Energy Efficiency and Spectral Efficiency Tradeoff in Type-I ARQ Systems

Jingxian Wu, Member, IEEE, Gang Wang, Student Member, IEEE, and Yahong Rosa Zheng, Senior Member, IEEE

Abstract—The optimum energy efficient and spectral efficient designs for type-I automatic-repeat-request (ARQ) systems in Rayleigh flat fading channels are studied in this paper. Three optimum designs are considered: the first scheme maximizes the energy efficiency (EE), or equivalently, minimizes the total energy per information bit without considering the spectral efficiency (SE); the second scheme minimizes a new metric, the energy per information bit normalized by the SE; and the third scheme maximizes the EE under the constraint of a minimum SE. For given physical and link layer parameters, such as hardware power consumption, coding rate, modulation, bit rate, number of overhead bits, and communication distance, the three schemes are optimized with respect to the average transmission energy and the number of information bits per frame. The fundamental EE-SE tradeoff curve is analytically identified via the optimization of the third scheme, and it is shown that the optimum EE is quasiconcave in SE in a type-I ARQ system. The optimum solutions to the first two schemes are special operating points on the EE-SE tradeoff curve. Computer simulation results verify the analytical solutions.

Index Terms—Cross-layer design, ARQ, energy efficiency, spectral efficiency, green communications.

I. INTRODUCTION

ENERGY efficiency (EE) and spectral efficiency (SE) are two essential design metrics for wireless communication systems. Energy efficient communication reduces energy consumption and extends the battery life of wireless terminals [1]. On the other hand, spectral efficient communication aims at supporting more simultaneous users or achieving a higher data rate with the scarce spectrum resource. However, EE and SE are often conflicting goals, featuring one of the most fundamental tradeoffs in communication system designs. Therefore, it is imperative to understand the tradeoff and develop optimum schemes that are efficient in terms of both energy and spectrum.

A large number of works are devoted to the development of energy efficient communication systems [2]–[7]. In the physical (PHY) layer, energy efficient communication techniques are mainly developed through coding, modulation, and signal processing techniques [2] and [3]. In the medium access control (MAC) layer, the energy consumption can be reduced in a number of ways, such as decreasing the transmission duty cycle [4], or reducing collisions via careful scheduling [5], etc. A PHY/MAC cross-layer design framework is considered in [6] for hybrid automatic-repeat-request (HARQ) in fading channels, where the optimum power assignment is studied to reduce the total average power consumption. Similarly, [7] investigates the power control of incremental redundancy HARQ with constraints on packet drop probability and peak power. However, the optimization in [6] and [7] are performed under the constraint of a targeted outage probability without considering the effects of practical system parameters such as overhead, modulation, data rate, bit error rate (BER), and practical codes, etc.

Recently, the research interests have been shifted to the EE-SE tradeoff [8]–[11]. The information theoretic study in [8] identifies the fundamental tradeoff between EE and SE in the wideband and low energy regime, and it is shown that SE is a decreasing function in EE for various channels. Incorporating more practical parameters, such as circuit power and orthogonal frequency division multiple access (OFDMA), Xiong et al. discovered that the maximum EE is strictly quasiconcave in SE if there are sufficiently many subcarriers in a downlink OFDMA system [9]. A similar result is presented in [10], where it was shown that EE and SE can increase simultaneously for a band unlimited system under a rate constraint, and EE is a decreasing function in SE when the bandwidth is limited. In [11], the throughput of an incremental HARQ system is maximized under a power constraint with partial channel state information obtained through feedback. All works use the Shannon channel capacity to measure data rate without considering practical operations such as modulation, channel coding, or frame overhead, etc.

In this paper, we study the optimum energy- and spectral-efficient designs of type-I ARQ systems, where a package is retransmitted if it cannot be recovered at the receiver. Although many varieties of ARQ and HARQ systems exist that offer different tradeoffs between the performance and complexity [6], [7], [11]–[13], type-I ARQ has a wide range of applications due to its simplicity that is especially important for low cost and low energy communication systems. Three

Manuscript received April 10, 2012; revised October 31, 2012 and Feb. 17, 2013. The work of J. Wu and G. Wang was supported in part by U.S. National Science Foundation under Grant ECCS-1202075. The work of Y.R. Zheng was supported in part by the U.S. National Science Foundation under Grant ECCS-0846486. Part of the material in this paper has been presented at the IEEE Global Telecommunication Conference, Anaheim, CA, 2012.

J. Wu and G. Wang are with the Department of Electrical Engineering, University of Arkansas, Fayetteville, AR 72701, USA (e-mail: wu@uark.edu; wangi@email.uark.edu).

Y.R. Zheng is with the Department of Electrical and Computer Engineering, Missouri University of Science and Technology, Rolla, MO 65409, USA (e-mail: zhengyr@mst.edu).

Digital Object Identifier 10.1109/JSAC.2014.141215.
optimization schemes are considered for type-I ARQ in flat fading channels: the first scheme aims to minimize the energy per information bit $E_t$, which is the energy required to successfully deliver one information bit from a transmitter to a receiver; the second scheme minimizes the normalized energy per information bit $E_m = \frac{E_t}{r}$, where $r$ is the SE in bps/Hz; and the third scheme minimizes $E_t$ under the constraint of a minimum SE, $\eta_0 \leq \eta_m$, with $\eta_m$ being a constant threshold. The first scheme minimizes the overall energy consumption at the cost of the SE. The second metric $E_m$ can be minimized by either reducing $E_t$ or increasing $\eta_m$, thus providing one possible tradeoff between EE and SE. The third optimization scheme allows us to flexibly adjust the EE-SE tradeoff based on specific system requirements, and it includes the $E_t$ or $E_m$ minimization as special cases.

The optimum designs are performed by jointly optimizing the transmission energy in the PHY layer and the frame length in the MAC layer. The system designs incorporate a large number of practical system parameters, such as the efficiency of the power amplifier, the power consumption of digital hardware, data rate, coding and modulation, frame length, and protocol overhead, frame error rate (FER), and frame retransmission, etc. To quantify the impacts of transmission energy and frame length, a new log-domain threshold approximation protocol overhead, frame error rate (FER), and frame retransmission are considered for type-I ARQ in flat fading channel and the discrete-time samples at the receiver can be represented as

$$y_i = h \cdot \sqrt{E_t} x_i + z_i, \quad i = 1, 2, \ldots, L_s,$$

where $i$ is the symbol index in the frame, $x_i \in S$ is the modulated symbol with unit energy, with $S$ being the modulation constellation set of cardinality $M = |S|$, $E_t$ is the average energy of the symbol observed at the receiver, and $y_i$, $h$, and $z_i$ are the received sample, the fading channel coefficient, and additive white Gaussian noise (AWGN) with one-sided power spectral density $N_0$, respectively. It is assumed that the system undergoes quasi-static Rayleigh fading, such that the fading coefficient is constant within one frame, but changes from frame to frame.

