

Department of Electrical Engineering
University of Arkansas



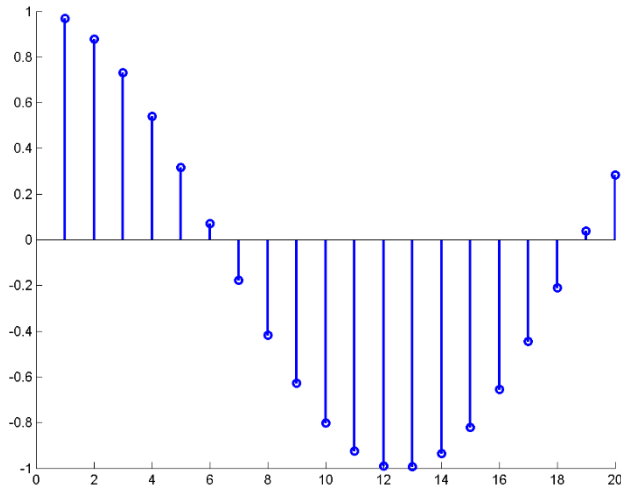
ELEG 3124 Signals & Systems

Ch. 6 Discrete-Time System

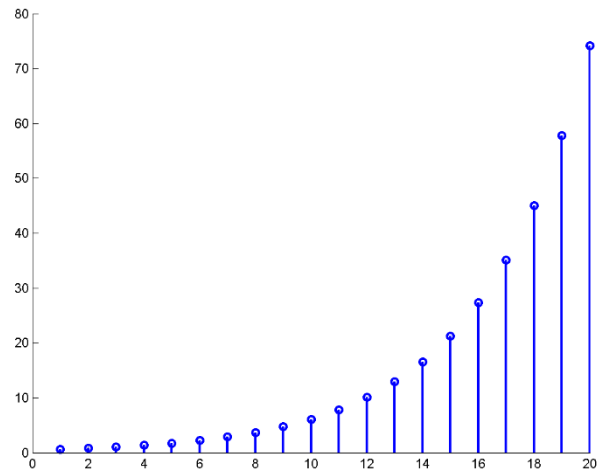
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SIGNAL

- **Discrete-time signal**
 - The time takes discrete values



$$x(n) = \cos\left(\frac{n}{4}\right)$$



$$x(n) = \frac{1}{2} \exp\left(\frac{n}{4}\right)$$

SIGNAL: CLASSIFICATION

- **Energy signal v.s. Power signal**

- Energy:

$$E = \lim_{N \rightarrow \infty} \sum_{n=-N}^N |x(n)|^2$$

- Power:

$$P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x(n)|^2$$

- Energy signal: $E < \infty$

- Power signal: $P < \infty$

SIGNAL: CLASSIFICATION

- **Periodic signal v.s. aperiodic signal**

- Periodic signal $x(n) = x(n + N)$
 - The smallest value of N that satisfies this relation is the fundamental periods.
- Is $\cos(\omega n)$ periodic?

$\cos(\omega n)$ is periodic if $\frac{2k\pi}{\omega}$ is integer for integer k .

