Collision-Tolerant Media Access Control with On-Off Accumulative Transmission

Jingxian Wu, *Member, IEEE*, and Geoffrey Ye Li, *Fellow, IEEE*

***Abstract*—In this paper, a cross-layer *collision-tolerant* (CT) *media access control* (MAC) scheme is proposed for wireless networks. Unlike conventional MAC schemes that discard and retransmit signals colliding at a receiver, the CT-MAC extracts the salient information from the colliding signals with a new *on-off accumulative transmission* (OOAT) scheme in the physical layer. Users employing OOAT deliver information to the *base station* (BS) through uncoordinated on-off transmissions of multiple identical sub-symbols (accumulative transmission). Silence periods are inserted between sub-symbols inside a frame to reduce collision probability and render a special signal structure for physical layer detection. Algebraic properties of the on-off transmission patterns, which are represented as cyclic-shifted binary vectors, are analyzed, and the results provide guidelines on the design of OOAT systems and other systems that rely on cyclic-shifted binary vectors. Then, we demonstrate that the structure of the on-off transmission patterns enables a sub-optimum iterative detection method, which improves performance by iteratively exchanging extrinsic soft information between a forward and a backward *soft interference cancellation* (SIC). Both analytical and simulation results show that the new CT-MAC with OOAT scheme significantly outperforms many existing cross-layer MAC schemes in terms of the number of users supported and the normalized throughput.**

***Index Terms*—Collision-tolerant media access control, random on-off accumulative transmission, cyclic-shifted binary vectors.**

I. INTRODUCTION

CROSS-LAYER design improves system performance by actively exploiting interactions and dependence among various protocol layers in a communication system [1]-[3]. It has attracted great attentions during the past decade for its potential to provide significant improvement of end-to-end throughput and scalable network performance.

A large family of cross-layer techniques for wireless networks are developed by performing joint design across the physical (PHY) and media access control (MAC) layers [3]-[16]. We will focus on using the signal processing capability in the PHY layer to improve the performance in the MAC layer. Many related works are heralded by the concept of multipacket reception [4]-[6], where the PHY layer can correctly decode a fraction of signals colliding at the receiver, with multiuser detection and/or interference cancellation. This is different from conventional MAC schemes that simply discard the colliding signals, and result in a waste of the precious transmission power and spectrum resource. One notable example of the multipacket reception technique is the *collision resolution diversity slotted-ALOHA* (CRDSA) [7], where each packet is transmitted twice at two random slots within a frame. If one of the transmission is successfully decoded, then the decoded information can be used to remove the interference caused by its twin replica. The interference cancellation is performed iteratively and this results in improved normalized throughput compared to the conventional slotted-ALOHA scheme. The idea of packet repetition in CRDSA is extended to *irregular repetition slotted-ALOHA* (IRSA) in [8] and [9], which allows a variable repetition rate of the packets and is designed with the assistance of graph-based analysis. The performance of CRDSA and IRSA is further improved by replacing the packet repetition with a linear block code in a *coded slotted-ALOHA* (CSA) scheme [10], or with a rateless code in a *rateless multiple access* (RMA) scheme [11]. The normalized throughput of CRDSA, IRSA, CSA and RMA increases almost linearly at low offered loads, but drops dramatically once the normalized offered load exceeds their respective saturation points. The sharp drop in throughput after the saturation point is due to the fact that the iterative interference cancellation is implemented without multiuser detection. The operation of the iterative interference cancellation relies on the assumption that there are sufficient number of uncollided packets at the receiver, such that these packets can be decoded first and used to initiate the interference cancellation process, which is not true when the offered load is high.

Multiuser detection can be used for multipacket reception, such as *code division multiple access* (CDMA) [12], *time hopping ultra-wide band* (TH-UWB) [13], and multiple-input multiple-output systems [14]-[16], with minimum mean-square-error or zero-forcing detectors. In order to exploit the full potential of the PHY/MAC interaction, it is desirable to have new PHY layer structures tailored for cross-layer design.