Let $E_b$ denote the average energy per uncoded bit, then the average $E_b/N_0$ at the receiver is

$$\gamma_b \triangleq \frac{E_b}{N_0} = \frac{E_t}{r N_0 \log_2 M}.$$  

The average transmission energy for each symbol is [2]

$$E_s = E_t G_d,$$

where $G_d = G_1 d^k M_1$ with $k$ being the path-loss exponent, $G_3$ being the gain factor (including path-loss and antenna gain) at unit distance, and $M_1$ being the link margin compensating the variations in hardware process and other additive background noise or interference.

In addition to the actual transmission energy, we also consider the hardware energy per symbol in the transmitter and receiver, which can be modelled as [2],

$$E_c = \left( \frac{\zeta_m}{\mu} - 1 \right) E_s + \frac{\beta}{R_b},$$

where $R_b$ is the gross symbol rate, $\mu$ is the drain efficiency of the power amplifier, $\zeta_m$ is the peak-to-average power ratio (PAPR) of an $M$-ary modulation signal, $\beta$ incorporates the effects of baseband processing in the transmitter and receiver, such as signal processing, encoding and decoding, modulation and demodulation, and it can be treated as a constant determined by the particular transceiver structure. For MQAM systems, $\zeta_m \approx 3(\sqrt{M} - \frac{1}{\sqrt{M}} + 1)$ for $M \geq 4$ [14].

From (2), (3), and (4), the average energy required to transmit an information bit during one transmission attempt is

$$E_0 = \frac{L_s}{L} (E_s + E_c) = \frac{L + L_0}{L} \gamma_b A + B,$$

where $A = \frac{\zeta_m N_0 G_d}{\beta}$ and $B = \frac{\beta}{L} \frac{L_0}{L} = \frac{\beta}{R_b}$ with $R_b$ being the information bit rate.

Due to the effects of channel fading and noise, the receiver might not be able to successfully recover the transmitted signal. The probability that a transmitted frame cannot be recovered equals to FER, which is a function of $\gamma_b$ at the receiver, the frame length $L_s$, the modulation level $M$, and the channel code. For a system employing type-I ARQ, the receiver will send a negative acknowledgment (NACK) to the transmitter if a packet cannot be recovered, and the packet will be retransmitted. Since the retransmissions are independent, the number of retransmissions $K$ is a geometric
random variable with the probability mass function \( P_K(k) = FER^{-k}(1 - FER) \). The average number of retransmissions is
\[
\Lambda = \frac{1}{1 - FER}. \tag{6}
\]

The result in (6) is obtained by assuming no limit on the number of retransmissions, which is often used to model delay tolerant communication systems. Even though there is no retransmission limit, the average number of retransmissions \( \Lambda \) is usually very small in reasonable communication systems, for example, \( \Lambda = 1.25 \) for an FER as high as 0.25. The average number of retransmissions for systems with retransmission limits falls between \([1, \Lambda]\). Therefore, imposing a retransmission limit has a relatively small impact on the average number of retransmissions.

With the effects of retransmissions, the total average energy required to successfully deliver an information bit from the transmitter to the receiver is \( E_t = \Lambda E_0 \), which can be expanded by combining (5) and (6) as
\[
E_t = \frac{1}{1 - FER} \left( \frac{L + L_0}{L} \right) \gamma_b A + B. \tag{7}
\]

The metric, energy per information bit \( E_t \), measures the EE of the ARQ system. A smaller \( E_t \) means a better EE. The value of \( E_t \) relies on a number of system parameters, including \( E_0/N_0 \) at the receiver, \( \gamma_b \), the number of information bits per frame \( L \), the number of overhead bits per frame \( L_0 \), the modulation level \( M \), the symbol rate \( R_s \), the code rate \( r \), and the FER that inherently depends on all of the above parameters, etc. Optimizing with respect to \( E_t \) will minimize the total energy consumption, but at the cost of reduced SE.

The SE, denoted as \( \eta_b \), is defined as the net data rate of the successfully transmitted information bit divided by the required bandwidth. Since \( L \) information bits are successfully transmitted in \( L_s \) symbols with \( \Lambda \) retransmissions, the SE is
\[
\eta_b = \frac{L}{L_s} \Lambda (1 + \alpha) = \frac{L}{L + L_0} \frac{\eta_m}{\Lambda}. \tag{8}
\]

where \( \eta_m = \frac{r \log_2 M}{1 + \alpha} \) is the maximum spectrum efficiency of an \( M \)-ary modulation scheme, and \( \alpha \) is the roll-off factor of the pulse shaping filter, which expands the bandwidth of each symbol from \( R_s \) to \((1 + \alpha)R_s \). The unit of the SE is bps/Hz.

To achieve a balanced tradeoff between the EE and the SE, we define a new metric, the normalized energy per information bit, as \( E_m = \frac{E_t}{\eta_b} \), which can be minimized by either reducing \( E_t \) or increasing \( \eta_b \), yielding one possible EE-SE tradeoff.

Combining (7) and (8), we can express the normalized energy per information bit as
\[
E_m = \frac{1}{(1 - FER)^2} \left( \frac{L + L_0}{L} \right) \gamma_b A + B. \tag{9}
\]

From (7) and (9), we can see that the value of \( \gamma_b \) has two opposite effects on \( E_t \) and \( E_m \). On one hand, FER is a decreasing function in \( \gamma_b \). Therefore, increasing \( \gamma_b \) will decrease the average number of retransmissions \( \Lambda \), thus reduce \( E_t \) and \( E_m \). On the other hand, \( E_0 \) is a strictly increasing function in \( \gamma_b \), thus it translates a positive relationship between \( \gamma_b \) and \( E_t \) or \( E_m \). A similar observation can also be obtained for the relationship between \( L \) and \( E_t \) or \( E_m \). Therefore, it is critical to identify the optimum values of \( \gamma_b \) and \( L \) that can minimize \( E_t \) or \( E_m \). The optimum system design requires the knowledge of FER, which is discussed in the next subsection.

B. FER with a Log-Domain Linear Threshold Approximation

In this subsection, an accurate approximation of the FER of systems operating in quasi-static Rayleigh fading is obtained with the threshold-based method originally presented in [15]. In addition, we propose a new log-domain linear approximation method for the calculation of the threshold value required for the FER approximation. The threshold-based method with the newly proposed log-domain linear approximation explicitly build a connection between the FER and the various system parameters.

The FER of a coded system in a quasi-static Rayleigh fading channel can be accurately approximated as [15]
\[
\text{FER} \simeq 1 - \exp \left( -\frac{\gamma}{\gamma_b} \right), \tag{10}
\]

where \( \gamma_b \) is a threshold value that depends on the actual channel code, the modulation scheme, and the number of symbols per frame, etc.

