

Lab 6 Linear System

I. Pre-lab

1. Calculate the following convolutions by-hand
 - A. $p(t) \otimes p(t-1)$
 - B. $\exp(-t)p(t) \otimes p(t)$
 - C. $tp(t) \otimes p(t)$
2. Repeat the example in Section II. Understand the Matlab program (bring the Matlab script with you to the lab).

II. Example

The convolutional integral is approximated by using the Matlab function `conv(a,b)`, which calculates $y(m) = \sum_n a(m-n)b(n)$. We can get an approximation of $y(t) = \int x(t-\tau)h(\tau)d\tau$ as $y(m \cdot dt) \approx \sum_n x(m \cdot dt - n \cdot dt)h(n \cdot dt)dt$ if the sampling interval dt is small enough.

The following examples perform numerical convolution $p(t) \otimes p(t)$, where $p(t) = \begin{cases} 1, & -1 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$, and compares the numerical result with the theoretical results

The unit pulse function: `upulse.m`

```
function y = upulse(x)
% function y = upulse(x)
%   y = 1 if -1 <= x <= 1,
%   y = 0 otherwise
%
% the number of elements in the vector x
N_element = length(x);

% initialize y as an all zero vector
y = zeros(1, N_element);

% if -1 <= x <= 1, then y = 1
one_idx_logic = and(x >= -1, x <= 1);
y(one_idx_logic) = 1;
```

The main program: `main.m`

```

% This is the main file that performs the convolution
clear all;

% sampling interval dt
dt = 0.01;

% the time variable vector
t = [-5:dt:5];

% signal x(t)
x = upulse(t);
% signal h(t)
h = upulse(t);

% convolution between x(t) and h(t)
y = conv(x, h, 'same')*dt;

% plot the convolution result
plot(t, y)

% label the x-axis and y-axis
xlabel('time (second)');
ylabel('y(t)');
% turn the grid on
grid on;

% compare the numerical result with the theoretical result
% calculating p(t) convolving with p(t) by hand, we have the
% theoretical results as
% y(t) = t+2, -2 <= t <= 0,
% y(t) = -t+2, 0 <= t <= 2,
% y(t) = 0, otherwise

% keep the original result
hold on;
% plot the theoretical result in the same figure
t1 = [-2:dt:0];
plot(t1, t1+2, 'r--');
t2 = [0:dt:2];
plot(t2, -t2+2, 'r--');

legend('Numerical Result', 'Theoretical Result');

```

III. Lab Assignments

Part A: Convolutions

1. Use Matlab to perform numerical convolutions of the three questions in the pre-lab. For each of the convolution:
 - 1) Plot the numerical convolution results.
 - 2) Plot, in the same figure as the numerical results, the theoretical results based on your calculation.

Part B: Ideal Low Pass Filter

2. Consider a LTI system with impulse response $h(t) = 20\text{sinc}(20t)$. It is an ideal low pass filter with a cut-off frequency 10 Hz, that is, it will only allow signals with frequency less than or equal to 10 Hz to pass through. We will verify this low pass properties of the filter by passing sinusoidal signals at different frequencies through it.
 - 1) Plot $h(t)$ in the range of $-10 \leq t \leq 10$ with a sampling interval $dt = 0.001$. Rescale the figure to show the result between the range $-1 \leq t \leq 1$ (Use the Matlab function *axis*).
 - 2) If the input $x_1(t)$ is a sinusoidal signal with frequency 9 Hz, use numerical convolution to find the output (use $-10 \leq t \leq 10$ and $dt = 0.001$.) Use subplot to plot the input and output of the system, and rescale the figure with $-1 \leq t \leq 1$ and $-1.5 \leq y \leq 1.5$. Write down the amplitude.
 - 3) If the input $x_2(t)$ is a sinusoidal signal with frequency 11 Hz, repeat the above step.
 - 4) If the input is $x = x_1(t) + x_2(t)$, repeat the above step. Find the frequency of the output signal by counting the number of cycles per second.
 - 5) **Amplitude Response.** Find the amplitudes of the output when the input signal is a sinusoidal with frequencies 1 Hz. Repeat this procedure for sinusoidal signals with frequencies 3:2:15 Hz, respectively. Tabulate and plot the amplitudes as a function of the frequency. This is the amplitude response of the LTI system.

Part C: Frequency and Music Notes

3. The frequencies of the music notes are given as follows

Table 1. Note Frequencies

Note	C	D	E	F	G	A	B
Freq (Hz)	262	294	330	349	392	440	494

Each music note is a sinusoid signal with a certain frequency. The following example shows how to play a music note with Matlab.

```
% sampling frequency is 100 KHz
Fs = 100e3;
% sampling interval Ts = 1/Fs
Ts = 1/Fs;
```

```

% each note will be played for 0.5 second
t = [0:Ts:0.5];

% The frequency for C note is 262 Hz
Fc = 262;

% the sinusoidal signal with frequency Fc
notec = cos(2*pi*Fc*t);

% play the sound
sound(notec, Fs);

```

4. Write a Matlab program to play the following music piece:
CCGGAAG--, FFEEDDC--,
You can adjust the length of each note to get better result.

IV. Homework Assignments

Part A: Convolution

1. Calculate the following convolution both manually and with the Matlab numerical convolution, and plot the theoretical and numerical results in the same figure: $\exp(-t)p(t) \otimes p(t)$.

Part B: Low Pass Filter

2. An RC circuit can be considered as a low pass filter: it allows low frequency signal to pass through, and attenuates high frequency signal. The impulse response of the circuit is $h(t) = \frac{1}{RC} \exp\left(-\frac{t}{RC}\right)u(t)$. Assume $R = 1K\Omega$, $C = 20\mu F$. Following the same procedure as in Step 2(5), tabulate and plot the amplitude response of the RC circuit at frequencies [1:2:21] Hz. (use $-10 \leq t \leq 10$, $dt = 0.001$, and measure the output amplitude in $-1 \leq t \leq 1$).