In this paper, we propose a new cross-layer *collision-tolerant MAC* (CT-MAC) scheme for one-hop wireless networks, such as the uplinks of the cellular or satellite networks, where multiple users transmit to a single *base station* (BS). The CT-MAC scheme can effectively extract the originally transmitted information by performing detection over the signals colliding at the BS with our newly developed *on-off accumulative transmission* (OOAT) scheme. With OOAT, each data symbol is transmitted in the form of multiple

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J. Wu is with the Department of Electrical Engineering, University of Arkansas, Fayetteville, AR 72701 USA (e-mail: wuj@uark.edu).

G. Y. Li is with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332 USA (e-mail: liye@ece.gatech.edu).

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identical sub-symbols (accumulative transmission), and two consecutive sub-symbols are separated with a silence period (on-off transmission). The on-off transmission patterns and their cyclic shifted versions for different users are different from each other. In the MAC layer, transmitted data are divided into frames. Unlike conventional MAC schemes that use random intervals between frames to reduce collision, the frames in the OOAT are transmitted consecutively with silence periods *inside* a frame. The structured silence periods not only reduce the probability of collision among the active users but also render a special signal structure at the receiver that can facilitate PHY layer signal detection. A new sub-optimum iterative detection algorithm is developed by utilizing the unique structure of the OOAT, and it almost achieves the performance of the optimum trellis-based algorithm with a much lower complexity. The performance of the proposed CT-MAC system with OOAT is theoretically analyzed in the MAC layer and the relationship between the normalized throughput and the offered load of the system is obtained.

Another important contribution of this paper is the development of a set of new theoretical results on the algebraic structure of the position vectors, which are represented as cyclic-shifted binary vectors. Due to the relative transmission delays, all the cyclic-shifted versions of a binary vector correspond to the same position vector, and they are defined as cyclic-identical in this paper. Two vectors that are not cyclic-identical are defined as cyclic-exclusive. The maximum number of users that can be supported by an OOAT system is thus determined by the maximum number of cyclic-exclusive binary vectors. It is well known that the number of non-identical length-M binary vectors with Hamming weight R is $\binom{M}{R}$. However, to the best of our knowledge, no result is available if cyclic-exclusiveness is considered. In this paper, we develop theoretical properties of the set containing cyclic-exclusive binary vectors, which provide guidelines to the development of OOAT systems and other systems that rely on cyclic-shifted binary vectors.

The rest of this paper is organized as follows. The model and principles of CT-MAC with the OOAT are described in Section II. The theoretical properties of the cyclic-exclusive position vector set are analyzed in Section III. Section IV presents the sub-optimum iterative detection algorithm. The performance analysis in the MAC layer is addressed in Section V. Simulation results are presented in Section VI, followed by conclusions in Section VII.

II. On-off Accumulative Transmission

A. System Model

Consider a wireless communication system with N active users transmitting to one BS. The proposed OOAT scheme in the PHY layer is shown in Fig. 1. The k-th data symbol of the n-th user, s_{nk}, in the OOAT scheme is transmitted through R weighted sub-symbols, as $x_{nk}^{(r)} = w_n^r s_{nk}$, for $r = 0, \cdots, R-1$ and $n = 1, \cdots, N$, where $w_n = \frac{1}{\sqrt{R}} e^{-j\frac{2\pi(n-1)}{N_R}}$ is the normalized weight coefficient for the n-th user, and $N_R \geq \max(N, R)$ is an integer. Each sub-symbol has a period of T_0. The symbol period is $T_s = MT_0$ with $M \geq R$. Consequently, R out of M sub-symbol positions are occupied during one symbol period. The positions of the occupied sub-symbols or the transmission pattern for the n-th user can be expressed by a binary vector of length M, $\mathbf{p}_n = [p_n(1), \cdots, p_n(M)]^T \in \mathcal{B}^{M\times 1}$, where $\mathcal{B} = \{0, 1\}$, with $p_n(m) = 1$ if a sub-symbol is transmitted at the m-th sub-symbol position, and $p_n(m) = 0$ otherwise[1]. To ensure that the receiver can distinguish between information from any two users with an arbitrary relative delay, each user in the OOAT scheme uses a unique transmission pattern that is cyclic-exclusive to patterns of other active users.

