Lab 5 Fourier Series

I. Pre-lab

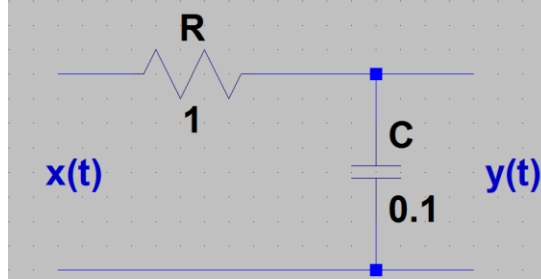1. Find the transfer function of the following RC circuit.

Fig. 1 an RC circuit

2. If the input to the above RC circuit is $x(t) = \cos(2\pi f_0 t)$, find the output $y(t)$.
3. Repeat the example in Section II. Understand the Matlab program (bring the Matlab script with you to the lab).

II. Example

The following code calculates the Fourier series of the following signal with Matlab symbolic calculation, with $T_0 = 5, \tau = 1$.

(Please note that the symbolic function for the unit step function u(t) in Matlab is *heaviside*.)
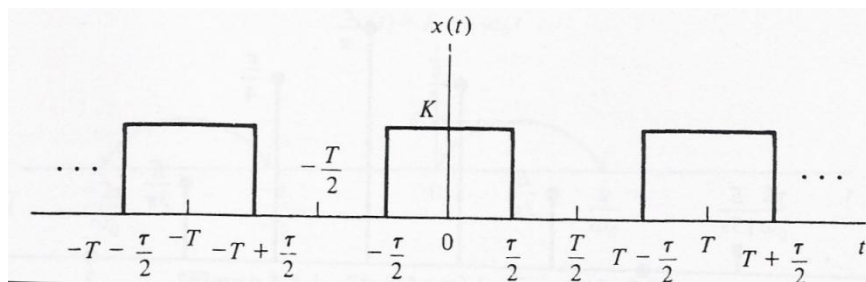

Fig. 2 rectangular pulses

The Fourier Series function: FourierSeries.m

```
function [X, w] = FourierSeries(x, T0, k_vec)
% function [X, w] = FourierSeries(x, T0, k_vec)
%
% symbolically calculate the Fourier Series, and return the
% numerical results
%
% x: the time domain signal within one period;
%    it must have definition over [0, T0]
%    it must be a symbolic function of t
```

```matlab
%
% T0: the period of the signal
% k_vec: the range of Harmonics to be calculated

syms t

for mm = 1:length(k_vec)
    k = k_vec(mm);
    % Fourier series
    X1(mm) = int(x*exp(-j*2*pi*k*t/T0), t, 0, T0)/T0
    % change the symbolic value to numerical value
    X(mm) = subs(X1(mm));
    % angular frequency
    w(mm) = k*2*pi/T0;
end
```

The main program: main.m

```matlab
% main.m
% Calculates the Fourier series through symbolic cacluations

clear all

% numerical Fourier Series
syms t;

tau = 1;
T0 = 5;
k_vec = [-3:3];

% !!!IMPORTANT!!!: the signal definition must cover [0 to T0]
% the signal is defined over [-T0, 2T0], which covers [0, T0]
xt = heaviside(t+tau/2)-heaviside(t-tau/2) + heaviside(t-(T0-tau/2))-
heaviside(t-(T0+tau/2));
[Xw, w] = FourierSeries(xt, T0, k_vec);

% plot the results from Matlab calculation
stem(w,abs(Xw), 'o-');
hold on;

% the results based on manual calculation (slides # 16)
Xw_manual = tau/T0*sinc(k_vec*tau/T0);
stem(w, abs(Xw_manual), 'r--');

legend('Matlab calculation', 'Manual calculation');
xlabel('\omega (rad/sec)');
ylabel('|c_k|');
```

III. Lab Assignments

Part A: Symbolic Fourier Series Calculation

1. Use Matlab to perform symbolic Fourier series calculation of the following signals. Plot both the amplitude and phase of the harmonics. Plot in the same figure the results from both Matlab calculation and manual calculation (use the manual calculation results from Assignment # 6, question 2).
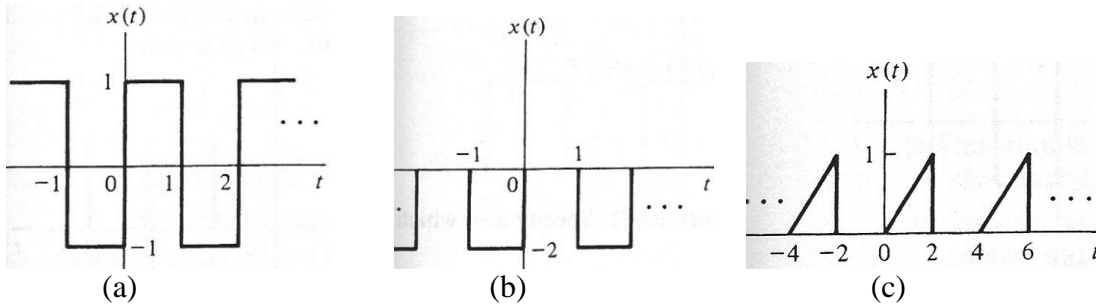

(a) (b) (c)

Fig. 3 Fourier series calculation

Part B: Gibbs Phenomenon

Let $\{c_n\}$ be the Fourier series coefficients of $x(t)$. Define

$$x_N(t) = \sum_{n=-N}^{+N} c_n e^{jn\Omega_0 t}$$ as a truncated version of the original signal. The

following code shows how to plot $x_N(t)$ for the signal in Fig. 3(a).

```
% Plot Fig. 3(a)
N = 99;
n_vec = [-N:N];
T0 = 2;
Omega0 = pi;

% the Fourier series coefficients
Cn = zeros(size(n_vec));
% odd indexed component
Cn(1:2:end) = 2./(j*n_vec(1:2:end)*pi);
% even indexed component
Cn(2:2:end) = 0;

dt = 0.01;
t = [-3:dt:3];

xt = 0;
for m = 1:length(n_vec)
    xt = xt + Cn(m)*exp(j*n_vec(m)*Omega0*t);
end
```

2. Plot $x_N(t)$ in Fig. 3 (b) and (c) as a function of time for $N = 3, 9, 19,$ and 99.

Part C: Linear Time Invariant System with Periodic Inputs

For LTI systems with periodic inputs, the output can be calculated in two ways: convolution, or transfer function. We will perform both operations in this part of the lab, and compare their results.

3. Input: $x(t) = \cos(2\pi f_0 t)$

   (a) Use convolution to find the output of the RC circuit in Fig. 1, tabulate and plot the amplitude response of the RC circuit at frequencies [1:10:100] Hz.
   (use $-1 \le t \le 1$, $dt = 0.01$, and measure the output amplitude in $-1 \le t \le 1$). Refer to Lab 4 for procedures.

   (b) Use transfer function to find the output of the RC circuit in Fig. 1, tabulate and plot the amplitude response of the RC circuit at frequencies [1:10:100] Hz.

   $$y(t) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\Omega_0 t} H(n\Omega_0)$$

   (c) Plot the amplitude and phase response of the RC transfer function.

IV. **Homework Assignments**

Part B: Gibbs Phenomenon

1. Repeat Step 2 for Fig. 3(b).

Part B: Linear Time Invariant System with Periodic Inputs

1. If the input to the RC circuit in Fig. 1 is the signal shown in Fig. 3(a), use transfer function to find the output, and plot the results between t between [-3:dt:3], where dt = 0.01. Use $x_N(t)$ and $N = 200$ to approximate x(t).