

ELEG 5633: Detection and Estimation: Homework 1

1. Two dice, one white with black dots and the other black with white dots, are tossed. The number of dots facing up on each die is counted and noted, recording the number of dots on the black die first and the number of dots on the white die second.

- Find the sample space.
- Find the set A corresponding to the event “the total number of dots showing is even”.
- Find the set B corresponding to the event “both dice are even”.
- Does A imply B or does B imply A ? Find $A \cap B^c$ and describe this event in words.
- Let C be the event “the number of dots on the two dice differ by one”. Find $A \cap C$.

2. Let $\mathbb{P}(A) = 0.7$, $\mathbb{P}(B^c) = 0.4$, and $\mathbb{P}(A \cup B) = 0.7$. (Note that $A \cup A^c = \Omega$, and $\mathbb{P}(A) = 1 - \mathbb{P}(A^c)$.)

Find

- $\mathbb{P}(A^c|B^c)$.
- $\mathbb{P}(B^c|A)$.

3. Consider a communication system with a binary communication channel. Denote the input to the channel as X and the output of the channel as Y . The transition probability of the channel is: $\mathbb{P}(Y = 1|X = 0) = 0.1$ and $\mathbb{P}(Y = 0|X = 1) = 0.2$. Assume that the inputs are equiprobable as $\mathbb{P}(X = 0) = \mathbb{P}(X = 1) = 0.5$.

- Find the probability that the output is 0.
- Find the probability that the input was 0 given that the output is 1.

4. Consider two RVs X, Y with joint pdf

$$p_{XY}(x, y) = \begin{cases} x + y, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

- Find $p_{X|Y}(x|y)$
- Find $p_X(x)$
- Find the covariance σ_{XY}
- Are they independent?
- Are they uncorrelated?