ELEG 5633 Detection and Estimation
Project 1: Neyman-Pearson Test

Consider N i.i.d. samples generated from one of the two hypotheses

$$\begin{aligned} H_0 :& \quad X \sim \mathcal{N}(0, \sigma_0^2) \\ H_1 :& \quad X \sim \mathcal{N}(0, \sigma_1^2) \end{aligned}$$

where $\sigma_1^2 > \sigma_0^2$.

1 Project Procedures

1. Design the Neyman-Pearson (NP) test under the constraint $P_{\mathrm{FA}} \leq \alpha$, and find P_{D} (Homework 5, Problem 2).

2. Implement the NP-test in the form of a Matlab function.

   - Input: the observed sample vector X of length N, σ_0^2, σ_1^2, and α.
   - Output: the binary decision.

3. Use simulation, find P_{D} when $N = 10, \sigma_0^2 = 1$, $\sigma_1^2 = 4$, and $\alpha = [0 : 0.1 : 1]$. (See Hints below for more detailed simulaton descriptions.)

4. Repeat Step 3 with $N = 1$ and $N = 20$. Plot the ROC curves with $N = 1$, $N = 10$, $N = 20$ in the same figure. Plot both the simulation and analytical ROC curves. Explain your results (e.g. does the simulation results match the analytical curves? what are the trend of the curves and why?).

5. Repeat Step 3 with $(\sigma_0^2 = 1, \sigma_1^2 = 8)$, and with $(\sigma_0^2 = 1, \sigma_1^2 = 12)$. Plot the ROC curves with $\sigma_1^2 = 4, 8$, and 12 in the same figure. Plot both the simulation and analytical ROC curves. Explain your results.

6. Use simulation, find P_{FA} when $N = 10, \sigma_0^2 = 1$, $\sigma_1^2 = 4$, and $\alpha = [0 : 0.1 : 1]$. Plot the simulation P_{FA} as a function of α. Explain your results.

2 Hints:

1. Use the Matlab function randn to generate i.i.d. Gaussian random variables.

2. In the simulation, for each value of N and σ_1^2, you should generate at least $M = 1,000$ independent length-N sample vectors. As a result, you will need to call the NP-test function M times, one for each sample vector, and get M decisions. Calculate P_{D} by using the results from the M trials.