Figure 1 shows \( \gamma_b \) as a function of \( L + L_0 \) under various modulation schemes, where the values of \( \gamma_b \) are estimated by matching the FER obtained through simulations with the analytical approximation in (10) using the least squares (LS) method. The channel code is the rate \( r = \frac{1}{2} \) convolutional code with generator polynomial \([5, 7]_8 \) and constraint length \( 3 \). It is observed from the figure that \( \gamma_b \) can be approximated as a linear function of \( \log(L + L_0) \), with the slope and intercept determined by different modulation schemes.

Similar linear relationships are also observed for other channel codes. Therefore, we propose to model \( \gamma_b \) as
\[
\gamma_b \simeq k_m \log(L + L_0) + b_m, \tag{11}
\]

where \( k_m \) and \( b_m \) are, respectively, the slope and intercept, the values of which for several codes are listed in Table 1.
<table>
<thead>
<tr>
<th>QPSK</th>
<th>16QAM</th>
<th>64QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_M$</td>
<td>$b_M$</td>
<td>$k_M$</td>
</tr>
<tr>
<td>Uncoded</td>
<td>0.931</td>
<td>-1.223</td>
</tr>
<tr>
<td>Code 1</td>
<td>0.374</td>
<td>-0.310</td>
</tr>
<tr>
<td>Code 2</td>
<td>0.207</td>
<td>0.140</td>
</tr>
<tr>
<td>Code 3</td>
<td>0.472</td>
<td>-0.860</td>
</tr>
<tr>
<td>Turbo Code</td>
<td>0.053</td>
<td>0.836</td>
</tr>
</tbody>
</table>

Code 1 is a rate $r = \frac{1}{2}$ convolutional code with generator polynomial $[5, 7]_8$ and constraint length 3. Code 2 is a rate $r = \frac{1}{4}$ convolutional code with generator polynomial $[171, 133]_8$ and constraint length 7. Code 3 is a rate $r = \frac{3}{4}$ convolutional code with generator polynomial $[51, 31, 13]_8$ and constraint length 3. Soft Viterbi decoders are employed for decoding the convolutional codes. The turbo code is a rate $r = \frac{1}{2}$ code with generator polynomial $[1, 5/7, 5/7]_8$ and a block interleaver. The turbo decode is performed with six iterations. It is noted that turbo code with a random interleaver might yield different results [18].

Combining (10) and (11) leads to a new approximation for the FER

$$\text{FER} \simeq 1 - (L + L_0)^{-\frac{k_M}{b_M} \exp \left( -\frac{b_M}{\gamma_b} \right)}.$$  \hspace{1cm} (12)

Figure 2 compares the actual FER obtained through simulations with the analytical approximation (12) under different values of $L + L_0$ for systems with $M = 4$. The comparisons are performed for systems with various coding schemes. Excellent agreements are observed between the actual simulation results and their analytical approximations.

III. MINIMIZING ENERGY PER INFORMATION BIT

This section derives the optimum values of $\gamma_b$ and $L$ that minimize the energy per information bit $E_i$.

A. Optimum $\gamma_b$ for minimum $E_i$

Before proceeding to the actual optimization, we present the following lemma about convexity of the cost function.

**Lemma 1:** Consider a decreasing function $f(x)$ with $f'(x) \leq 0$, and an increasing function $g(x)$ with $g'(x) \geq 0$. If both $f(x)$ and $g(x)$ are convex, then $f(x)g(x)$ is convex.

**Proof:** The proof is in Appendix A.

Since $E_i$ is the product of the number of retransmissions $\Lambda$ and the average energy in one transmission $E_0$ in (5), we can prove that $E_i$ is convex in $\gamma_b$ if $\Lambda$ and $E_0$ satisfy the conditions stated in Lemma 1. The results are stated as follows.

**Corollary 1:** For the FER given in (12), the total energy per information bit $E_i$ in (7) is convex in $\gamma_b$.

**Proof:** The proof is in Appendix B.

Once the convexity of $E_i$ in $\gamma_b$ is established, the optimum $\gamma_b$ can be obtained through convex optimization and the result is in Proposition 1.

**Proposition 1:** In a quasi-static Rayleigh fading channel, if the FER is given in (12), then the optimum $\gamma_b$ that minimizes $E_i$ is given by

$$\hat{\gamma}_b = \frac{1}{2} \left( \gamma_\omega + \sqrt{\gamma_\omega^2 + 4 \gamma_\omega \frac{B}{A} L} \right),$$ \hspace{1cm} (13)

where $A$ and $B$ are defined after (5).

**Proof:** The proof is in Appendix C.

It should be mentioned here that the optimum $\gamma_b$ is the average $E_b / N_0$ at the receiver. Correspondingly, the optimum energy per symbol required at the transmitter is

$$\hat{E}_s = \frac{\gamma_b N_0 G_d r \log_2 M}{\lambda}.$$ \hspace{1cm} (14)

B. Optimum $L$ for minimum $E_i$

Similar to the results in Proposition 1, the optimum solution of $L$ relies on the convexity of $E_i$. However, the direct proof of the convexity of $E_i$ in $L$ is quite tedious. To simplify the analysis, we show that $E_i$ is convex in $\xi = \log (L + L_0)$. Therefore, we can solve the optimum $L$ by indirectly minimizing $E_i$ with respect to $\xi$ through $\frac{\partial E_i}{\partial L} = 0$. It is exactly the same as solving $\frac{\partial E_i}{\partial \xi} = 0$ because $\frac{\partial E_i}{\partial \xi} = \frac{1}{L + L_0} \neq 0$.

**Corollary 2:** For the FER given in (12), the total energy per information bit $E_i$ in (7) is convex in $\xi = \log (L + L_0)$.

**Proof:** The proof is in Appendix D.

Once the convexity in $\xi$ is established, the optimum $L$ can be solved with result stated in Proposition 2.

**Proposition 2:** In a quasi-static Rayleigh fading channel, if the FER is given in (12), then the optimum $L$ that minimizes $E_i$ is calculated as

$$L = \sqrt{A^2 (k_M + \gamma_b)^2 + 4Ak_M B - A(k_M - \gamma_b)} \gamma_b L_0.$$ \hspace{1cm} (15)

**Proof:** The proof is in Appendix E.

C. Joint Optimum $\gamma_b$ and $L$ for minimum $E_i$

The joint optimum values of $\gamma_b$ and $L$ can be obtained by solving the system of two equations defined by (13) and (15). Since both (13) and (15) are in closed forms, we can
solve them by substituting (13) into (15) and then solve $L$ numerically. This method requires the numerical solution of a nonlinear equation, and it might incur a high complexity.

Alternatively, the joint optimum values of $\gamma_b$ and $L$ can be calculated by iteratively invoking (13) and (15). Given an initial value $L$, we calculate the optimum $\gamma_b$ using (13), the output of which is then used to update the value of $L$ with (15). This procedure is performed iteratively and it will efficiently converge to the joint optimum value of $\gamma_b$ and $L$ that achieves the global minimum energy consumption.