The OOAT scheme is defined by the triplet (N, R, M), where N is the number of users, R is the number of repetitions, and M is the number of sub-symbol positions inside each symbol period. Fig. 1 shows an example of a system with $N = 5$, $M = 12$, and $R = 4$, respectively. Due to the on-off transmission pattern, only a subset of the users will mutually interfere with each other at a given sub-symbol position.

Define the collision order at the sub-symbol position m as $N_c(m) = \sum_{n=1}^{N} p_n(i_{nm})$, where $i_{nm} = \text{mod}_M(m - p_{n0}) + 1$, with $\text{mod}_M(m)$ being the modulo M operator, and p_{n0} is the relative starting position of the n-th user. For example, in Fig. 1, $p_{10} = 3$ and $p_{20} = 2$. The collision order of the multiuser system is defined as $N_c = \max_m N_c(m)$. We have $N_c = 2$ for the system shown in Fig. 1.

Based on the above discussion, a CT-MAC system with the OOAT can be represented by

$$y(m) = \sum_{n=1}^{N} p_n(i_{nm}) w_n^{r_{nm}} h_n s_{nk_{nm}} + z(m) \tag{1}$$

where $y(m)$ and $z(m)$ are the received sample and additive white Gaussian noise (AWGN) at the m-th sub-symbol, respectively, $r_{nm} = \sum_{i=1}^{i_{nm}} p_n(i) - 1$ counts the number of transmitted sub-symbols for the current symbol, h_n is the channel coefficient between user n and the BS, s_{nk} is the k-th symbol transmitted by user n, and $k_{nm} = \lceil \frac{m - p_{n0} + 1}{M} \rceil$, with $\lceil a \rceil$ being the smallest integer greater than or equal to a. It is assumed that the channel experiences quasi-static fading, *i.e.*, the fading coefficient is constant within one frame, and changes from one frame to another.

For a CT-MAC system with the OOAT, each received sample is the weighted superposition of symbols from up to N_c different users at any moment. In the mean time, each symbol, s_{nk}, is embedded in R received samples at the receiver. This is equivalent to an N_c-input R-output system. Due to linear independence among the coefficient vectors of different users, each symbol is embedded in R linearly independent equations, and each equation has at most N_c unknown variables.

Since each user in the OOAT employs a unique transmission pattern, the PHY layer structure of the OOAT scheme can be considered as a special case of the sparsely spread CDMA systems [18] and [19]. The OOAT is a cross-layer technique designed to achieve collision-tolerance through joint design across the PHY and MAC layers. It will be shown through

[1]It should be noted that on-off transmission has been used in [17] to achieve virtual full duplex, where each user can receive from other users during its off-slots among the on-slots. In this paper, on-off transmission in OOAT is used instead to reduce the probability of collisions at the receiver.

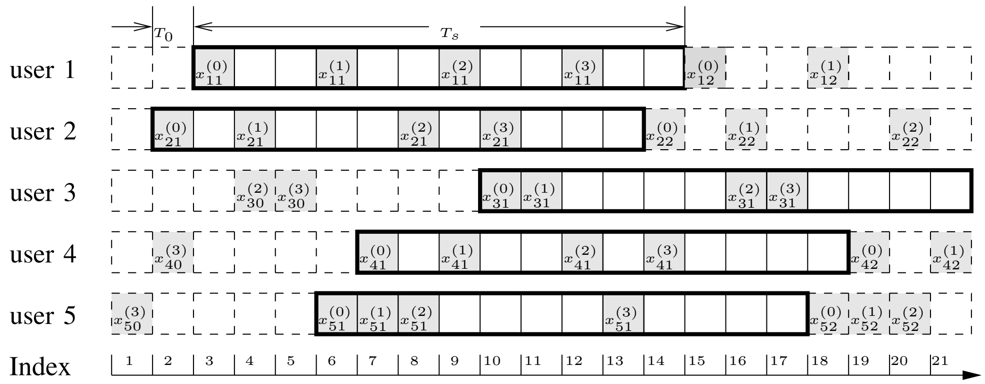

Fig. 1. An OOAT system with $N = 5$ users, $R = 4$ repetitions per symbol period, and each symbol contains $M = 12$ possible sub-symbol positions.

simulations that, by jointly utilizing the multiuser detection in the PHY layer and collision-tolerance in the MAC layer, the OOAT scheme with M sub-symbols per symbol period can support $N > M$ simultaneous users.

