Euclidean distance between the vector $\mathbf{r}$ and $\mathbf{s}_i$

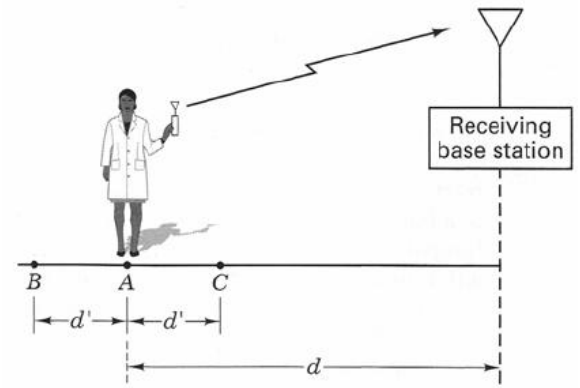
$$\|\mathbf{r} - \mathbf{s}_i\| = \sqrt{(r_I - s_{iI})^2 + (r_Q - s_{iQ})^2}$$

OUTLINE

- Introduction
- Bandpass Modulation
- Coherent Detection
- **Noncoherent Detection**
- Complex Envelope

NONCOHERENT DETECTION

- **Why noncoherent detection?**
  - Coherent detection requires the exact knowledge of the phase of the received signal
  - Example: BPSK
    - The distance between Tx and Rx is d
    - At Tx, the signal is $s(t)=\sqrt{\frac{2E_b}{T}}\cos(2\pi f_0 t)$
    - At Rx, the signal is $r(t)=\sqrt{\frac{2E_b}{T}}\cos[2\pi f_0(t+T_d)]=\sqrt{\frac{2E_b}{T}}\cos[2\pi f_0 t+\theta(t)]$
    - $T_d = d/c$: the amount of time the signal travels from Tx to Rx
    - $\theta(t)=2\pi f_0 T_d$: the phase of the signal at the receiver
    - In order to perform coherent detection, the Rx needs to know $\theta(t)$
      - $\theta(t)$ can be estimated through a circuitry called phase locked loop (PLL)



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NONCOHERENT DETECTION

- **Why noncoherent detection? (Cont'd)**
  - What if the Rx doesn't have the knowledge of $\theta(t)$?
  - Example:
    - Assume a system operates a 1GHz. If the distance between Tx and Rx is 24.075m, find out the phase of the Rx signal.

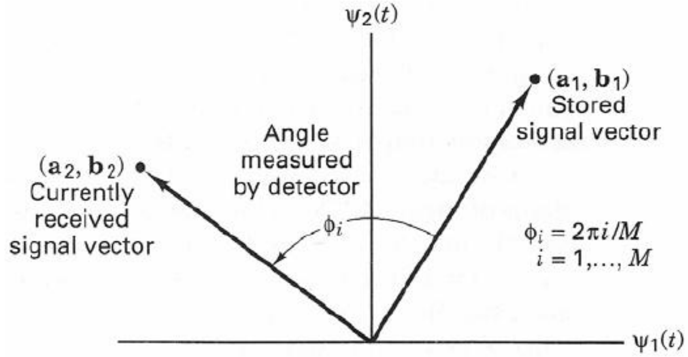
- **Noncoherent detection**
  - The Rx doesn't require the knowledge of the absolute phase of the Rx signal.

NONCOHERENT DETECTION: DPSK

- **Differential PSK**
  - The information is carried by the phase difference between the current symbol and the previous symbol
    - Recall: for coherent PSK, the information is carried by the absolute phase of one symbol.
  - Example:
    - The kth Rx symbol is $r(t)=\sqrt{\frac{2E}{T}}\cos[2\pi f_0 t+\theta_k+\alpha]$
    - The (k+1)th Rx symbol is $r(t)=\sqrt{\frac{2E}{T}}\cos[2\pi f_0 t+\theta_{k+1}+\alpha]$
    - The phase difference between the two consecutive symbols

$$(\theta_{k+1}+\alpha)-(\theta_k+\alpha)=\phi_i$$

    - The information is carried by the phase difference $\phi_i=\frac{2\pi i}{M}$
  - The Rx doesn't need the knowledge of the absolute phase α. The information is carried by the phase difference.



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NONCOHERENT DETECTION: DPSK

- **Binary DPSK**
  - The essence of differential detection is that the data is carried in the phase difference between two consecutive symbols
  - Tx: differential encoding; Rx: differential decoding.
  - Binary differential encoding

$$c(k) = \overline{c(k-1) \oplus m(k)}$$

- m(k): information
- c(k): differentially encoded bit
- $\oplus$: modulo-2 addition
- $-$: complement
- The information, m(k), is carried by the difference between c(k) and c(k-1)

Sample index, k	0	1	2	3	4	5	6	7	8	9	10
Information message, $m(k)$		1	1	0	1	0	1	1	0	0	1
Differentially encoded message (first bit arbitrary), $c(k)$	1	1	1	0	0	1	1	1	0	1	1
Corresponding phase shift, $\theta(k)$	π	π	π	0	0	π	π	π	0	π	π