**Corollary 3:** Define vector $\mathbf{\pi} = [\gamma_b, L]^T$. The iterative algorithm described above will always converge to the globally optimum value $\hat{\mathbf{\pi}} = [\gamma_b, \hat{L}]^T$ that minimizes $E_t$.

**Proof:** The proof is in Appendix F.

Figure 3 visualizes the convergence of the proposed iterative procedure by plotting (13) and (15) in the same figure. The intersection of the two curves gives the optimum values. In the figure, we used $M = 4$ and $d = 100$ m. Given the initial value $L + L_0 = 10^4$ bits, the iteration follows the trace and converges quickly to the optimum value within three iterations.

### IV. Minimizing Normalized Energy Per Information Bit

This section derives the optimum system design by minimizing the normalized energy per bit $E_m$, which can achieve a balanced tradeoff between EE and SE. To simplify notations, denote $\Delta_1 = \frac{L+L_0}{L\eta_M}$, $\Delta_2 = \frac{L+L_0}{\eta_M} = \frac{\gamma_b A + B}{L}$, then $E_m = \Delta_1 \Delta_2$.

#### A. Optimum $\gamma_b$ for minimum $E_m$

Following the similar procedures as in Section III, we first prove the convexity of $E_m$ with respect to $\gamma_b$.

**Corollary 4:** For the FER given in (12), the normalized energy per bit $E_m$ defined in (9) is convex in $\gamma_b$.

**Proof:** The proof is in Appendix G.

The optimum $\gamma_b$ that minimizes $E_m$ can be obtained by solving $\frac{\partial E_m}{\partial \gamma_b} = 0$, and the result is given as follows.

**Proposition 3:** In a quasi-static Rayleigh fading channel, if the FER is given in (12), then the optimum $\gamma_b$ that minimizes $E_m$ is

$$\gamma_b = \gamma_0 + \sqrt{\frac{\gamma_0^2 + 2 \gamma_0 B L}{A L + L_0}}.$$  

**Proof:** The proof is in Appendix H.

Comparing the results in Propositions 1 and 3, it is clear that $\gamma_0 > \gamma_b$. Therefore, the system that is optimum for $E_m$ requires a slightly higher energy than that optimum for $E_t$. The extra energy is used to achieve a higher spectral efficiency.

#### B. Optimum $L$ for minimum $E_m$

**Corollary 5:** For the FER given in (12), the normalized energy per information bit $E_m$ in (9) is convex in $\xi = \log(L + L_0)$.

**Proof:** The proof is in Appendix I.

The following proposition states the optimum value of $L$ that minimizes $E_m$.

**Proposition 4:** In a quasi-static Rayleigh fading channel, the optimum $L$ that minimizes $E_m$ is given by

$$\hat{L} = \frac{2A\gamma_b - 2Ak_M + B}{4k_M(A\gamma_b + B)} - \gamma_b L_0 + \frac{\sqrt{4A^2 k_M^2 + 4Ak_M (2A\gamma_b + 3B) + (2A\gamma_b + B)^2}}{4k_M(A\gamma_b + B)} - \gamma_b L_0.$$  

**Proof:** The proof is in Appendix J.

Comparing the results in Propositions 2 and 4, we have

$$\hat{L} - \hat{L}_0 = \frac{\sqrt{D + 12Ak_M B + 4A\gamma_b B + B^2 + B}}{4k_M(A\gamma_b + B)} - \frac{\sqrt{D + 16Ak_M B}}{4k_M(A\gamma_b + B)} - \gamma_b L_0,$$

where $D = 4A^2 (k_M + \gamma_b)^2$.

From (11) and (16), we have $\gamma_b > \gamma_0 > k_M$ if $L + L_0 > \exp\left(1 + \frac{b_M}{k_M}\right)$, which are true for all practical system configurations. Therefore, $\hat{L} > \hat{L}_0$, which means the system that is optimum with respect to $E_m$ requires a longer frame than that optimum with respect to $E_t$.

The joint optimum values of $\gamma_b$ and $L$ that achieve the globally minimum $E_m$ can be obtained with a similar iterative method as described in Sec. III-C.

### V. Energy and Spectral Efficient Designs for Type-I ARQ Systems

The metric $E_m$ combines $E_t$ and $\eta_M$ in the same equation to reflect their tradeoff relationship. To gain better insights on the separate effects of $E_t$ and $\eta_M$ on the system performance, we minimize $E_t$ under the constraint of the spectral efficiency $\eta_M$ in this section. The results allow us to flexibly adjust the EE-SE tradeoff based on specific system design requirements.

The constrained optimization problem is written as

$$\min E_t \text{ with respect to } \gamma_b > 0, L > 0$$

subject to $\eta_M \geq \eta_M^\text{th}$,

where $\eta_M^\text{th}$ is a constant threshold.
A. Minimizing $E_t$ under constraint $\eta_s = \eta_h$

We first transform (18) by setting a more restrict constraint $\eta_h = \eta_h$, yielding a new optimization problem

$$
\begin{aligned}
\text{minimize} & \quad E_t \\
\text{subject to} \quad & \eta_s = \eta_h.
\end{aligned}
$$

(19)

The results will help us to identify the EE-SE tradeoff and solve the original optimization problem in (18).

The Lagrangian objective function of (19) can be expressed by

$$
\psi(\gamma_b, L) = \frac{1}{1 - \text{FER}} \left( \frac{L + L_0}{L} \gamma_b A + B \right) + \lambda \left[ \frac{L}{L + L_0} \eta_M - \eta_h \right],
$$

(20)

where $\lambda$ is the Lagrangian multiplier.

Taking the first derivative of $\psi(\gamma_b, L)$ with respect to $\lambda$ and setting the result to zero, we have

$$
\exp \left( \frac{\gamma_b}{\gamma_s} \right) = \frac{L}{L + L_0} \eta_M.
$$

(21)

The first derivatives of $\psi(\gamma_b, L)$ with respect to $\gamma_b$ and $L$ are

$$
\frac{\partial \psi(\gamma_b, L)}{\partial \gamma_b} = \exp \left( \frac{\gamma_s}{\gamma_b} \right) \left( -\frac{\gamma_s}{\gamma_b^2} \right) \left( \frac{L + L_0}{L} \gamma_b A + B \right) + \exp \left( \frac{\gamma_s}{\gamma_b} \right) \frac{L + L_0}{L} \gamma_b A + \lambda \frac{L}{L + L_0} \eta_M \exp \left( \frac{\gamma_s}{\gamma_b} \right) \frac{\gamma_s}{\gamma_b} \frac{\gamma_s}{\gamma_b^2},
$$