B. Collision Tolerance

A multiuser system with OOAT is called collision-tolerant if the N_c-dimensional signal at the input can be recovered by the R-dimensional signal at the output.

If $N_c \leq R$, the system is always collision-tolerant because it is equivalent to a consistent over-determined linear system with more equations than unknowns ($R > N_c$), or a symmetric system with equal number of equations and unknowns ($R = N_c$). The R system equations are always linearly independent due to the linear independence of the weight vectors from different users. Therefore, the $N_c \leq R$ unknowns in the system of R linearly independent equations can always be solved.

If $N_c > R$, then the OOAT system is equivalent to an under-determined linear system. In this case, the system is not collision-tolerant if the transmitted signal, s_{nk}, is analog, because an under-determined linear system has infinite analog solutions. On the other hand, if the transmitted signal has a finite alphabet, then the system may be still collision-tolerant as long as there is a unique intersection between the solution set of the under-determined system and the finite alphabet set.

Intuitively, the number of repetition has two opposite effects on the collision tolerance. For a given number of users and the number of sub-symbol positions per symbol, a smaller repetition number means fewer potential collisions among the users, and this contributes positively to the collision tolerance of the system. On the other hand, a smaller R means a smaller dimension of the received signal, and this contributes negatively to the collision tolerance. The choice of R depends on which of the two effects is dominating, which in turn is determined by the parameters M and N.

In summary, the OOAT scheme contributes to the collision tolerance of a system in two aspects. First, the on-off transmission will reduce the collision order, N_c, at the BS. Second, the transmission of R identical sub-symbols results in an R-dimensional received signal in the time-domain, which can be used for the detection of the N_c-dimension signal in the spatial-domain.

III. Properties of Position Vector Set

The position vectors play a critical role on the collision tolerance of the OOAT system. We can reduce or minimize the collision order, N_c, by carefully constructing the set of available position vectors. Theoretical properties of the position vectors, $\{\mathbf{p}_n\}_{n=1}^N$, are analyzed in this section, which provide a guideline for the design of OOAT systems.

Denote $\mathcal{P}_M^R = \left\{\mathbf{p} | \mathbf{p} \in \mathcal{B}^{M\times 1}, w(\mathbf{p}) = R \right\}$, with $w(\mathbf{p})$ being the Hamming weight of the binary vector $\mathbf{p}$. The set $\mathcal{P}_M^R$ contains all the length-M weight-R binary vectors, and its cardinality is $|\mathcal{P}_M^R| = \binom{M}{R}$. The position vector set is a subset of $\mathcal{P}_M^R$.

Due to the relative delays among the users, the receiver observes a cyclic-shifted version of $\mathbf{p}$. Define the set that contains all the cyclic-shifted versions of $\mathbf{p}$ as $\mathcal{O}(\mathbf{p}) = \left\{\mathbf{q} | \mathbf{q} \in \mathcal{P}_M^R; \exists k, (\mathbf{q})_k = \mathbf{p}\right\}$, where $(\mathbf{q})_k = [q(M-k+1), \cdots, q(M), q(1), \cdots, q(M-k)]^T$ is obtained by cyclically shifting $\mathbf{q}$ to the right by k positions. To ensure that the receiver can distinguish between signals from any two users with an arbitrary relative delay, we should have $\mathcal{O}(\mathbf{p}_n) \bigcap \mathcal{O}(\mathbf{p}_m) = \emptyset$ if $m \neq n$. Define a subset $\mathcal{Q}_M^R \subseteq \mathcal{P}_M^R$ as

$$\mathcal{Q}_M^R = \left\{\mathbf{q}_n | \mathbf{q}_n \in \mathcal{P}_M^R; \mathcal{O}(\mathbf{q}_m) \bigcap \mathcal{O}(\mathbf{q}_n) = \emptyset, \forall m \neq n \right\}. \quad (2)$$

Definition 1: (*Cyclic-Identical*) Two length-M weight-R binary vectors, $\mathbf{p}, \mathbf{q} \in \mathcal{P}_M^R$, are cyclic-identical if and only if $\mathbf{p} \in \mathcal{O}(\mathbf{q})$. In that case, $\mathcal{O}(\mathbf{p}) = \mathcal{O}(\mathbf{q})$.