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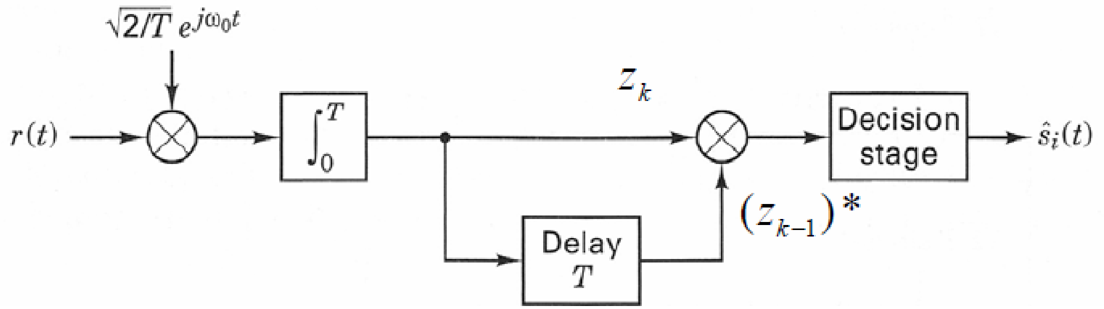
NONCOHERENT DETECTION: DPSK

- **Binary DPSK**
  - Binary differential decoding



  - After the correlation detector, the k-th sample is

$$z_{kI} = \int_0^T r(t)\psi_1(t)dt = c_{kI} + n_{kI} \qquad z_{kQ} = \int_0^T r(t)\psi_2(t)dt = c_{kQ} + n_{kQ}$$

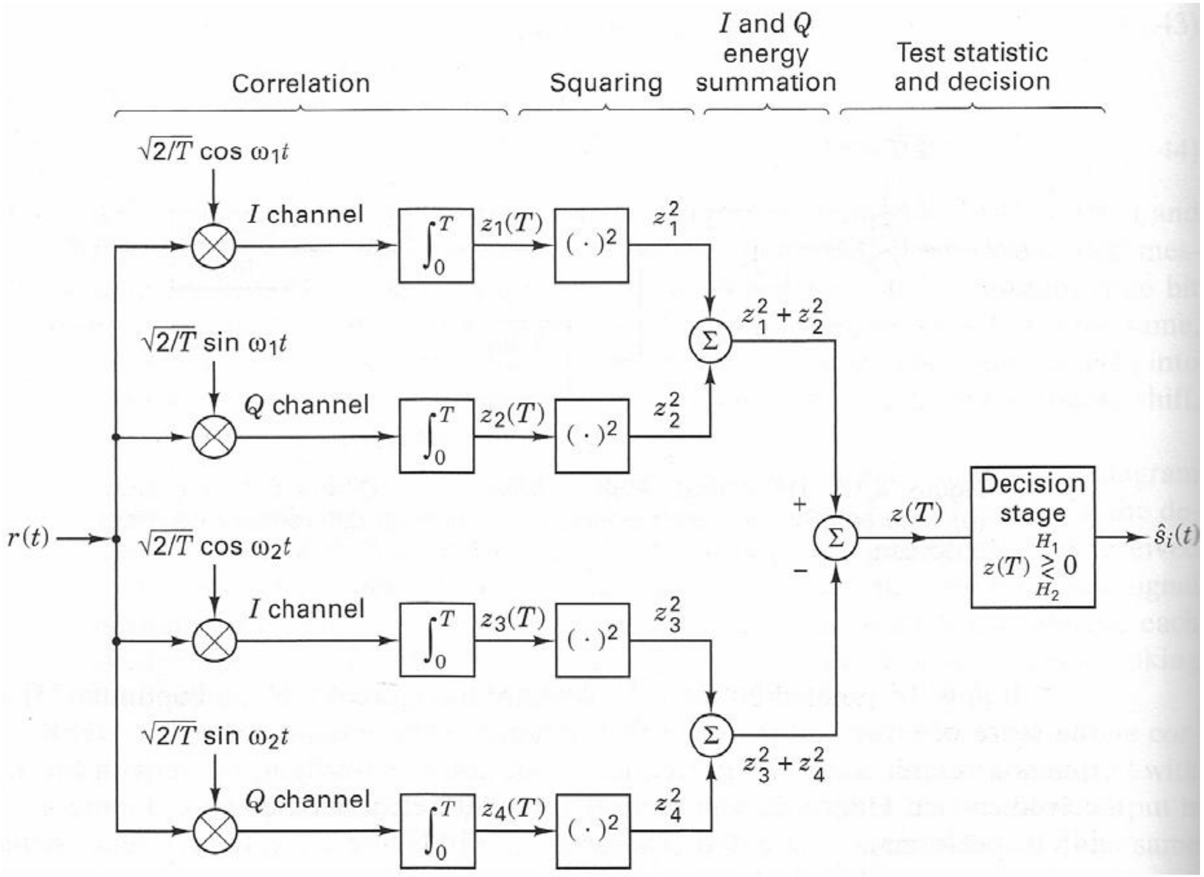
  - Performing detection by compare the phase of (z_{kI}, z_{kQ}) and $(z_{(k-1)I}, z_{(k-1)Q})$

