ELEG 3124 Assignment # 10

1. Consider a system with transfer function $H(\omega) = \text{rect}\left(\frac{\omega}{2\omega_f}\right)$. The input signal is $x(t) = \frac{\sin(\omega_1 t)}{t} + \frac{\sin(\omega_2 t)}{t}$, where $0 < \omega_1 < \omega_2$.
   (a) Find the impuse response $h(t)$.
   (b) Find $X(\omega)$.
   (c) Find $y(t)$ is $0 < \omega_f < \omega_1$.
   (d) Find $y(t)$ is $\omega_1 < \omega_f < \omega_2$.
   (e) Find $y(t)$ is $\omega_2 < \omega_f$.

2. The Fourier transform of $x(t)$ is $X(\omega)$. The pulse train is $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$. Define $x_s(t) = x(t)p(t)$ as the sampled signal of $x(t)$ with a sampling period of T_s.
   (a) Find the Fourier transform of $x_s(t)$.
   (b) Assume the highest frequency of $x(t)$ is ω_0 and it satisfies $2\omega_0 \leq \omega_s = \frac{2\pi}{T_s}$. Pass $x_s(t)$ through a low pass filter with transfer function $H(\omega) = \text{rect}\left(\frac{\omega}{\omega_s}\right)$, what is the time domain signal at the output of the filter?

3. The amplitude modulation can be represented as $s(t) = m(t)\cos(\omega_c t)$, where $m(t)$ is the message signal with the highest frequency ω_0 and $\cos(\omega_c t)$ is the carrier signal. The carrier frequency is ω_c and $\omega_c >> \omega_0$. The Fourier transform of $m(t)$ is $M(\omega)$.
   (a) Find the Fourier transform of $s(t)$.
   (b) At the receiver, the coherent demodulator will perform $r(t) = s(t)\cos(\omega_c t)$, then pass the signal through a low pass filter with transfer function $H(\omega) = \text{rect}\left(\frac{\omega}{2\omega_0}\right)$. Find the Fourier transform of $r(t)$. Find the output of the low pass filter.