ELEG 5633 Detection and Estimation
Project 2: MMSE Estimation

1 Problem

Consider

$$x_n = a\cos(2\pi f_0 n) + b\sin(2\pi f_0 n) + w_n, n = 0, 1, \ldots, N-1$$

where $f_0 = 1/N$, and $w[n]$ is WGN with variance σ^2. It is desired to estimate $\boldsymbol{\theta} = [a, b]^T$, under the assumption that $\boldsymbol{\theta} \sim \mathcal{N}(\mathbf{0}, \sigma_\theta^2 \mathbf{I})$, and $\boldsymbol{\theta}$ is independent of $w[n]$.

2 Theory

1. Find the MMSE estimator of $\boldsymbol{\theta}$.
2. Find the error covariance matrix Σ_e and BMSE $\sigma_e^2 = \mathrm{tr}\Sigma_e$, as a function of σ_θ^2 and σ^2.

3 Procedures

3.1 Impacts of SNR

Set $\sigma_\theta^2 = 1$. Find the simulated BMSE at $\gamma = \frac{\sigma_\theta^2}{\sigma^2} = 0 : 5 : 20$ dB with $N = 10$. Plot the analytical BMSE and simulated BMSE in the same figure as a function of SNR (in dB).

Hints:

1. For each value of SNR, get the simulated BMSE by averaging over 10,000 trials.

2. For each trial, randomly generate a, b and w_n. That is, the values of a, b, and w_n should be different at different trials.

3. Plot the BMSE in log-scale by using the Matlab function semilogy.

4. Use a curve to represent the analytical BMSE and marks (such as '*') to represent simulation results.

5. turn on the grids in the figure by using the Matlab function "grid on"

3.2 Impacts of N

Find the simulated BMSE at $N = 5 : 5 : 20$ with $\sigma_\theta^2 = 1$ and $\gamma = 5$ dB. Plot the analytical BMSE and simulated BMSE in the same figure as a function of N.