NONCOHERENT DETECTION: DPSK

- **DPSK: pros and cons**
  - Pro:
    - Doesn't require the absolute value of the signal phase ➔ simpler receiver
  - Con:
    - Two noisy signal are compared to detect the signal ➔ there are twice as much noise as in coherent detection ➔ the performance is worse compared to coherent detection
  - Trade-off between complexity and performance

NONCOHERENT DETECTION: FSK

- **Non-coherent detection of FSK**



$\psi_1(t)=\sqrt{\frac{2}{T}}\cos\omega_1 t$ $\psi_2(t)=\sqrt{\frac{2}{T}}\sin\omega_1 t$ $\psi_3(t)=\sqrt{\frac{2}{T}}\cos\omega_2 t$ $\psi_4(t)=\sqrt{\frac{2}{T}}\sin\omega_2 t$ are mutually orthonormal

NONCOHERENT DETECTION: FSK

- **Non-coherent detection of FSK**
  - If $s_1(t) = \sqrt{\frac{2E}{T}} \cos(\omega_1 t + \theta)$ has been transmitted
    - What are the values at the output of the non-coherent detector of FSK?

NONCOHERENT DETECTION: FSK

- **Non-coherent detection of FSK**
  - The minimum tone space for non-coherent orthogonal FSK

The minimum tone space for non-coherent orthogonal FSK is $f_1 - f_2 = \frac{1}{T}$

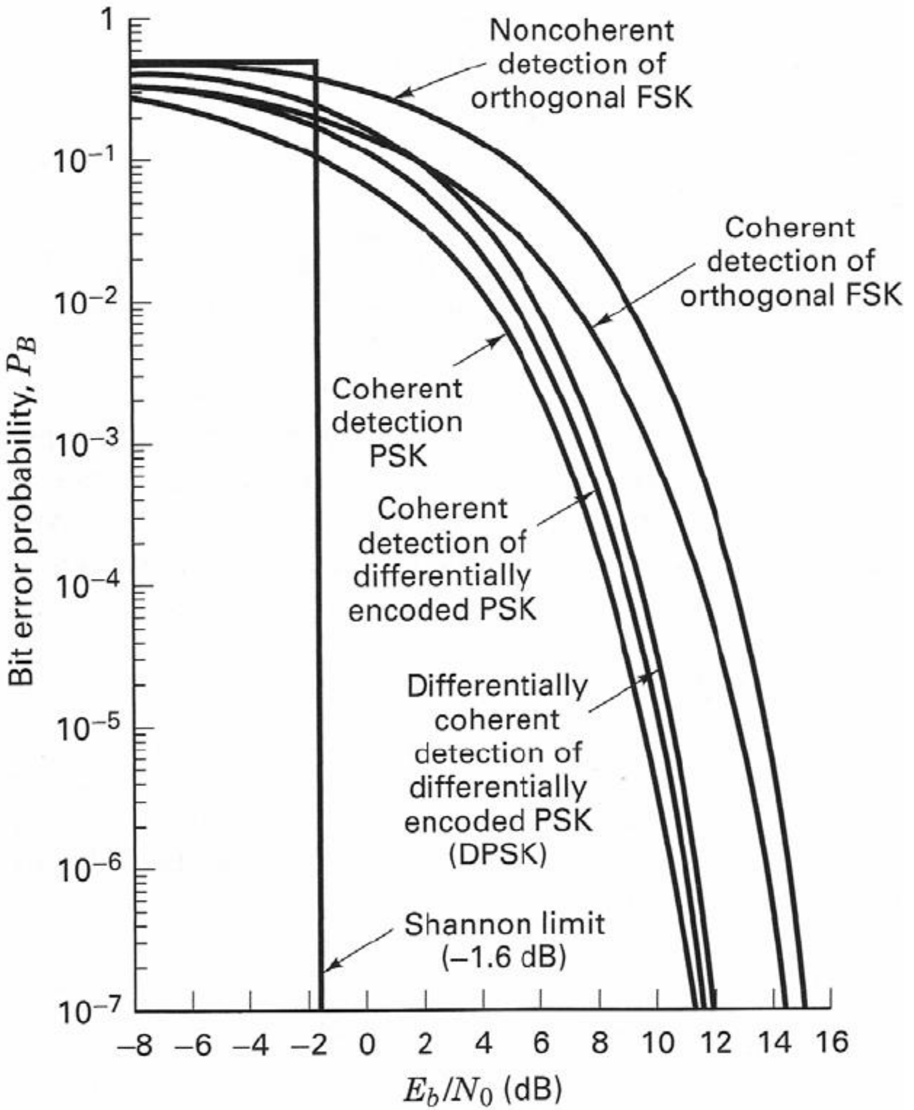
NONCOHERENT DETECTION: FSK

- **Minimum Tone Spacing for Coherent Orthogonal FSK**

The minimum tone space for coherent orthogonal FSK is $f_1 - f_2 = \dfrac{1}{2T}$

NONCOHERENT DETECTION

- **Coherent detection v.s. non-coherent detection**



OUTLINE

- Introduction
- Bandpass Modulation
- Coherent Detection
- Noncoherent Detection
- **Complex Envelope**

