ELEG 3143 Assignment # 13

1. A sample function from a random process is generated by rolling a die five times. During the interval from $i-1$ to i the value of the function is equal to the outcome of the i-th roll of the die.

   (a) Sketch the resulting sample function if the outcomes of the five rolls are 5, 2, 6, 4, 1.

   (b) How many different sample functions does the ensemble of this random process obtain?

   (c) What is the probability that the particular sample function observed in part (a) will occur?

   (d) What is the probability that the sample function consisting entirely of threes will occur?

2. Sample functions from a random process are described by

$$X(t) = \begin{cases} At + B, & t \geq 0 \\ 0, & t < 0 \end{cases} \tag{1}$$

   where A is a Gaussian RV with zero mean and a variance of 9, and B is a RV that is uniformly distributed between 0 and 6. A and B are independent.

   (a) Find the mean function of this process.

   (b) Find the variance function of this process.

   (c) If a particular sample function is found to have a value of 10 at $t = 2$ and a value of 20 at $t = 4$, find the value of the sample function at $t = 8$.

3. A random process has sample function in the following form

$$X(t) = A\left[\cos(2\pi f t + \Theta) + \sin(2\pi f t + \Theta)\right]$$

where A is a random variable with mean 2 and variance 4, and Θ is uniformly distributed in $[-\pi/2, 3\pi/2]$. A and Θ are independent. Find

   (a) the mean function of this process.
   (b) the variance function of this process.
   (c) the autocorrelation function of this process.
   (d) the autocovariance function of this process.

4. The random process $Z(t)$ is defined by $Z(t) = 2Xt - Y$, where X and Y are two correlated random variables with means m_X and m_Y, variances σ_X^2 and σ_Y^2, and correlation coefficient ρ. Find

   (a) The mean function of $Z(t)$
   (b) The auto-covariance function of $Z(t)$