ELEG 4603/5173L Lab # 3
Z-Transform Part I

1. (a) Read the help file of the Matlab function `residuez()` and `filter()`.

   (b) Read, repeat, and understand the following example, which performs inverse Z-transform of the following signal with partial fraction expansion.

$$X(z) = \frac{1 + z^{-1} + 2z^{-2}}{1 - 0.25z^{-2}} \quad (1)$$

```
% the unit impulse function defined as an inline function
uimpulse = inline('double(n==0)');

% the unit step function defined as an inline function
ustep = inline('double(n>=0)');

% numerator coeffcients
b = [1, 1, 2];

% denominator coefficients
a = [1, 0, -0.25];

 % partial fraction expansion
[r, p, k] = residuez(b, a)

% find the poles
z = roots(b);

```

```matlab
% plot the poles and zeros
figure(1);
zplane(b, a);

%-----------------------------------
% Plot x(n): Method 1: Inverse Z-Transform
% Based on the partial fraction expansion,
% find the expression of x(n) and plot it
% the following code is based on the results
% r = [5.5, 3.5], p = [0.5, -0.5], k = -8
n = [-5:10];
xn = 5.5*(0.5).^n.*ustep(n)+3.5*(-0.5).^n.*ustep(n)-8*uimpulse(n);
figure(2);
stem(n, xn);
hold on;

%-----------------------------------
% Plot x(n): Method 2: Filter
% we can alternatively find x(n) by performing convolution
% between x(n) and delta(n), i.e., passing detla(n) through
% a system with
% transfer function defined by b and a
xn_2 = filter(b, a, uimpulse(n));
stem(n, xn_2, 'r+:-');
```

2. (a) Find the inverse Z-transform of the following signals by using partial fraction expansion.
   (b) Plot the poles and zeros on the z-plane.
   (c) Plot the time domain signal x(n) with both of the two methods in the example

$$X(z) = \frac{8 - 4z^{-1}}{z^{-2} + 6z^{-1} + 8} \tag{2}$$

$$X(z) = \frac{z^3 + 1}{z^3 - z^2 - z - 2} \tag{3}$$

$$X(z) = \frac{0.6z^2}{z^3 - 0.5z^2 - 0.25z + 0.125} \tag{4}$$

3. The difference equation representation of a discrete-time LTI system is

$$y(n) - 1.94y(n-1) + 0.94y(n-2) = 4.43 \times 10^{-4}x(n) + 8.86 \times 10^{-4}x(n-1) + 4.43 \times 10^{-4}x(n-2) \tag{5}$$

   (a) Manually find the transfer function
   (b) Plot the impulse response by using the inverse-Z transform method
   (c) Plot the impulse response on the same figure by using the filter method.
   (d) If the input is $x(n) = u(n) - u(n-100)$, plot $x(n)$ and $y(n)$ for `n = [0: 200]`. Based on the output, is this a low pass filter or high pass filter?
   (e) Load music.wav from Lab 2, and pass it through the filter by using the filter method. Listen to the output to verify whether it is a low pass or high pass filter.