Definition 2: (*Cyclic-Exclusive*) Two length-M weight-R binary vectors, $\mathbf{p}, \mathbf{q} \in \mathcal{P}_M^R$, are cyclic-exclusive if they are not cyclic-identical.

Definition 3: (*Cyclic Order*) The cyclic order of a sequence $\mathbf{q} \in \mathcal{Q}_M^R$ is defined as the smallest integer k that satisfies $\mathbf{q} = (\mathbf{q})_k$, and it is denoted as $C(\mathbf{q}) = k$.

The set, $\mathcal{Q}_M^R$, contains all the length-M weight-R binary vectors that are pairwise cyclic-exclusive. The binary position sequences need to be selected from $\mathcal{Q}_M^R$ to make sure that no two sequences belong to the same cyclic-shift set, $\mathcal{O}(\mathbf{p})$. The maximum number of users that can be supported by an OOAT system with parameters M and R is thus equal to the cardinality of $\mathcal{Q}_M^R$, which can be recursively calculated as stated in the following theorems.

Theorem 1: Define $P(M, R) = |\mathcal{Q}_M^R|$ as the cardinality of

$\mathcal{Q}_M^R$, then

$$P(M,R) = \frac{\binom{M}{R}}{M} + \sum_{c\in\mathcal{D}_M^R} V_M^R(c)\left(1-\frac{1}{c}\right), \tag{3}$$

where $\mathcal{D}_M^R$ is the set of all the common divisors of M and R, and $V_M^R(c)$ is the number of unique sequences in $\mathcal{Q}_M^R$ with a cyclic order $k = \frac{M}{c}$, $\forall c \in \mathcal{D}_M^R$.

The calculation of $P(M,R)$ requires the value of $V_M^R(c)$, which in turn can be calculated from $P(M,R)$ as stated in the following theorem.

Theorem 2: For any $c \in \mathcal{D}_M^R$, the value of $V_M^R(c)$ can be calculated as

$$V_M^R(c) = P\left(\frac{M}{c},\frac{R}{c}\right) - \sum_{u>1, u\in\mathcal{D}_{M/c}^{R/c}} V_M^R(uc). \tag{4}$$

The proofs of Theorems 1 and 2 are in the Appendix.

The values of $P(M,R)$ and $V_M^R(c)$ can be calculated recursively by employing Theorems 1 and 2. To initialize the recursion, first find the greatest common divisor (gcd) of M and R, and denote it as c'. From (3) and (4), we have $V_M^R(c') = P\left(\frac{M}{c'},\frac{R}{c'}\right) = \frac{1}{M/c'}\binom{M/c'}{R/c'}$, which can be used as the initial value to recursively calculate $P(M,R)$ and $V_M^R(c)$, $\forall c \in \mathcal{D}_M^R$.

Based on Theorems 1 and 2, the maximum number of users, $\max(N) = P(M,R)$, that can be supported by an OOAT system is shown in Fig. 2. $P(M,R)$ grows rapidly with M, with the slope increases as R increases.

The position vector set can be generated through induction. The induction is initialized with $\mathcal{Q}(M,1) = \{[1, \mathbf{0}_{M-1}]\}$, where $\mathbf{0}_{M-1}$ is a length-$(M-1)$ all-zero row vector. Assume that $\mathcal{Q}(M,R-1)$ has been constructed. Initialize $\mathcal{Q}(M,R)$ as an empty set. Then for a vector, $\mathbf{q} \in \mathcal{Q}(M,R-1)$, we can obtain $M-R+1$ weight-R vectors by replacing one of the $M-R+1$ 0s in $\mathbf{q}$ with 1. Each of the new vector will be compared against all the existing vectors in $\mathcal{Q}(M,R)$ and only the one that is cyclic exclusive with all existing vectors will be added to $\mathcal{Q}(M,R)$. The above procedure will be repeated until all the vectors in $\mathcal{Q}(M,R-1)$ has been processed. Since the position vector set for a given M and R only needs to be constructed once offline, the complexity incurred by the position vector construction will not affect the performance of the OOAT scheme.

