ELEG 5633: Detection and Estimation
Homework 3

1. Let $x_1 = \begin{bmatrix} \sqrt{2} \\ 0 \\ \sqrt{2} \end{bmatrix}$ and $x_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, find the angle θ between the two vectors.

2. Let $x_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $x_3 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$. Answer the following questions and explain why.

   a) Are they linearly independent?
   b) Are they orthogonal to each other?
   c) Is $\{x_1, x_2, x_3\}$ a basis for $\mathbb{R}^3$?
   d) Is $\{x_1, x_2, x_3\}$ an orthonormal basis for $\mathbb{R}^3$? If not, convert it to an orthonormal basis using Gram-Schmidt Orthogonalization.
   e) What is the projection of x_3 on the subspace spanned by $\{x_1\}$? $\{x_2\}$? $\{x_1, x_2\}$? Verify that the difference between the projection and x_3 is orthogonal to the corresponding subspace.

3. Let $X \sim \mathcal{N}(0, \Sigma)$, where $\Sigma = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$. Find the eigen-decomposition of Σ.