# Lab 8 Fourier Transform

## I. Pre-lab

1. Consider an ideal band pass filter with the transfer function $H(\omega) = rect\left(\frac{\omega - \omega_0}{\omega_B}\right) + rect\left(\frac{\omega + \omega_0}{\omega_B}\right)$, where $\omega_0$ is the center frequency (in rad/sec) of the pass band, and $\omega_B < 2\omega_0$ is the bandwidth. Find the impulse response $h(t)$.
2. Read and understand the following example.

## II. Example

The following code calculates the symbolic Fourier transform of the rectangular function $x(t) = rect\left(\frac{t}{\tau}\right)$. The symbolic results are compared to the theorectical result.
(Please note that the symbolic function for the unit step function u(t) in Matlab is *heaviside*.)

The main program: main.m

```
% use the Matlab function fourier.m to symbolically find the
% fourier transform
syms t w

tau = 1;
% symbolic rect(t/tau)
rect_sym = heaviside(t+tau/2)-heaviside(t-tau/2);

% sybmolic F[rect(t)]
fourier_rect_sym = fourier(rect_sym);

% use subs( ) to get the numerical rect(t)
t_vec = [-3:0.01:3];
rect_t = subs(rect_sym, t, t_vec);
figure(1)
plot(t_vec, rect_t);
axis([-3 3 -0.1 1.1]);
xlabel('t (sec)');
ylabel('x(t)');

% use subs( ) to get the numerical value of the Fourier transform
w_vec = [-50:0.01:50];
fourier_rect_w = subs(fourier_rect_sym, w, w_vec);
figure(2);
% plot the amplitude
subplot(2, 1, 1);
plot(w_vec, abs(fourier_rect_w));
```

```
axis([-50, 50, -0.5 1.5]);
xlabel('\omega (rad/s)');
ylabel('|X(\omega)|')
% plot the phase
subplot(2, 1, 2);
plot(w_vec, phase(fourier_rect_w));
xlabel('\omega (rad/s)');
ylabel('phase (rads)')


% theorectical expression of the fourier transform
fourier_rect_w_theo = tau*sinc(w_vec*tau/(2*pi));
% plot the amplitude
subplot(2, 1, 1);
hold on;
plot(w_vec, abs(fourier_rect_w_theo), 'r--');
legend('symbolic', 'theorectical');
% plot the phase
subplot(2, 1, 2);
hold on;
plot(w_vec,% phase(fourier_rect_w_theo), 'r--');
legend('symbolic', 'theorectical');
```

## III. Lab Assignments

### Part A: Symbolic Fourier Transform

1. Use Matlab to perform symbolic Fourier transform calculation of the following signals. Plot both the amplitude and phase response. Plot in the same figure the theoretical Fourier transform results (use the theoretical results from the Fourier transform table).

(1) $x_1(t) = \exp(-t)u(t)$

(2) $x_2(t) = \exp(-|t|)$

(3) $x_3(t) = t\exp(-|t|)$

### Part B: Band Pass Filtering of Audio Signals

The following code shows an example of passing an audio signal through a low pass filter with cutoff frequency 1500 Hz.

```
% read the file
[data, Fs, Nbits]=wavread('female_voice.wav');
data = data(:, 1).';
% Fs: sampling frequency; Ts: sampling period
Ts = 1/Fs;

% playback the corrputed sound
sound(data, Fs);
```

```
% time vector
t = [-10:Ts:10];

% cutoff of the low pass filter is 1500 Hz
wb = 1500*2*pi;

% ideal low pass filter with cutoff frequency wb
% fourier transform: rect(w/wb)
ht = wb/(2*pi)*sinc(wb*t/(2*pi));

% input: data, LTI impulse response: ht, output: y = convolution
between data and ht
y = conv(data, ht, 'same');

% normalize the processed sournd to avoid clipping
y = y/max(abs(y));
% playback the processed sound
sound(y, Fs);
```

1. Change the cutoff frequency of the low pass filter and see how it affects the sound.
2. The file corrupted_male_voice.wav contains a male voice distorted by two tones at 100 Hz and 6000 Hz, respectively. Based on the results in the pre-lab, design a band pass filter with passband [250, 3750] Hz. What are the values of w0 and wB?
3. Pass the corrupted sound through the band pass filter, and play the output of the filter.

**Part C: Low Pass Filter**

4. Consider a system with transfer function $H(\omega) = rect\left(\frac{\omega}{\omega_f}\right)$. The input signal is $x(t) = 5sinc\left(\frac{5t}{\pi}\right) + 10sinc\left(\frac{10t}{\pi}\right)$. Write convolution in Matlab to find the signal, y(t), at the output of the filter for the following values of $\omega_f$, and compare the results with the theorectical results (you can find the theoretical results in HW 10). Use dt = 0.001 and t = [-10:dt:10].
   a) $\omega_f = 7$ rad/sec
   b) $\omega_f = 3$ rad/sec

## IV. Homework

1. Repeat the question in Part C for $\omega_f = 12$.