

ELEG 5633: Detection and Estimation

Homework 6

1. Consider a binary hypothesis:

$$H_0 : X \sim \text{Bernoulli}(0.3)$$

$$H_1 : X \sim \text{Bernoulli}(0.6)$$

- Find $D(P_1||P_0)$ and $D(P_0||P_1)$
 - Based on a sequence of i.i.d. observations x_1, \dots, x_n , design the ML test
 - What is the upper bound of P_{MD} ?
 - How many i.i.d. samples do we need to achieve $P_{\text{MD}} < 0.1$?
2. It is desired to detect the known signal $s[n] = Ar^n$ for $n = 0, 1, \dots, N - 1$ in WGN with variance σ^2 . Find the NP detector and its detection performance. Explain what happens as $N \rightarrow \infty$ for $0 < r < 1$, $r = 1$ and $r > 1$.
3. A radar signal $s[n] = A \cos 2\pi f_0 n$ for $n = 0, 1, \dots, N - 1$ is received embedded in white Gaussian noise with variance σ^2 . A detector is to be designed that maintains $P_{FA} = 10^{-8}$. If $f_0 = 0.25$ and $N = 25$, find the probability of detection versus A .
4. We wish to design a signal for the best detection performance in WGN. Two competing signals are proposed. They are

$$s_1[n] = A, \quad n = 0, 1, \dots, N - 1$$

$$s_2[n] = A(-1)^n, \quad n = 0, 1, \dots, N - 1$$

Which one will yield the better detection performance?