ELEG 5633: Detection and Estimation

Homework 2

1. Suppose that $X = [X_1, X_2, X_3]^T \sim \mathcal{N}(\mu, \Sigma)$ with $\mu = [1, 5, 2]^T$ and

$$\Sigma = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix} \tag{1}$$

   Find the pdfs of

   a) X_1,
   b) $2X_1 + X_2 + X_3$,
   c) X_3 given $[X_1, X_2]^T$,
   d) $[X_2, X_3]^T$ given X_1,
   e) $Y = \mathbf{A}X$, where $\mathbf{A} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$.

   Find

   f) $\mathbb{P}(2X_1 + X_2 + X_3 < 0)$. Express this probability in terms of $\Phi(x)$, the cdf of a standard normal RV.

2. Show that the following collection of random variables is a vector space. (**Hint:** use Cauchy-Schwarz inequality for the first property)

$$L_2(\Omega) := \left\{ X : \mathbb{E}[X^2] < +\infty \right\}$$

3. Given two random variables X, Y. Prove that their correlation coefficient ρ_{XY} satisfies $|\rho_{XY}| \leq 1$. (**Hint:** use Cauchy-Schwarz inequality.)