(22a)

$$
\frac{\partial \psi(\gamma_b, L)}{\partial L} = \exp \left( \frac{\gamma_s}{\gamma_b} \right) \frac{k_M}{\gamma_b} \frac{L + L_0}{L} \gamma_b A + B + \exp \left( \frac{\gamma_s}{\gamma_b} \right) \frac{L + L_0}{L} \gamma_b A + \frac{\lambda \eta_M}{\gamma_b} \left[ \frac{L_0}{(L + L_0)^2} \exp \left( \frac{\gamma_s}{\gamma_b} \right) \frac{\gamma_s}{\gamma_b} \frac{L_0}{L + L_0}\right] + \exp \left( \frac{\gamma_s}{\gamma_b} \right) \frac{L}{L + L_0} \exp \left( \frac{\gamma_s}{\gamma_b} \right) \left( -\frac{1}{\gamma_b} \right) \frac{k_M}{\gamma_b} \left( \frac{1}{L + L_0} \right).\n$$

(22b)

The joint optimum $\gamma_b$ and $L$ can be obtained by solving the equation system, $\partial \psi(\gamma_b, L)/\partial \gamma_b = 0$ and $\partial \psi(\gamma_b, L)/\partial L = 0$, with the condition in (21). The result is given in Lemma 2.

**Lemma 2:** For an ARQ system operating in a quasi-static Rayleigh fading channel, given a constraint on the spectral efficiency $\eta_s = \eta_h$, the optimum values, $\hat{\gamma}_b$ and $\hat{L}$ that minimize $E_t$ must satisfy the following equations

$$
\hat{\gamma}_b = \frac{1}{2} \frac{\hat{L}}{L_0} \left( k_M + \sqrt{k_M^2 + 4 \gamma_s B \frac{L_0^2}{L(L + L_0)}} \right),
$$

(23)

and

$$
\hat{L}^{-1} \left( L + L_0 \right) \frac{\hat{\gamma}_b}{\gamma_s} = \eta_M \exp \left( \frac{b_M}{\gamma_s} \right).
$$

(24)

**Proof:** The proof is in Appendix K.

The optimum $\hat{L}$ can be obtained by substituting (23) into (24) and solving for $\hat{L}$ numerically. The optimum $\hat{\gamma}_b$ can then be calculated by using (23) and the optimum $\hat{L}$.

**Corollary 6:** With the optimum $\hat{\gamma}_b$ and $\hat{L}$ obtained from Lemma 2, we conclude: 1) $\hat{\gamma}_b$ is a strictly increasing function in $\hat{L}$; 2) $\hat{\eta}_b$ is a strictly increasing function in $\hat{\gamma}_b$; and 3) $\hat{\eta}_b$ is a strictly increasing function in $\hat{L}$.

**Proof:** The proof is in Appendix L.

B. EE-SE tradeoff

To illustrate the tradeoff between the energy efficiency and spectral efficiency, we plot the optimum $(\hat{\gamma}_b, \hat{E}_t)$ values obtained from (23) and (24) by varying the threshold $\eta_h = \eta_h$. The curve, denoted as the $\hat{E}_t(\eta_h)$ curve, is shown in Fig. 4 for the case where Code 1 in Table I and 16QAM modulation are used, and the maximum spectral efficiency is $\eta_M = 1.64$ for $\alpha = 0.22$, which corresponds to the case that the transmission is successful with the first attempt. The $\hat{E}_t(\eta_h)$ curve contains two special points: the point $\hat{E}_t(\eta_h)$ corresponds to the optimum solution obtained by minimizing $E_t$ from Section III; the point $\hat{E}_t(\eta_h)$ corresponds to the optimum solution of minimizing $E_m$ obtained from Section IV.

The EE-SE tradeoff for a type-I ARQ system is given in Theorem 1.

**Theorem 1:** For a given $\eta_h$ in a type-I ARQ system, the minimum $E_t$ can be computed by using the parameters in Lemma 2. The resulting $\hat{E}_t(\eta_h)$ curve is quasiconvex in $\eta_h$.

**Proof:** The proof of the theorem is in Appendix M.

Based on Theorem 1, we know that $(\hat{\eta}_h, \hat{E}_t)$ corresponds to the point with the global minimum $\hat{E}_t$, and its corresponding $\hat{\eta}_h$ divides the curve into two regions: Region 1 with $\eta_h < \hat{\eta}_h$ (with a negative slope) and Region 2 with $\eta_h \geq \hat{\eta}_h$ (with a positive slope). Furthermore, minimizing $E_m$ guarantees that the operation point $(\hat{\eta}_h, \hat{E}_t)$ falls in Region 2 of the $\hat{E}_t(\eta_h)$ curve, which further divides Region 2 into two subregions: $\eta_h < \hat{\eta}_h$ (denoted as region 2L) and $\eta_h \geq \hat{\eta}_h$ (denoted as region 2R).

Now we are ready to solve our original optimization problem with the practical constraint $\eta_h \geq \eta_h$ by noting which
region the threshold $\eta_b$ falls in. The solution to the original problem (18) is stated as follows.

**Corollary 7:** Denote $\hat{\eta}_b$ as the spectral efficiency corresponding to the global minimum $E_t$ as obtained in Propositions 1 and 2. If $\eta_b < \hat{\eta}_b$, then the optimum solution to (18) coincides with the results in Propositions 1 and 2. If $\eta_b \geq \hat{\eta}_b$, then the optimum solution can be calculated using Lemma 2.

Proof: The results can be directly obtained by using the $E_t-\eta_b$ tradeoff relation described in Theorem 1.

Corollary 7 provides a generalized optimum system design for type-1 ARQ systems by considering both EE and SE. The results from Sections III and IV are special cases of Corollary 7. Given the two regions on the $\tilde{E}_t(\eta_b)$ curve, none of the points in Region 1 is optimum with respect to (18). Consequently, no practical systems should operate in Region 1. In Region 2, $E_t$ is a strictly increasing function in $\eta_b$, which means the EE has to be improved at the cost of SE or vice versa. Each point in Region 2 of the $\tilde{E}_t(\eta_b)$ curve corresponds to one possible EE-SE tradeoff, allowing flexible designs according to the requirement of the specific applications. In general, the $E_t-\eta_b$ slope is relatively small in Region 2L. Therefore, we can achieve a relatively large improvement in $\eta_b$ with only a small cost in $E_t$ in Region 2L. In contrast, Region 2R has a large slope, which means that a small improvement in SE is achieved as a large cost of the EE. Therefore, Region 2L is the most desirable operation region for the EE-SE tradeoff.

**VI. Numerical Results**

An ARQ system was simulated in the PHY and MAC layers, including channel coding and decoding, baseband modulation and demodulation, protocol overhead, and retransmissions, etc. Most of the simulation parameters followed those in [2] with $\mu = 3.5$, $\beta = 310.014$ mw, $N_0/2 = -174$ dBm/Hz, $G_1 = 30$ dB, $k = 3.5$, and $M_I = 40$ dB. The rest of the parameters are $L_0 = 48$, $\alpha = 0.22$, and a fixed bit rate $R_b = 300$ kbps. The channel code used in the simulation was a rate $r = \frac{1}{2}$ convolutional code with generator polynomial [5, 7] and constraint length of 3.

