# ELEG 5633: Detection and Estimation
# Homework 7

1. Find the matrix prewhitener $\mathbf{D}$ for the covariance matrix $\mathbf{C} = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$.

2. A binary communication system with $N = 2$ employs $s_0 = [1 \ -1]^T$ and $s_1 = [1 \quad 1]^T$. The received signal is embedded in WGN with variance $\sigma^2 = 1$. Draw the decision regions in $\mathbb{R}^2$ to minimize $P_e$. Do not assume that $P(H_0) = P(H_1)$. Explain your results.

3. Find the NP detector for the problem of a random signal $s[n]$ with mean zero and covariance matrix $\mathbf{C}_s = \text{diag}(\sigma_1^2, \sigma_2^2, \ldots, \sigma_{N-1}^2)$ embedded in WGN with variance $\sigma^2$. Assume that the data samples observed are $x[n]$ for $n = 0, 1, \ldots, N-1$

4. We wish to detect the $N \times 1$ Gaussian random signal $\mathbf{s}$ with zero mean and covariance matrix $\mathbf{C}_s = (\sigma_s^2/2)(\mathbf{1}\mathbf{1}^T + \mathbf{1}^-(\mathbf{1}^-)^T)$ in WGN with variance $\sigma^2$. The vectors are defined as $\mathbf{1} = [1, 1, \ldots, 1]^T$ and $\mathbf{1}^- = [1, -1, 1, -1, \ldots, -1]^T$ (assume $N$ is even). If $N = 10$, plot $P_D$ versus $\sigma_s^2/\sigma^2$ for $P_{\text{FA}} = 10^{-2}$. Hint: the eigenvectors of $\mathbf{C}_s$ with nonzero eigenvalues are $\mathbf{1}/\sqrt{N}$ and $\mathbf{1}^-/\sqrt{N}$.