

ELEG 5633: Detection and Estimation

Homework 8

1. We wish to detect a DC level with unknown amplitude A in WGN, i.e.,

$$\begin{aligned}H_0 : x[n] &= w[n], \quad n = 0, 1, \dots, N - 1 \\H_1 : x[n] &= A + w[n], \quad n = 0, 1, \dots, N - 1\end{aligned}$$

Assume that $A < 0$ and $w[n]$ has variance σ^2 . Derive the NP test. If, in fact, $A > 0$, find P_D as a function of A .

2. Consider the detection problem

$$\begin{aligned}H_0 : x[n] &= w[n], \quad n = 0, 1, \dots, N - 1 \\H_1 : x[n] &= A + w[n], \quad n = 0, 1, \dots, N - 1\end{aligned}$$

where A is unknown with $A \neq 0$, $w[n]$ is WGN with variance σ^2 . Show that the GLRT has detection probability

$$P_D = Q \left(Q^{-1} \left(\frac{P_{FA}}{2} \right) - \sqrt{\frac{NA^2}{\sigma^2}} \right) + Q \left(Q^{-1} \left(\frac{P_{FA}}{2} \right) + \sqrt{\frac{NA^2}{\sigma^2}} \right)$$