Digital Signal Processing Assignment # 8

1. Consider a continuous-time signal $x_a(t)$ with Fourier transform

$$X_a(\omega) = \begin{cases} 1, & |\omega| \leq 200\pi, \\ 0, & |\omega| > 200\pi \end{cases} \tag{1}$$

   Define $x_s(t) = x_a(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)$, and $x(n) = x_a(nT)$. Denote $X_s(\omega)$ as the Fourier transform of $x_s(t)$, and $X(\Omega)$ as the DTFT of $x(n)$. Manually plot the following frequency domain signals.

   (a) $X_a(\omega)$.

   (b) If $T = 2.5$ ms, plot $X_s(\omega)$ and $X(\Omega)$.

   (c) If $T = 5$ ms, plot $X_s(\omega)$ and $X(\Omega)$.

   (d) If $T = 8$ ms, plot $X_s(\omega)$ and $X(\Omega)$.

   (e) In order to avoid aliasing, what is the maximum value of T?

2. Consider a discrete-time signal $x(n) = x_a(nT)$. Prove $X(\Omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X_a\left(\frac{\Omega}{T} - n\frac{2\pi}{T}\right)$, where $X(\Omega)$ is the DTFT of $x(n)$, and $X_a(\omega)$ is the Fourier transfor of $x_a(t)$.

3. An analog signal with spectrum

$$X_a(\omega) = \begin{cases} 1 - \frac{|\omega|}{1000}, & |\omega| \leq 1000, \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

   is sampled at a frequency $\omega_s = 10,000$ rad/s.

   (a) Sketch the DTFT of $x(n) = x_a(nT)$, where $T = \frac{2\pi}{\omega_s}$.

   (b) If the signal is decimated by a factor M, what is the largest value of M that can be used without introducing aliasing?

(c) Sketch the DTFT of the decimated signal if $M = 4$.

(d) The decimated signal in (c) is processed by an interpolator to obtain a sampling frequency of 7500 rads/s. Sketch the spectrum of the DTFT of the interpolated signal.