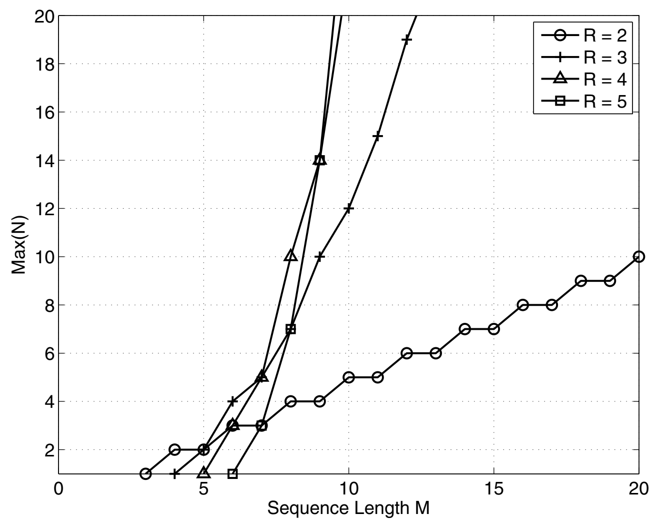

Fig. 2. Maximum number of users supported by a CT-MAC system with the OOAT. (N: number of users; M: number of sub-symbols/symbol; R: number of repetitions per symbol.)

IV. Iterative Detection of OOAT Signals

An iterative detection method is presented in this section to recover the transmitted symbols from signals colliding at the BS. The new detection method is developed by exploiting the unique structures of the OOAT.

In OOAT, the symbol from one user usually overlaps with two consecutive symbols from another user in the time-domain. Therefore, each received sample is a superposition of symbols from multiple users and noise, and the composition of the received sample changes from sample to sample. As a result, the *co-channel interference* (CCI) structure changes with respect to time. The presence of CCI dictates that the optimum detection should be performed in terms of a sequence-based detection, such as the *maximum likelihood sequence estimation* (MLSE) with a trellis-based detection method. However, the well known trellis-based Viterbi algorithm [21] cannot be directly applied to the detection of OOAT signals, mainly due to the time-varying CCI structure, which results a trellis structure that varies with time. An extended Viterbi algorithm with a time-varying trellis was proposed in [20] for the optimum detection of the OOAT signals. The complexity of the optimum detection grows exponentially with the number of users and the modulation size. To further reduce the complexity of the receiver, we develop a sub-optimum iterative detection method with soft interference cancellation (SIC).

The iterative detection method is developed by utilizing the signal structure of the OOAT. For an OOAT system with M sub-symbol positions per symbol period, the received sample stream is first divided into blocks, such that each block contains M received samples. The received sample vector in the k-th block can then be written as

$$\mathbf{y}_k = \mathbf{C}_0 \cdot \mathbf{F} \cdot \mathbf{s}_{k-1} + \mathbf{C}_1 \cdot \mathbf{F} \cdot \mathbf{s}_k + \mathbf{z}_k \tag{5}$$

where $\mathbf{y}_k = [y_{kM+1}, \cdots, y_{kM+M}]^T \in \mathcal{C}^{M\times 1}$, $\mathbf{z}_k = [z_{kM+1}, \cdots, z_{kM+M}]^T \in \mathcal{C}^{M\times 1}$ are the signal sample vector and AWGN vector, respectively, $\mathbf{s}_k = [s_{1k}, s_{2k}, \cdots, s_{Nk}]^T \in \mathcal{C}^{N\times 1}$ is the multiuser signal vector of the k-th symbol, and $\mathbf{F} = \text{diag}\{\mathbf{h}\} \in \mathcal{C}^{N\times N}$ is a diagonal matrix with $\mathbf{h} = [h_1, \cdots, h_N]^T$ on its main diagonal. The effects of the position vectors and the weight coefficients are represented in the position matrices, $\mathbf{C}_i \in \mathcal{B}^{M\times N}$, for $i = 0, 1$. The (m,n)-th element of $\mathbf{C}_0$ is $w_n^{r_{nm}}$ if $s_{n(k-1)}$ from the n-th user is transmitted at the sub-symbol position $kM+m$, and 0 otherwise. $\mathbf{C}_1$ is defined similarly based on s_{nk}.

