ELEG 3143 Assignment # 14

1. A discrete-time random process is defined by $X(n) = Y^{2n+1}$, where Y is uniformly distributed in (0, 1).
   (a) Find the CDF of $X(n)$.
   (b) Find the mean function and variance function of $X(n)$.
   (c) Find the autocorrelation function of $X(n)$.

2. A stationary random process $X(t)$ has an autocorrelation function $R_x(\tau) = 5e^{-5|\tau|}$. Define another random process

$$Y(t) = X(t) + bX(t - 0.1)$$

   (a) Find the autocorrelation function of $Y(t)$
   (b) Find the power of $Y(t)$.
   (c) Is $Y(t)$ WSS?

3. For each of the autocorrelation functions given below, state whether the process it represents might be WSS or cannot be WSS.
   (a) $R_x(t_1, t_2) = e^{t_1} e^{-t_2}$
   (b) $R_x(t_1, t_2) = \cos(t_1)\cos(t_2) + \sin(t_1)\sin(t_2)$
   (c) $R_x(t_1, t_2) = e^{t_1^2 - t_2^2}$
   (d) $R_x(t_1, t_2) = \frac{\sin(t_1)\cos(t_2) - \cos(t_1)\sin(t_2)}{t_1 - t_2}$

4. A random process is defined as

$$X(t) = Y\cos(\omega_0 t + \theta)$$

   where Y, ω_0, and θ are independent random variables. Assume Y has a mean of 3 and a variance of 9, θ is uniformly distributed from $-\pi$ to π, and ω_0 is uniformly distributed from -6 to 6.

(a) Find the mean $\mathbb{E}[X(t)]$.

(b) Find the autocorrelation function and mean-square value $\mathbb{E}[X^2(t)]$

(c) Is it WSS?

5. Determine the mean value and the variance of each of the random process with the following auto-correlation functions

   (a) $10e^{-\tau^2}$

   (b) $10\frac{\tau^2+8}{\tau^2+4}$

6. A random process having sample function of the form $X(t) = A + B\sin(\omega t + \theta)$, where A is an exponential RV with parameter $\lambda = 3$, B is a Gaussian RV with mean 3 and variance 4, ω is uniformly distributed between 100 and 110, and θ is uniformly distributed between 0 and 2π. All the RVs are mutually independent. Is the random process mean ergodic?

7. In the above problem, if A is a constant instead of a RV, is the random process mean ergodic?