ELEG 4603/5173L Lab # 7
Discrete Fourier Transform

1. Write a function of DFT and IDFT, respectively, by using the matrix representation. Calculate the DFT and IDFT of the following sequences, and compare your results with the Matlab built-in function `fft()`.

   (a) $x(n) = [3, 5, 7, 2, 1, 6]$;

   (b) $x(n) = \exp(-n/3), n = 0, 1, \cdots, 15$

2. The frequency resolution domain resolution of DFT can be improved by increasing the length of the time domain signal. For an aperiodic signal with finite support, the frequency domain resolution can be improved by padding zeros to the end of the signal. Consider the following aperiodic signal

$$x(n) = u(n) - u(n - 7)$$

   (a) Find the DFT of $x(n)$ by using $N = 10$ and your own DFT function. Plot the amplitude of the DFT as a function of the digital frequency $\Omega_k = \frac{2\pi k}{N}$.

   (b) Find the DFT of $x(n)$ by using $N = 20$ and your own DFT function. Plot the amplitude of the DFT as a function of the digital frequency $\Omega_k = \frac{2\pi k}{N}$.

   (c) Find the DFT of $x(n)$ by using $N = 100$ and your own DFT function. Plot the amplitude of the DFT as a function of the digital frequency $\Omega_k = \frac{2\pi k}{N}$.

   (d) Find the frequency resolutions of the above DFTs.

3. For a periodic signal, the frequency domain resolution can be imporved by including multiple periods of the time domain signal in the DFT. Consider a periodic signal with fundamental period being $N_0 = 10$. The signal in one period is defined as

$$x(n) = \begin{cases} 1, & 0 \leq n \leq 6 \\ 0, & 7 \leq n \leq 9 \end{cases}$$

   (a) Compute the DFT of $x(n)$ by using exactly one period of $x(n)$. Plot the amplitude of the DFT as a function of the digital frequency $\Omega_k = \frac{2\pi k}{N}$.

   (b) Compute the DFT of $x(n)$ by using two periods of $x(n)$. Plot the amplitude of the DFT as a function of the digital frequency $\Omega_k = \frac{2\pi k}{N}$.

   (c) Compute the DFT of $x(n)$ by using ten periods of $x(n)$. Plot the amplitude of the DFT as a function of the digital frequency $\Omega_k = \frac{2\pi k}{N}$.

   (d) Find the frequency resolutions of the above DFTs.

   (e) Comparing the results in this question and the previous question, comment on the difference between the spectra of periodic and aperiodic signals.

4. The sampling rate of an analog signal determines the frequency range of DFT. Consider the following aperiodic analog signal

$$x(t) = u(t) - u(t-1)$$

   If we sample the signal with a sampling period of T_s, then the discrete-time signal is $x(n) = x(nT_s)$

   - Set the sampling frequency to be $f_s = \frac{1}{T_s} = 10$ Hz. Compute the DFT of $x(n)$ with $N = 100$, and plot the DFT as a function of the linear analog frequency $f_k = \frac{k}{N} f_s$. What is the highest frequency represented by DFT?
   - Set the sampling frequency to be $f_s = \frac{1}{T_s} = 20$ Hz. Compute the DFT of $x(n)$ with $N = 100$, and plot the DFT as a function of the linear analog frequency $f_k = \frac{k}{N} f_s$. Whati is the highest freuqency represented by DFT?

- Set the sampling frequency to be $f_s = \frac{1}{T_s} = 20$ Hz. Compute the DFT of $x(n)$ with $N = 200$, and plot the DFT as a function of the linear analog frequency $f_k = \frac{k}{N} f_s$. Whati is the highest freuqency represented by DFT?
- What are the frequency resolutions in the above DFTs?
- Comment how the sampling frequency and N affect the freuquency range and frequency resolution.