For example, for the system described in Fig. 1, if we group the samples from the first $M = 12$ sub-symbol positions into a block, then the position matrices $\mathbf{C}_0$ and $\mathbf{C}_1$ are

$$\mathbf{C}_0=\begin{pmatrix} 0 & 0 & 0 & 0 & w_5^3 \\ 0 & 0 & 0 & w_4^3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w_3^2 & 0 & 0 \\ 0 & 0 & w_3^3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \mathbf{C}_1=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & w_2^0 & 0 & 0 & 0 \\ w_1^0 & 0 & 0 & 0 & 0 \\ 0 & w_2^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ w_1^1 & 0 & 0 & 0 & w_5^0 \\ 0 & 0 & 0 & w_4^0 & w_5^1 \\ 0 & w_2^2 & 0 & 0 & w_5^2 \\ w_1^2 & 0 & 0 & w_4^1 & 0 \\ 0 & w_2^3 & w_3^0 & 0 & 0 \\ 0 & 0 & w_3^1 & 0 & 0 \\ w_1^3 & 0 & 0 & w_4^2 & 0 \end{pmatrix}.$$

In the k-th block, the matrices $\mathbf{C}_0$ and $\mathbf{C}_1$ correspond to the transmission positions of the previous symbol vector, $\mathbf{s}_{k-1}$, and the current symbol vector, $\mathbf{s}_k$, respectively. The sum of the number of non-zero elements on the n-th columns of $\mathbf{C}_0$ and $\mathbf{C}_1$ is always R. It should be noted that in order to write the system equation in the form of (5), the block should be chosen in such a way that the samples in each block are contributed by at most two symbol vectors, $\mathbf{s}_{k-1}$ and $\mathbf{s}_k$. This requirement is always satisfied since there are at most two different symbols for each user transmitted during M sub-symbol positions.

With the model in (5), there are both *inter-block interference* (IBI) between $\mathbf{s}_{k-1}$ and $\mathbf{s}_k$, and CCI among the symbols inside $\mathbf{s}_k$. We propose to detect each block separately, with one symbol vector, $\mathbf{s}_k$, detected by using one sample block, $\mathbf{y}_k$. With such a block-wise detection, the IBI is removed by subtracting the soft-decisions from the previous block. The CCI among the symbols inside $\mathbf{s}_k$ is mitigated by employing a *soft-input soft-output* (SISO) equalizer. The SISO equalizer can be implemented with any equalizer that supports soft information processing. In this paper, SISO *block decision feedback equalization* (BDFE) [22] is used as the SISO equalizer.

For the k-th block, the soft-input to the SISO equalizer is the *a priori* probability of the symbols, $P(s_{nk} = S_i)$, for $n = 1, \cdots, N$ and $i = 1, \cdots, S$, where $S_i \in \mathcal{S}$ with $\mathcal{S}$ being the modulation constellation set with cardinality S. The *a priori* information is obtained from the previous detection round with an iterative detection method, and details will be given later in this section. The soft-output of the equalizer is the *a posteriori* probability of the symbols, $P(s_{nk} = S_i|\mathbf{y}_k)$, for $n = 1, \cdots, N$ and $i = 1, \cdots, S$. With the soft-output at the equalizer output, define the *a posteriori* mean, $\hat{s}_{nk}$, and the extrinsic information, β_{nk}, of the symbol s_{nk} as

$$\hat{s}_{nk} = \sum_{i=1}^{S} P(s_{nk} = S_i|\mathbf{y}_k)S_i, \tag{6a}$$

$$\text{and} \quad \beta_{nk}(i) = \log P(s_{nk}=S_i|\mathbf{y}_k) - \log P(s_{nk}=S_i). \tag{6b}$$

The extrinsic information will be used as the *a priori* input for the SISO equalization in the next round as $P(s_{nk} = S_i) = c_{nk}\exp[\beta_{nk}(i)]$, where c_{nk} is a normalization constant to make $\sum_{i=1}^{S} P(s_{nk} = S_i) = 1$.