COMPLEX ENVELOPE

- **Complex representation of bandpass modulated signal**

$$s(t) = s_I(t)\sqrt{\frac{2}{T}}\cos 2\pi f_0 t - s_Q(t)\sqrt{\frac{2}{T}}\sin 2\pi f_0 t$$

$$\tilde{s}(t) = [s_I(t) + js_Q(t)]\sqrt{\frac{2}{T}}e^{j2\pi f_0 t}$$

  – There is a one to one relationship between $s(t)$ and $\tilde{s}(t)$

$$s(t) = \mathrm{Re}\left[\tilde{s}(t)\right]$$

- **Complex envelope**

$$g(t) = \left[s_I(t) + js_Q(t)\right]$$

  – The complex baseband signal $g(t)$ is called complex envelope
    - The envelope of the bandpass signal.
  – The complex envelope is the same as the vector representation of the signal up to a scaling factor

$$s(t) \Leftrightarrow \left[s_I(t), s_Q(t)\right] \Leftrightarrow \left[s_I(t) + js_Q(t)\right]$$

COMPLEX ENVELOPE

- **Complex envelope**

$$g(t) = x(t) + jy(t)$$

  - Polar representation $$g(t) = |g(t)|e^{j\theta(t)}$$
    - Amplitude $$|g(t)| = \sqrt{x^2(t) + y^2(t)}$$
    - Phase $$\theta(t) = \tan^{-1}\frac{y(t)}{x(t)}$$

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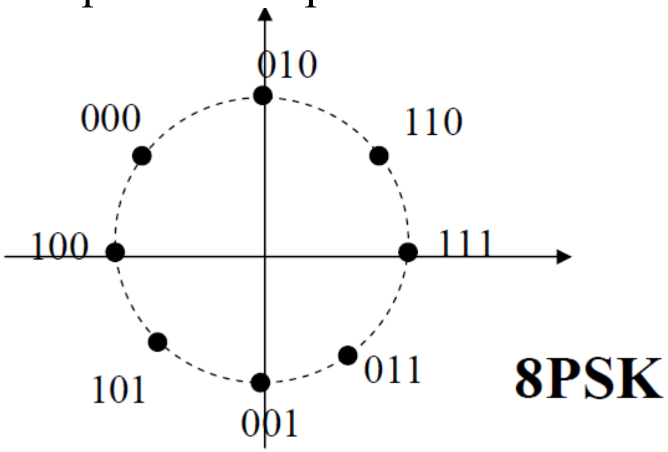
COMPLEX ENVELOPE

- **Bandpass modulation can be divided into two steps (Constellation)**
  - 1. Baseband modulation:
    - Transfer information (‘1’s and ‘0’s) into complex envelope
    - Example: 8PSK

      1 0 0 0 1 0 1 1 1 0 0 1

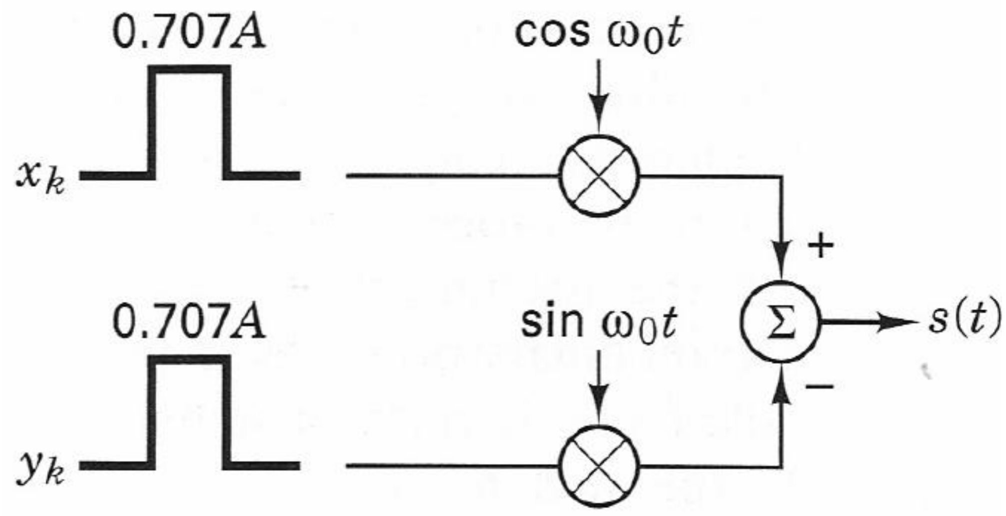


  - 2. Frequency upconversion
    - Multiply the complex envelope with $e^{j2\pi f_0 t}$