Figure 5 shows $E_t$ as a function of $\gamma_b$ with various values of $L + L_0$. The communication distance was $d = 100$ m. It can be seen from the figure that $E_t$ is a convex function in $\gamma_b$. The optimum values of $\gamma_b$ for different $L$ calculated from (13) are marked on the figure as “x”, which match nicely with the simulation results. If $\gamma_b < \hat{\gamma}_b$, then the FER is so high that the total energy consumption is dominated by retransmissions. In this case, we can reduce the total energy consumption by increasing $\gamma_b$. For example, for $L + L_0 = 1,000$ bits, increasing $\gamma_b$ from $-2$ dB to 5 dB can save 5.3 dB energy per bit. If $\gamma_b > \hat{\gamma}_b$, then $E_t$ increases almost linearly with $\gamma_b$ because the FER is low enough and the effect of retransmission is negligible. The result demonstrates that a higher $E_b/N_0$ does not necessarily translate into better performance in energy efficiency. Significant energy saving can be achieved by carefully choosing the operation point.

In Fig. 6, the normalized energy per information bit $E_m$ is plotted as a function of $L + L_0$ under several values of $\gamma_b$. Again, the communication distance was $d = 100$ m. As expected, $E_m$ is convex in $\log(L + L_0)$, as shown by the simulation curves. The optimum values $L$ calculated from (17) are marked as “x” in the figure. Excellent agreement is observed between the analytical optimum operation points and the simulation results. When $\gamma_b$ is small, increasing $L$ passing its optimum operation point leads to a fast performance degradation. On the other hand, when $\gamma_b$ is large, for example, $\gamma_b = 8$ dB, increasing $L$ passing its optimum operation point has a much smaller impact, because the FER only degrades slightly with $L$ at high $\gamma_b$. Therefore, the EE performance of a system operating at low $\gamma_b$ is more sensitive to the frame length.

In Figures 7 and 8, we compare the $E_t$ and $\eta_b$ obtained from the three different optimum designs: minimizing $E_t$, minimizing $E_m$, and minimizing $\eta_b$ with the constraint $\eta_b \geq \hat{\eta}_b$. The modulation scheme was 16QAM and the communication...
distance varied from 50 m to 400 m. For minimizing $E_t$ with the SE constraint, we set $\eta_{th} = 1.5 \text{ bps/Hz}$. It can be seen from the figures that, although the design from minimizing $E_t$ achieves the lowest energy consumption, the corresponding SE is also very low. In contrast, the design from minimizing $E_m$ can almost double the spectral efficiency with very small increase in energy consumption. At $d = 100$ m, the SE of the minimum $E_m$ scheme outperforms the minimum $E_t$ one by 80%, with only a 0.9 dB increase in energy consumption. For the optimum design with the SE constraint, it is possible to achieve a very high SE at the cost of increased energy consumption. At $d = 100$ m, increasing the SE from 0.9 bps/Hz to 1.5 bps/Hz results in a 7 dB increase in energy consumption. Other EE-SE tradeoffs can be obtained by setting different thresholds in the constrained optimization scheme.

**VII. CONCLUSIONS**

The energy efficient and spectral efficient designs have been studied for type-I ARQ systems operating in quasi-static Rayleigh fading channels. A new log-domain threshold approximation method has been proposed to facilitate the system design. The optimum transmission energy and frame length for various design criteria have been identified, and most are in closed-form expressions. With insights gained through the design results, the fundamental EE-SE tradeoff in type-I ARQ systems has been theoretically identified. It has been shown that the minimum $E_t$ is quasi-convex in $\eta_t$, and the optimum operation points are always in the positive slope region of the $E_t$-$\eta_t$ curve. From the analytical and simulation results, we have the following observations: 1) The total energy per information bit could be reduced by increasing the energy used in one transmission attempt if the system operates in the negative slope region of the $E_t$-$\eta_t$ curve; 2) systems operating at higher $E_t/N_0$ are less sensitive to the frame length change; 3) optimizing with respect to $E_m$ instead of $E_t$ can almost double the spectral efficiency with less than 1 dB more cost in $E_t$; 4) increasing the SE beyond the value corresponding to the minimum $E_m$ might lead to significant cost in $E_t$.

**APPENDIX**

**A. Proof of Lemma 1**

Consider $0 < x_1 < x_2$ and $\varepsilon \in [0, 1]$. Define $\theta_1 = \varepsilon f(x_1)g(x_1) + (1 - \varepsilon) f(x_2)g(x_2)$, and $\theta_2 = f(x_1) + (1 - \varepsilon)x_2)g(x_1 + (1 - \varepsilon)x_2)$. It is sufficient to prove that $\theta_1 \leq \theta_2$.

Since $f(x)$ and $g(x)$ are convex, we have $\theta_2 \geq \theta_3$ with $\theta_3$ defined as

$$\theta_3 = [\varepsilon f(x_1) + (1 - \varepsilon)f(x_2)](\varepsilon g(x_1) + (1 - \varepsilon)g(x_2))$$

Since $\theta_1 = \theta_1(1 - \varepsilon + \varepsilon)$, the term $\theta_1$ can be alternatively represented as

$$\theta_1 = \varepsilon^2 f(x_1)g(x_1) + (1 - \varepsilon)^2 f(x_2)g(x_2) + \varepsilon(1 - \varepsilon)[f(x_1)g(x_1) + f(x_2)g(x_2)]$$

From (25) and (26), we have

$$\frac{\theta_3 - \theta_1}{\varepsilon(1 - \varepsilon)} = [f(x_1) - f(x_2)][g(x_2) - g(x_1)] \geq 0.$$

Therefore $\theta_2 \geq \theta_3 \geq \theta_1$, and this completes the proof.

