

ELEG 5633: Detection and Estimation

Homework 9

1. For the posterior PDF

$$p(\theta|x) = \frac{\epsilon}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(\theta - x)^2\right] + \frac{1 - \epsilon}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}(\theta + x)^2\right]$$

- (a) when $\epsilon = 1/2$, plot the pdf with Matlab for $x = 0.5$ and $x = 1.5$, respectively, and find the MMSE estimator.
 - (b) when $\epsilon = 3/4$, plot the pdf with Matlab for $x = 0.5$ and $x = 1.5$, respectively, and find the MMSE estimator.
2. Given μ , data $x[n]$, $n = 0, 1, \dots, N$ are i.i.d observations generated according to $x[n]|\mu \sim \mathcal{N}(\mu, \sigma^2)$. Assume σ^2 is deterministic and known, μ has prior PDF $\mu \sim \mathcal{N}(\mu_0, \sigma_0^2)$. Find the MAP estimators of μ .
 3. Given μ , data $x[n]$, $n = 0, 1, \dots, N$ are i.i.d observations generated according to $x[n]|\mu \sim \mathcal{N}(\mu, \sigma^2)$. Assume σ^2 is deterministic and known, μ has prior PDF $\mu \sim \mathcal{N}(\mu_0, \sigma_0^2)$.
 - (a) Find the MMSE estimator of μ .
 - (b) Comparing the MMSE estimator with the MAP estimator.
 - (c) What happens as $\sigma_0^2 \rightarrow 0$ and $\sigma_0^2 \rightarrow \infty$?