- Example: $\cos(3n)$

$$\cos(\pi n)$$

$$\cos\left(\frac{3}{4}\pi n\right)$$

SIGNAL: ELEMENTARY SIGNAL

- **Unit impulse function**

$$\delta(n) = \begin{cases} 1, & n = 0, \\ 0, & n \neq 0. \end{cases}$$

- **Unit step function**

$$u(n) = \begin{cases} 0, & n < 0, \\ 1, & n \geq 0. \end{cases}$$

- **Relation between unit impulse function and unit step function**

$$\delta(n) = u(n) - u(n-1)$$

$$u(n) = \sum_{k=-\infty}^n \delta(k)$$

SIGNAL: ELEMENTARY SIGNAL

- **Exponential function**

$$x(n) = \exp(\alpha n)$$

- **Complex exponential function**

$$x(n) = \exp(j\omega_0 n) = \cos(\omega_0 n) + j \sin(\omega_0 n)$$

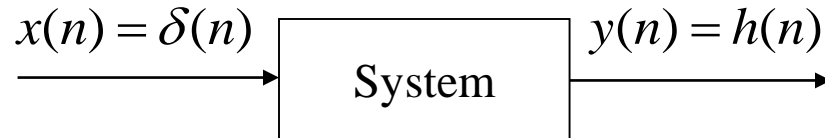
OUTLINE

- Discrete-time signals
- **Discrete-time systems**
- Z-transform

SYSTEM: IMPULSE RESPONSE

- **Impulse response of LTI system**

- The response of the system when the input is $\delta(n)$

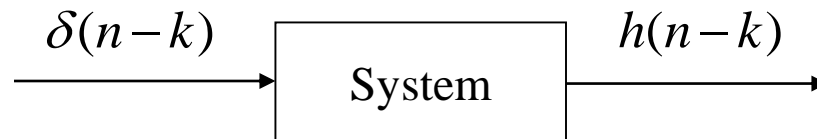


- **System response for arbitrary input**

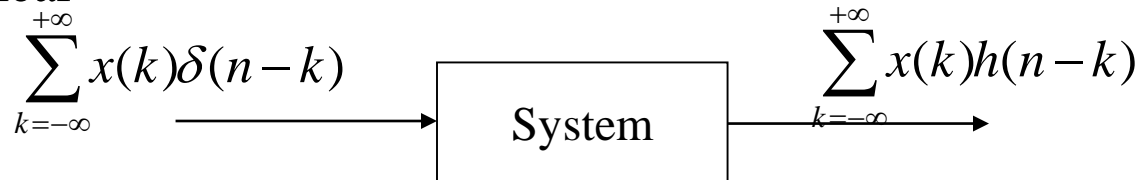
- Any signal can be decomposed as the sum of time-shifted impulses

$$x(n) = \sum_{k=-\infty}^{+\infty} x(k)\delta(n-k)$$

- Time invariant



- Linear



SYSTEM: CONVOLUTION SUM

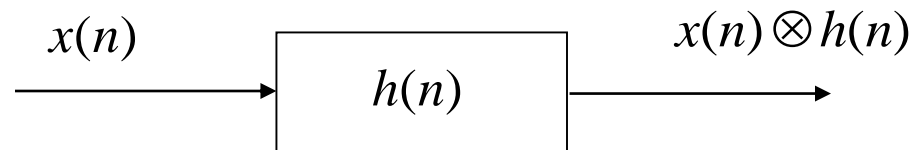
- **Convolution sum**

- The convolution sum of two signals $x(n)$ and $h(n)$ is

$$x(n) \otimes h(n) = \sum_{k=-\infty}^{+\infty} x(k)h(n-k)$$

- **Response of LTI system**

- The output of a LTI system is the convolution sum of the input and the impulse response of the system.



SYSTEM: CONVOLUTION SUM

- **Example**

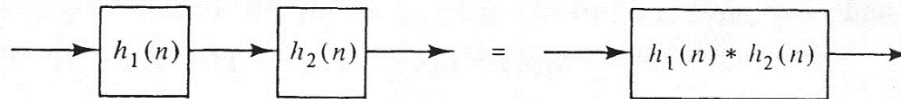
- 1. $x(n) \otimes \delta(n-m)$

- 2. $x(n) = \alpha^n u(n), \quad h(n) = \beta^n u(n)$

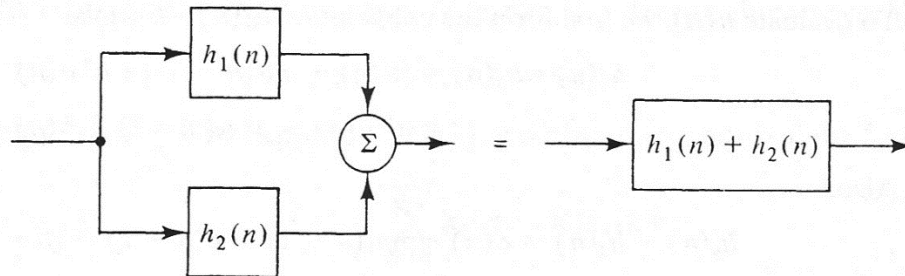
- $x(n) \otimes h(n) =$

SYSTEM: COMBINATION OF SYSTEMS

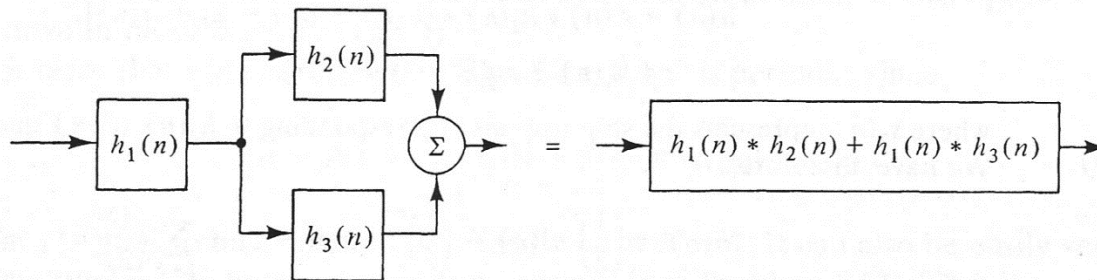
- Combination of systems



(a)