**B. Proof of Corollary 1**

We first prove that $\Lambda$ is convex in $\gamma_h$. The first derivative of $\Lambda$ with respect to $\gamma_h$ is

$$\frac{\partial \Lambda}{\partial \gamma_h} = -\frac{\gamma_h}{\eta_h} \exp \left( \frac{h_0}{\eta_h} \right) \leq 0.$$  

(28)

Denote $\text{FER}'(\gamma_h) = \frac{\partial \text{FER}}{\partial \gamma_h}$ and $\text{FER}''(\gamma_h) = \frac{\partial^2 \text{FER}}{\partial \gamma_h^2}$. The second derivative of $\Lambda$ with respect to $\gamma_h$ can be expressed as

$$\frac{\partial^2 \Lambda}{\partial \gamma_h^2} = \frac{\text{FER}''(\gamma_h)(1 - \text{FER}) + 2 [\text{FER}'(\gamma_h)]^2}{[1 - \text{FER}]^3}.$$  

(29)
From (12), we have
\[ \text{FER}'(\gamma_b) = -\frac{\gamma_0}{\gamma_b} \exp \left( -\frac{\gamma_0}{\gamma_b} \right), \]  
(30a)
\[ \text{FER}''(\gamma_b) = \frac{\gamma_0}{\gamma_b^2} \left( 2 - \frac{\gamma_0}{\gamma_b} \right) \exp \left( -\frac{\gamma_0}{\gamma_b} \right). \]  
(30b)
Substituting the above results into (29) and simplifying lead to
\[ \frac{\partial^2 \Delta}{\partial \gamma_b^2} = \frac{\gamma_0}{\gamma_b^2} \left( 2 + \frac{\gamma_0}{\gamma_b} \right) \exp \left( \frac{\gamma_0}{\gamma_b} \right) \geq 0. \]  
(31)
Therefore $\Delta$ is decreasing and convexity function in $\gamma_b$. It is straightforward to show that $E_0$ is a linear increasing function in $\gamma_b$. Based on Theorem 1, $E_t = \Delta E_0$ is convex in $\gamma_b$.

C. Proof of Proposition 1
Since $E_t$ is convex in $\gamma_b$, the optimum $\gamma_b$ that minimize $E_t$ can be obtained by solving $\frac{\partial E_t}{\partial \gamma_b} = 0$, which yields
\[ \gamma_b^2 - \gamma_b \gamma_0 - \frac{B}{A L + L_0} = 0. \]  
(32)
The result in (13) can be obtained by solving (32).

D. Proof of Corollary 2
From (6), we have
\[ \frac{\partial^2 \Delta}{\partial \xi^2} = \left( \frac{k_m}{\gamma_b} \right)^2 \exp \left( -\frac{k_m \xi}{\gamma_b} \right) \geq 0. \]  
(33)
Therefore $\Delta$ is convex in $\xi$. It is also straightforward to show that $\Delta$ is an increasing function in $\xi$. From (5), we have $E_0 = A \gamma_b \left( 1 + \frac{L_0}{e^{-\xi} - L_0} \right) + B$, and
\[ \frac{\partial^2 E_0}{\partial \xi^2} = A \gamma_b L_0 \exp \left( \frac{\xi + L_0}{e^{\xi} - L_0} \right)^2 \geq 0. \]  
(34)
Therefore $E_0$ is convex in $\xi$. In addition, $E_1$ is a decreasing function in $\xi$. The convexity of $E_t$ is then proved by using Theorem 1.

E. Proof of Proposition 2
The optimum $L$ is obtained by solving $\frac{\partial E_1}{\partial \xi} = 0$, which yields
\[ k_m (A \gamma_b + B) L^2 + A \gamma_b L_0 (k_m - \gamma_b) L - A \gamma_b^2 L_0^2 = 0. \]  
(35)
The result in (15) can be obtained by solving (35).

F. Proof of Corollary 3
Assume the values after the $i$-th iteration is $\pi^{(i)} = [\gamma_b^{(i)}, L^{(i)}]^T$. During the $(i + 1)$-th iteration, we can first calculate $\gamma_b^{(i+1)}$ by using (13) and $L^{(i)}$. Based on Proposition 1, $E_t(\gamma_b^{(i+1)}, L^{(i)}) \leq E_t(\gamma_b^{(i)}, L^{(i)})$. Then $L^{(i+1)}$ can be calculated by using (15) and $\gamma_b^{(i+1)}$. Based on Proposition 2, $E_t(\gamma_b^{(i+1)}, L^{(i+1)}) \leq E_t(\gamma_b^{(i+1)}, L^{(i)})$. If $\pi^{(i)} \neq \pi$, then $E_t(\gamma_b^{(i+1)}, L^{(i+1)}) < E_t(\gamma_b^{(i)}, L^{(i)})$, i.e., the value of $E_t$ will decrease after each iteration, and $\pi^{(i)}$ is converging to its globally optimum value given that there is a minimum $E_t$. If $\pi^{(i)} = \pi$, then $E_t(\gamma_b^{(i+1)}, L^{(i+1)}) = E_t(\gamma_b^{(i)}, L^{(i)})$, and the iteration converges.

G. Proof of Corollary 4
The first derivative of $\Delta_1$ with respect to $\gamma_b$ is
\[ \frac{\partial \Delta_1}{\partial \gamma_b} = -2 \frac{\gamma_0}{\gamma_b} \exp \left( \frac{2 \gamma_0}{\gamma_b} \right) \leq 0. \]  
(36)
Therefore $\Delta_1$ is a decreasing function in $\gamma_b$.
The second derivative of $\Delta_1$ with respect to $\gamma_b$ is
\[ \frac{\partial^2 \Delta_1}{\partial \gamma_b^2} = 2 \text{FER}''(\gamma_b) (1 - \text{FER}) + 6 \frac{\text{FER}'(\gamma_b)^2}{(1 - \text{FER})^4}. \]  
(37)
Substituting (30) into (37) and simplifying lead to
\[ \frac{\partial^2 \Delta_1}{\partial \gamma_b^2} = \frac{4 \gamma_0}{\gamma_b^2} \exp \left( \frac{2 \gamma_0}{\gamma_b} \right) \geq 0. \]  
(38)
Therefore $\Delta_1$ is convex in $\gamma_b$.

It is straightforward that $\Delta_2$ is a linear increasing function in $\gamma_b$. Based on Theorem 1, $E_m = \Delta_1 \Delta_2$ is convex in $\gamma_b$.

H. Proof of Proposition 3
Since $E_m$ is convex in $\gamma_b$, the optimum $\gamma_b$ that minimize $E_m$ can be obtained by solving $\frac{\partial E_m}{\partial \gamma_b} = 0$, which yields
\[ \frac{\partial \Delta_1}{\partial \gamma_b} \Delta_2 + \frac{\partial \Delta_2}{\partial \gamma_b} \Delta_1 = 0. \]  
(39)
Since $\frac{\partial \Delta_2}{\partial \gamma_b} = \left( \frac{L_0 + L_b}{L} \right)^2 AC$. Substituting (36) into (39) leads to
\[ \gamma_b^2 - 2 \gamma_0 \gamma_b - 2 \gamma_0 \frac{B}{A L + L_0} = 0. \]  
(40)
The result in (16) can be obtained by solving (40).

I. Proof of Corollary 5
From the definition of $\Delta_1$, we have
\[ \frac{\partial^2 \Delta_1}{\partial \xi^2} = \frac{4 k_m^2}{\gamma_b^2} \exp \left( \frac{2 \gamma_0}{\gamma_b} \right) \geq 0. \]  
(41)
Therefore $\Delta_1$ is convex in $\xi$. It is also straightforward to show that $\Delta_2$ is an increasing function in $\xi$.