$$s(t) = \mathrm{Re}\{g(t)e^{j2\pi f_0 t}\}$$

COMPLEX ENVELOPE

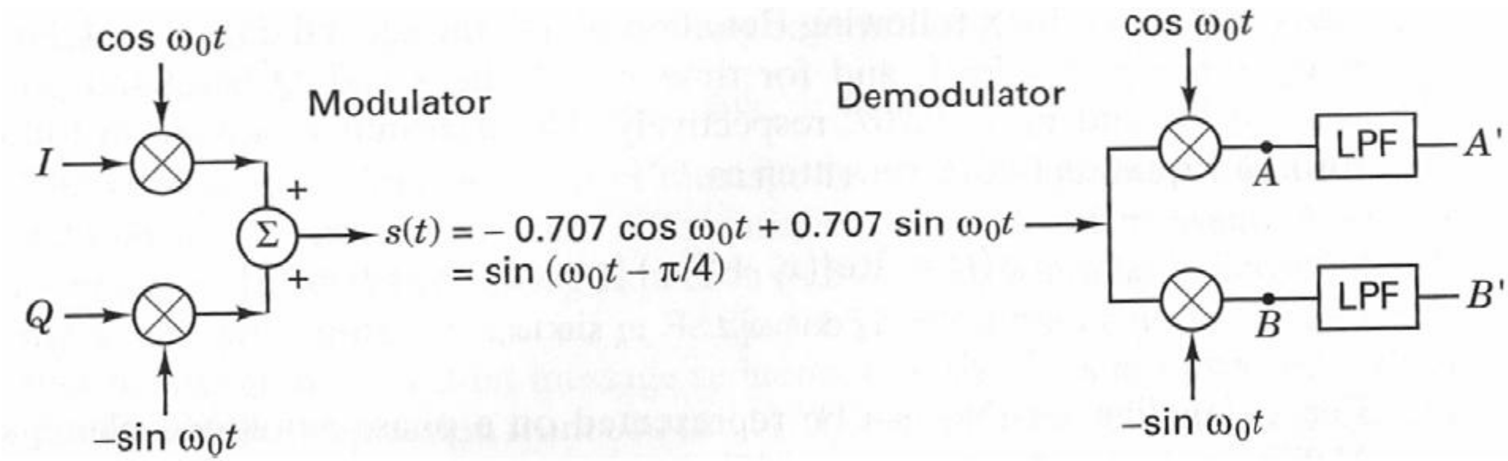
- **Quadrature implementation of a modulator**



  - Baseband modulation:
    - Mapping ‘0’s and ‘1’s to the values of x(t) and y(t).
  - Frequency upconversion:
    - Upconverting the frequency of the baseband signal through quadrature modulation.

COMPLEX ENVELOPE

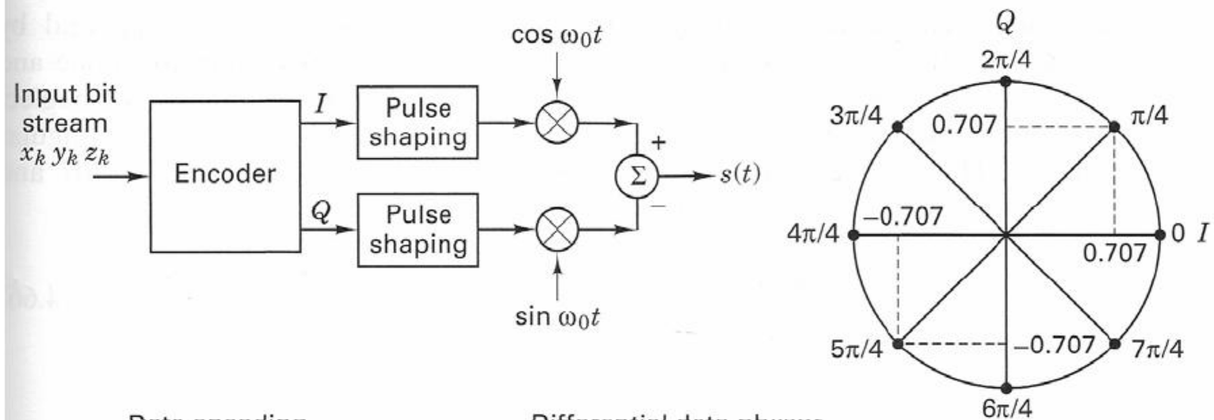
- **Quadrature implementation of a demodulator**



- Frequency downconversion:
  - Downconverting the frequency of the bandpass signal.
- Baseband demodulation:
  - Mapping the baseband signal to ‘0’s and ‘1’s

COMPLEX ENVELOPE: D8PSK

- **D8PSK (Differential 8PSK)**
  - Baseband modulation



Data encoding

x_k	y_k	z_k	$\Delta\phi_k$
0	0	0	0
0	0	1	$\pi/4$
0	1	1	$2\pi/4$
0	1	0	$3\pi/4$
1	1	0	$4\pi/4$
1	1	1	$5\pi/4$
1	0	1	$6\pi/4$
1	0	0	$7\pi/4$