(b)



SYSTEM: DIFFERENCE EQUATION REPRESENTATION

- Difference equation representation of system

$$\sum_{k=0}^N a_k y(n-k) = \sum_{k=0}^M b_k x(n-k)$$

OUTLINE

- Discrete-time signals
- Discrete-time systems
- **Z-transform**

Z-TRANSFORM

- **Bilateral Z-transform**

$$X(z) = \sum_{n=-\infty}^{+\infty} x(n)z^{-n}$$

- **Unilateral Z-transform**

$$X(z) = \sum_{n=0}^{+\infty} x(n)z^{-n}$$

- **Z-transform:**

- Ease of analysis
- Doesn't have any physical meaning (the frequency domain representation of discrete-time signal can be obtained through discrete-time Fourier transform)
- Counterpart for continuous-time systems: Laplace transform.

Z-TRANSFORM

- **Example: find Z-transforms**

- 1. $x(n) = \delta(n)$

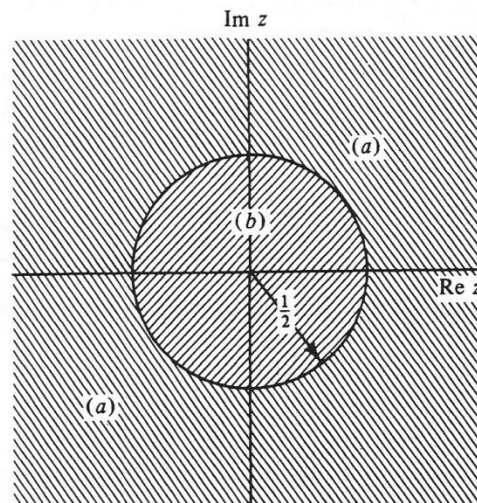
- 2. $x(n) = \left(\frac{1}{2}\right)^n u(n)$

Z-TRANSFORM

- **Example**

- 3. $x(n) = -\left(\frac{1}{2}\right)^n u(-n-1)$

- **Region of convergence (ROC)**



Z-TRANSFORM: CONVERGENCE

- **Convergence of causal signal**

$$x(n) = \alpha^n u(n)$$

- **Convergence of anti-causal signal**

$$x(n) = \beta^n u(-n-1)$$

Z-TRANSFORM: TIME SHIFTING PROPERTY

- **Time Shifting**

- Let $x(n)$ be a causal sequence with the Z-transform $X(z)$

- Then

$$Z[x(n + n_0)] = z^{n_0} X(z) - z^{n_0} \sum_{m=0}^{n_0-1} x(m) z^{-m}$$

$$Z[x(n - n_0)] = z^{-n_0} X(z) + z^{-n_0} \sum_{m=-n_0}^{-1} x(m) z^{-m}$$

Z-TRANSFORM: LTI SYSTEM

- **LTI System**

- Difference equation representation

$$\sum_{k=0}^N a_k y(n-k) = \sum_{k=0}^M b_k x(n-k)$$

- Z-domain representation

$$\left[\sum_{k=0}^N a_k z^{-k} \right] Y(z) = \left[\sum_{k=0}^M b_k z^{-k} \right] X(z)$$

- Transfer function

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\left[\sum_{k=0}^M b_k z^{-k} \right]}{\left[\sum_{k=0}^N a_k z^{-k} \right]}$$

Z-TRANSFORM: LTI SYSTEM

- **Example**

- Find the transfer function of the system described by the following difference equation

$$y(n) - 2y(n-1) + 2y(n-2) = x(n) + \frac{1}{2}x(n-1)$$

Z-TRANSFORM: STABILITY

- **Stability**

$$H(z) = \frac{z}{z-a}$$

$$h(n) = a^n u(n)$$

- A LTI system is BIBO stable if all the poles are within the unit circle ($|a| < 1$)
- A LTI system is unstable if at least one pole is on or outside of the unit circle ($|a| \geq 1$)