In addition, $\Delta_2$ can be alternatively expressed as
\[ \Delta_2 = \left( 1 + \frac{L_0}{e^{-\xi} - L_0} \right) \left[ A \gamma_b \left( 1 + \frac{L_0}{e^{-\xi} - L_0} \right) + B \right] \frac{1}{\eta_m}, \]  
and
\[ \frac{\partial^2 \Delta_2}{\partial \xi^2} = \frac{L_0 e^\xi \left[ 2 \gamma_0 A e^\xi (2 e^\xi + L_0 - 1) + B (e^{2 \xi} - L_0^2) \right]}{\eta_m (e^\xi - L_0)^4} \geq 0. \]  
Therefore $\Delta_2$ is convex in $\xi$. In addition, $\Delta_2$ is a decreasing function in $\xi$. The convexity of $E_m$ with respect to $\xi$ is then proved by using Theorem 1.
J. Proof of Proposition 4

The optimum $L$ is obtained by solving $\partial E_\gamma / \partial L = 0$, which yields

$$\frac{\partial \Delta_1}{\partial \xi} \Delta_2 + \frac{\partial \Delta_2}{\partial \xi} \Delta_1 = 0. \tag{42}$$

Since

$$\frac{\partial \Delta_1}{\partial \xi} = \frac{2k_M}{\gamma_b} \exp\left(\frac{2\omega_L}{\gamma_b}\right), \tag{43a}$$

$$\frac{\partial \Delta_2}{\partial \xi} = -\frac{L_0^b \ell_c^2}{\eta_M} \left(\frac{2A\gamma_b \ell_c + \ell_c - L_0^b + B}{\ell_c - L_0^b}\right). \tag{43b}$$

Substituting the above results into (42) leads to

$$2k_M (A\gamma_b + B)L^2 + \gamma_b L_0 (2A\gamma_b - B)L - 2A\gamma_b^2 L_0^b = 0.$$  

The result in (17) can be obtained by solving the above equation.

K. Proof of Lemma 2

Substituting (21) into the equation $\partial \psi(\gamma_b, L) / \partial \gamma_b = 0$ yields

$$\lambda \gamma_b - \frac{\eta_M}{\gamma_b} \left(\frac{\gamma_M L}{\gamma_b^2 L + L_0^b B - A + \gamma_b A}\right). \tag{44}$$

Similarly, combining (21) with $\partial \psi(\gamma_b, L) / \partial L = 0$, we have

$$\frac{\lambda L_0 \gamma_b - k_M \gamma_b L}{L_0 b} = \frac{\eta_M}{\gamma_b^2} \left(\frac{\gamma_M L_0}{L_0^b - k_M A - k_M \gamma_b B}{\gamma - \gamma_b A}\right). \tag{45}$$

The Lagrangian multiplier $\lambda$ can be removed by dividing (44) by (45) and the result is

$$\frac{\gamma_M L^2}{\gamma_b^2} \left(\frac{\gamma_M L_0}{L_0^b - k_M A - k_M \gamma_b B}{\gamma - \gamma_b A}\right) = \left(\frac{\gamma_M L}{\gamma_b^2 L + L_0^b B - A + \gamma_b A}\right).$$

Rearranging the terms, we obtain a second order linear equation in $\gamma_b$

$$\gamma_b^2 - k_M \gamma_b L + \gamma_b B + A \gamma_b = 0. \tag{46}$$

Solving (46) yields the desired solution (23).

To solve for optimum $L$, combining (11) and (21) yields

$$\frac{k_M}{\gamma_b} \log(1 + L_0 b) + \frac{b M}{\gamma_b} = \log \frac{L}{L_0} + \log \frac{\eta_M}{\gamma_b}. \tag{47}$$

which is simplified to (24).

L. Proof of Corollary 6

Eqn. (23) can be alternatively written as

$$\gamma_b = \frac{1}{2} \left(\frac{L_0 L}{L_0 b} + \sqrt{\left(\frac{L_0 L}{L_0 b}\right)^2 + 4 \gamma_b B + A \gamma_b (L_0 + 1)}\right). \tag{48}$$

Since $\gamma_\omega$ is a strictly increasing function in $\hat{L}$, it is straightforward that the optimum $\gamma_\omega$ strictly increases with $\hat{L}$.

Performing differentiation of $\gamma_\omega$ in (8) with respect to $\gamma_b$, we have $\partial \gamma_\omega / \partial \gamma_b > 0$, thus statement 2) is proved.


Jingxian Wu (S’02-M’06) received the B.S. (EE) degree from the Beijing University of Aeronautics and Astronautics, Beijing, China, in 1998, the M.S. (EE) degree from Tsinghua University, Beijing, China, in 2001, and the Ph.D. (EE) degree from the University of Missouri at Columbia, MO, USA, in 2005. He is currently an Assistant Professor with the Department of Electrical Engineering, University of Arkansas, Fayetteville. His research interests mainly focus on wireless communications and wireless networks, including ultra-low power communications, energy efficient communications, cognitive radio, and cross-layer optimization, etc. He served as a Cochair for the 2012 Wireless Communication Symposium of the IEEE International Conference on Communication, and a Cochair for the 2009 Wireless Communication Symposium of the IEEE Global Telecommunications Conference. He served as an Associate Editor of the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY from 2007 to 2011, and is now an Editor of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS.

Gang Wang received the B.S. degree from the Northwestern Polytechnical University, Xi’an, China, in 2005, and the M.S. degree from Beihang University (with honor), Beijing, China, in 2008, both in electrical engineering. He then served as a system engineer in New Postcom Equipment Co., Ltd, in Beijing, from 2008 to 2010. He is currently a Ph.D. student at the Department of Electrical Engineering, University of Arkansas. His current research interests include energy-efficient system, convex optimization, cross-layer design, and wireless networks, etc. He is a member of Golden Key International Honor Society.

Yahong Rosa Zheng received the B.S. degree from the University of Electronic Science and Technology of China, Chengdu, China, in 1987, and the M.S. degree from Tsinghua University, Beijing, China, in 1989, both in electrical engineering. She received the Ph.D. degree from the Department of Systems and Computer Engineering, Carleton University, Ottawa, Canada, in 2002. She was an NSERC postdoctoral fellow from January 2003 to April 2005 at the University of Missouri-Columbia. Since 2005, she has been assistant then associate professor with the Department of Electrical and Computer Engineering at the Missouri University of Science and Technology (formerly the University of Missouri-Rolla). Her research interests include array signal processing, space-time adaptive processing, wireless communications, and wireless sensor networks. She has served as associate editor for IEEE Transactions on Wireless Communications 2006–2008 and is currently associate editor for IEEE Transactions on Vehicular Technology. She is the recipient of an NSF CAREER award in 2009.