Differential data phasor

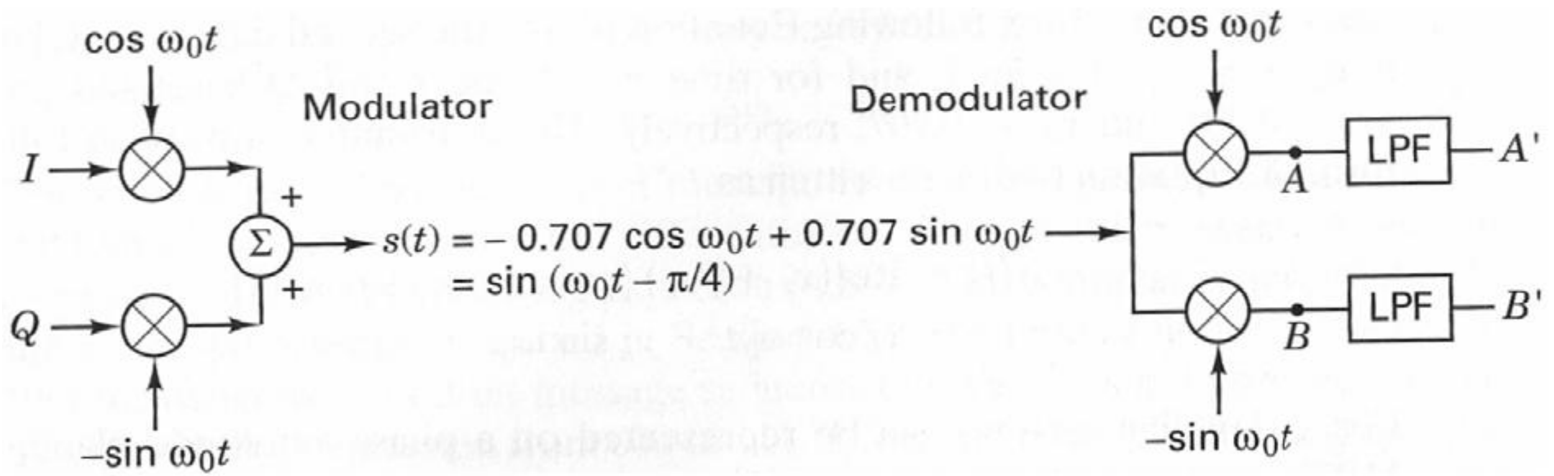
$$\phi_k = \phi_{k-1} + \Delta\phi_k$$

Let $\phi_0 = 0$	$k = 1$	$k = 2$	$k = 3$	$k = 4$
$x_k\ y_k\ z_k$	110	001	110	010
$\Delta\phi_k$:	π	$\pi/4$	π	$3\pi/4$
ϕ_k:	π	$5\pi/4$	$\pi/4$	π
I:	−1	−0.707	0.707	−1
Q:	0	−0.707	0.707	0

  - Frequency upconversion

COMPLEX ENVELOPE: D8PSK

- **D8PSK Demodulation**
  - Frequency downconversion
  - Baseband demodulation



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OUTLINE

- Introduction
- Bandpass Modulation
- Coherent Detection
- Noncoherent Detection
- Complex Envelope
- **Error Probability**

ERROR PROBABILITY: BINARY MODULATION

- **Comparison of error performance of binary system**
  - BER of BPSK

$$P_B = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

  - BER of coherently detected, differentially encoded binary PSK

$$P_B = 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left[1 - Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right]$$

  - BER of differentially detected, differentially encoded binary PSK (DPSK)

$$P_B = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$$

ERROR PROBABILITY: BINARY MODULATION

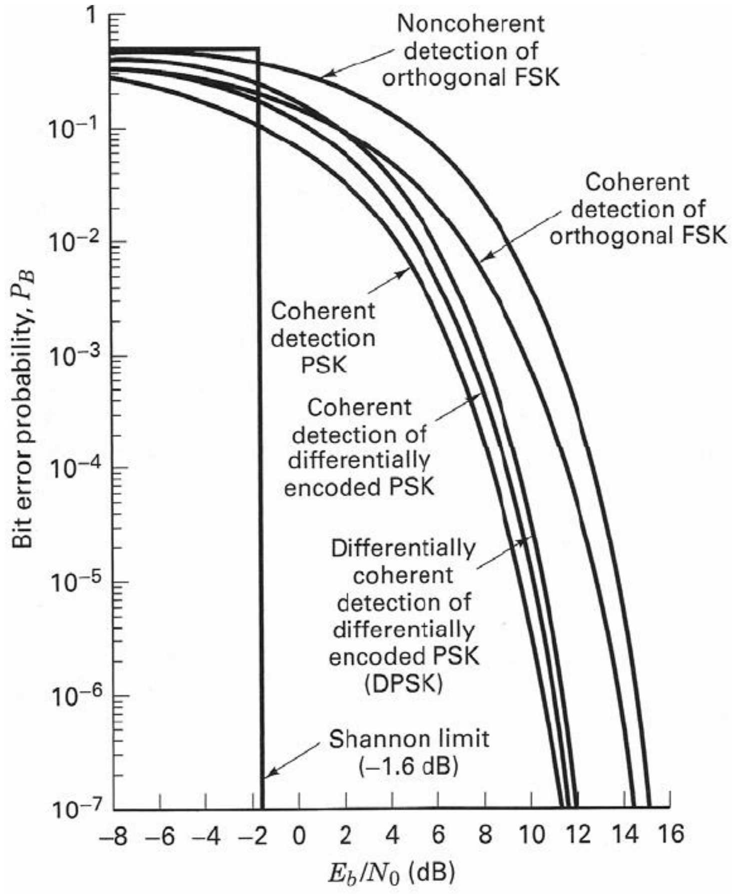
- **Comparison of error performance of binary system**
  - BER for coherently detected binary orthogonal FSK

