ELEG 5633: Detection and Estimation
Homework 10

1. Suppose we model our detected signal as $\mathbf{x} = \mathbf{H}\boldsymbol{\theta} + \mathbf{w}$, where $\mathbf{x} \in \mathbb{R}^n$, $\boldsymbol{\theta} \in \mathbb{R}^k$, $\mathbf{H}_{n\times k}$ is a known linear transformation, and $\mathbf{w}$ is a noise process. Furthermore assume that $\mathbb{E}[\mathbf{w}] = \mathbf{0}$, $\mathbb{E}[\mathbf{w}\mathbf{w}^T] = \sigma_w^2\mathbf{I}_{n\times n}$, $\mathbb{E}[\boldsymbol{\theta}] = \mathbf{0}$, $\mathbb{E}[\boldsymbol{\theta}\boldsymbol{\theta}^T] = \sigma_\theta^2\mathbf{I}_{k\times k}$. In addition, assume we know that the parameter and the noise process are uncorrelated, i.e., $\mathbb{E}[\boldsymbol{\theta}\mathbf{w}^T] = \mathbb{E}[\mathbf{w}\boldsymbol{\theta}^T] = 0$. Find the LMMSE estimator and the corresponding BMSE by using the orthogonality principal.

2. If we observe N i.i.d samples from a Bernoulli experiment (coin toss) with the probabilities

$$\mathbb{P}[x_i = 1] = p$$
$$\mathbb{P}[x_i = 0] = 1 - p$$

   find the MLE of p.

3. Find N i.i.d observations from a $\mathcal{N}(0, 1/\theta)$ pdf, where $\theta > 0$. Find the MLE of θ.