Digital Signal Processing Assignment # 7

1. Consider an LTI system with frequency response

$$H(\Omega) = \begin{cases} 1, & |\Omega| \leq \Omega_0, \\ 0, & \Omega_0 < |\Omega| \leq \pi \end{cases} \tag{1}$$

   (a) Find the impulse response.

   (b) Assume $\Omega_0 = 0.125\pi$. If the input signal is obtained by sampling $x_a(t) = 2\cos(50\pi t) + 3\sin(150\pi t + 6)$ with a sampling frequency 800 Hz. Find the output.

   (c) If the sampling frequency is 1.6 KHz, and all the other parameters are the same as (b), find the output of the system.

2. Consider an analog signal, $x_a(t)$, with spectrum

$$X_a(\omega) = \begin{cases} 1 - \frac{|\omega|}{1000\pi}, & |\omega| \leq 1000\pi, \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

   is sampled at a frequency $F_s = 1/T_s$ Hz. The sampled signal is denoted as $x_s(t) = x_a(t)\sum_{n=\infty}^{\infty}\delta(t - nT_s)$. The corresponding discrete-time signal is $x(n) = x_a(nT_s)$. Denote the Fourier transform of $x_s(t)$ as $X_s(\omega)$ and the DTFT of $x(n)$ as $X(\Omega)$. Manually sketch the spectrum of the following signals.

   (a) $X_a(\omega)$.

   (b) If $F_s = 2000$ Hz, $X_s(\omega)$ and $X(\Omega)$.

   (c) If $F_s = 1000$ Hz, $X_s(\omega)$ and $X(\Omega)$.

   (d) If $F_s = 750$ Hz, $X_s(\omega)$ and $X(\Omega)$.

3. The frequency response of a discrete-time system is

$$H(\Omega) = \frac{\frac{1}{2} + \frac{1}{12}\exp(-j\Omega)}{1 + \frac{5}{6}\exp(-j\Omega) + \frac{1}{6}\exp(-j2\Omega)} \tag{3}$$

(a) Find the impulse response

(b) Find the difference equation representation of the system.