$$P_B = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

  - BER for non-coherently detected binary orthogonal FSK

$$P_B = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)$$

ERROR PROBABILITY: BINARY MODULATION



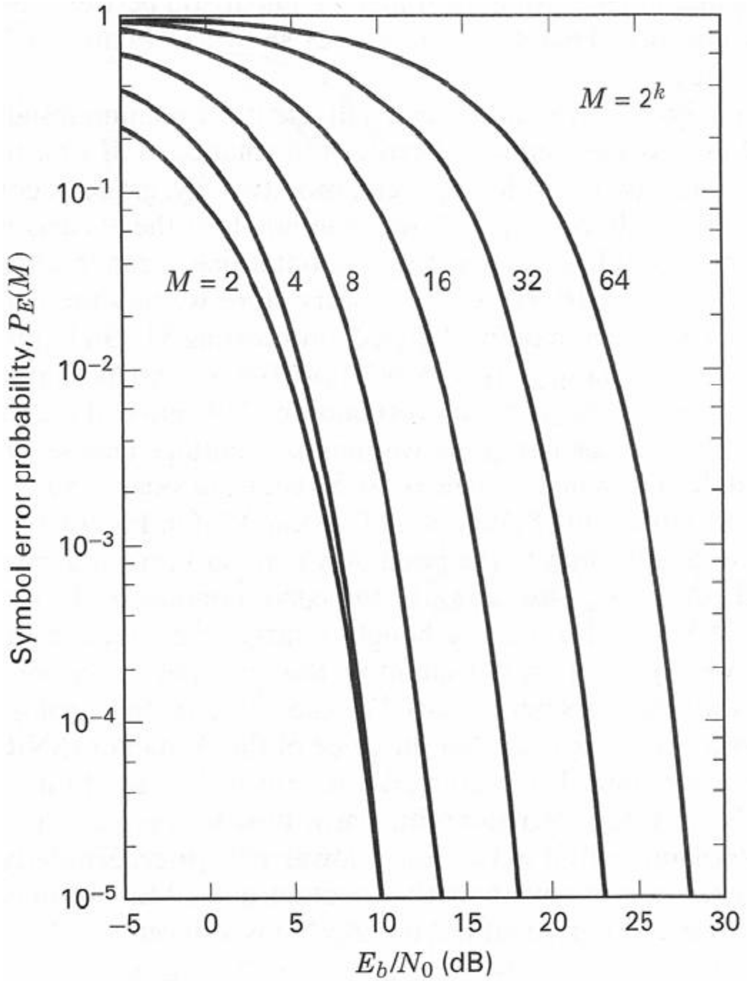
Coherent detection PSK > coherent detection of differentially encoded PSK > Differential detection of differentially encoded PSK (DPSK) > Coherent detection of orthogonal FSK > Noncoherent detection of orthogonal FSK.

ERROR PROBABILITY: MPSK

- **Symbol error rate for MPSK (Fig. 4.35)**
  - Symbol error rate (SER): # of error symbols/# of symbols transmitted

$$P_E(M) \approx 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\left(\frac{\pi}{M}\right)\right)$$

  - $E_s = E_b \log_2 M$: symbol energy



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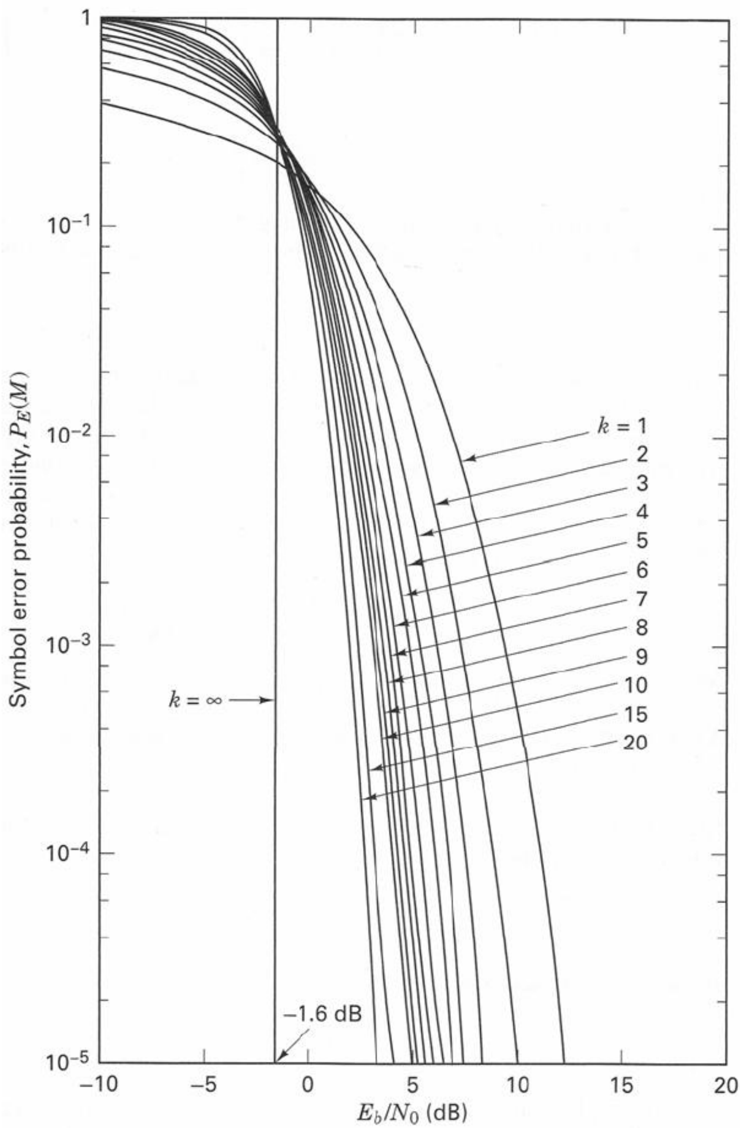
ERROR PROBABILITY: MFSK

