ELEG 5633: Detection and Estimation
Homework 4

1. Based on an observation of $X = [X_1, X_2]^T$, we wish to distinguish between two hypotheses

$$\begin{aligned} H_0 : &\quad X \sim \mathcal{N}(0, \sigma^2 I_{2\times 2}) \\ H_1 : &\quad X \sim \mathcal{N}(\mu, \sigma^2 I_{2\times 2}) \end{aligned}$$

   where $\mu = [\mu_1, \mu_2]^T$. If $\pi_0 = \pi_1$, find the decision regions that minimize P_e. Manually sketch the decision region if $\mu_1 = \mu_2 = 1$.

2. Consider binary hypothesis

$$\begin{aligned} H_0 : &\quad X \sim \mathcal{N}(0, 1) \\ H_1 : &\quad X \sim \mathcal{N}(0, 2) \end{aligned}$$

   (a) Find the MAP decision rule, which should be expressed as a function of $\pi_0 = \Pr(H_0)$.

   (b) Find the decision region if $\pi_0 = 1/2$. Find the probability of error P_e.

   (c) Find the decision region if $\pi_0 = 3/4$. Find the probability of error P_e.

   (d) Briefly explain the differences between the two decision regions in (b) and (c).

3. Consider an observed sample $X \sim \mathcal{N}(\mu, 1)$ with a binary hypotheis $H_0 : \mu = 0$ versus $H_1 : \mu = 1$.

   (a) Determine the Neyman-Pearson (NP) test for the hypothesis. Assume the constraint is $P_{\text{FA}} \leq \alpha$.

   (b) Express P_{D} as a function of the P_{FA} constraint α.

   (c) Use Matlab to plot P_{D} versus $P_{\text{FA}} = \alpha$.