- **SER for MFSK**

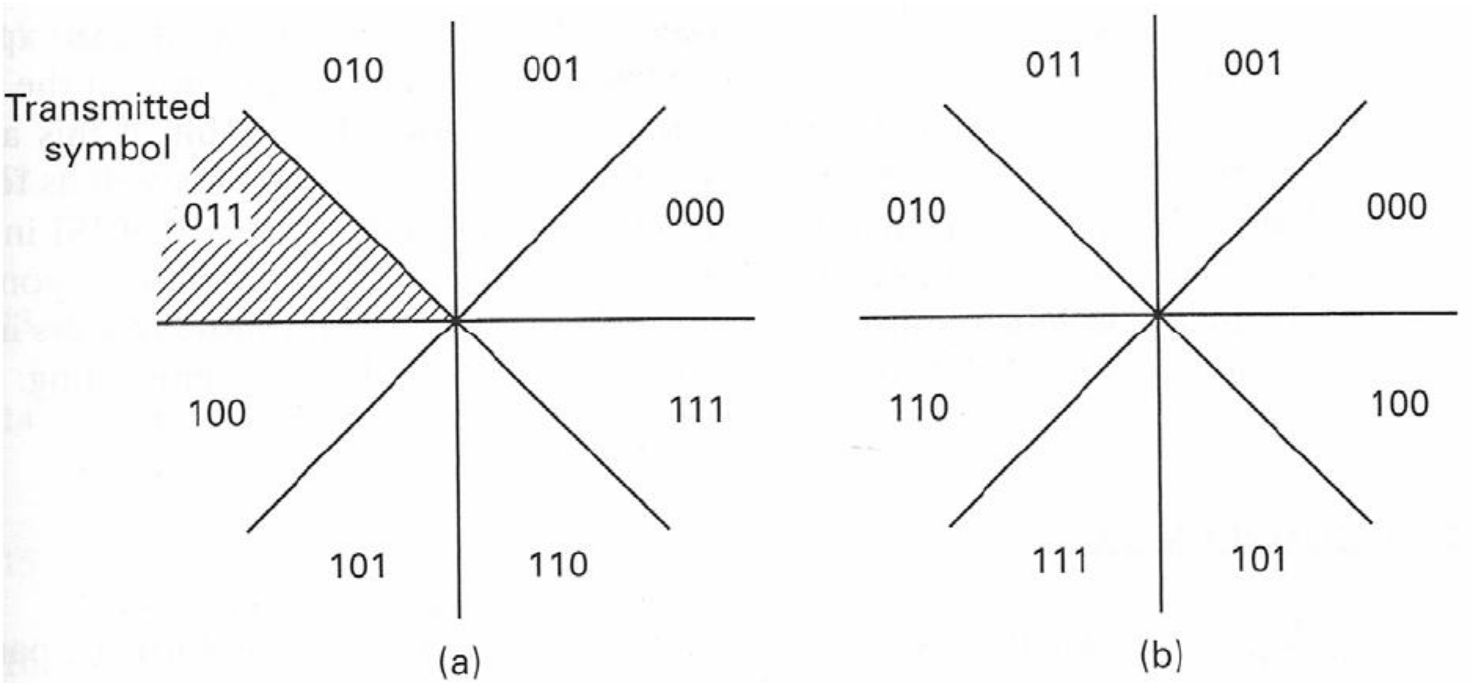
$$P_E(M) \le (M-1)Q\left(\sqrt{\frac{E_s}{N_0}}\right)$$



ERROR PROBABILITY: BER V.S. SER

- **Relationship between BER and SER for MPSK (Fig. 4.39)**
  - Gray encoding: two adjacent symbols differ in 1 bit
    - At high SNR, Most of the errors are the confusion between adjacent symbols
    - At high SNR, 1 symbol error approximately corresponds to 1 bit error

$$P_B \approx \frac{1}{\log_2 M} P_E$$



ERROR PROBABILITY: BER V.S. SER

- **Relationship between BER and SER for orthogonal signals**

$$P_B = \frac{M/2}{M-1} P_E$$

  - The relationship is exact