The *a posteriori* mean vector, $\hat{\mathbf{s}}_k = [\hat{s}_{1k}, \cdots, \hat{s}_{Nk}]^T \in \mathcal{C}^{N\times 1}$, is the soft decision, and it is employed during the IBI cancellation as

$$\hat{\mathbf{y}}_k = \mathbf{y}_k - \mathbf{C}_0 \cdot \mathbf{F} \cdot \hat{\mathbf{s}}_{k-1} \approx \mathbf{C}_1 \cdot \mathbf{F} \cdot \mathbf{s}_k + \mathbf{z}_k. \tag{7}$$

Then the SISO equalizer can be applied to $\hat{\mathbf{y}}_k$ in (7) by treating $\mathbf{H}_F = \mathbf{C}_1 \cdot \mathbf{F}$ as the channel matrix, and the result is the *a posteriori* mean and extrinsic information of $\mathbf{s}_k$ as in (6).

The above procedures will be repeated until the end of a detection window containing K blocks. Since $\mathbf{s}_{k-1}$ is detected before $\mathbf{s}_k$, we denote the IBI cancellation in (7) as forward SIC, or FSIC. In the above procedures, the matrix $\mathbf{C}_0$ is used for IBI cancellation, and the matrix $\mathbf{C}_1$ is used for CCI cancellation.

Once reaching the end of a detection window, we can reverse the detection order by detecting $\mathbf{s}_K$ first and $\mathbf{s}_1$ last. In this case, the IBI cancellation will be performed with a backward SIC, or BSIC, as

$$\hat{\mathbf{y}}_k^{(B)} = \mathbf{y}_k - \mathbf{C}_1 \cdot \mathbf{F} \cdot \hat{\mathbf{s}}_k \approx \mathbf{C}_0 \cdot \mathbf{F} \cdot \mathbf{s}_{k-1} + \mathbf{z}_k. \tag{8}$$

After the IBI cancellation, $\mathbf{s}_{k-1}$ can be detected by applying the SISO-BDFE on $\hat{\mathbf{y}}_k^{(B)}$ and $\mathbf{H}_B = \mathbf{C}_0 \cdot \mathbf{F}$, with the extrinsic information $\beta_{n(k-1)}(i)$ from the FSIC as the *a priori* information at the equalizer input. In the BSIC process, $\mathbf{C}_1$ is used for IBI cancellation and $\mathbf{C}_0$ is used for CCI cancellation. Replacing $\mathbf{C}_1$ with $\mathbf{C}_0$ during the SISO-BDFE will generate new information that is independent of what is obtained during the FSIC.

The combination of the FSIC and BSIC forms one iteration of the detection process. The extrinsic output of the SISO-BDFE during the BSIC will be used as the *a priori* input for the FSIC in the next iteration.

The iterative detection is summarized as follows.

I) **Initialization**
   Set $P(s_{nk}) = \frac{1}{S}$, for $n = 1, \cdots, N$ and $k = 1, \cdots, K$.

II) **FSIC**
   i) Set $k = 1$.
   ii) Perform forward IBI cancellation by subtracting the *a posteriori* mean, $\hat{\mathbf{s}}_{k-1}$, from $\mathbf{y}_k$ as in (7).
   iii) Apply SISO-BDFE on $\hat{\mathbf{y}}_k$ with the help of $\mathbf{C}_1$ and $\beta_{nk}(i)$ from the BSIC of the previous iteration.
   iv) Calculate the *a posteriori* mean, $\hat{s}_{nk}$, and the extrinsic information, $\beta_{nk}(i)$, with the SISO-BDFE.
   v) Increase k by 1. If $k < K$, go back to II-ii), otherwise go to BSIC in Step III).

III) **BSIC**
   i) Set $k = K$.
   ii) Perform backward IBI cancellation by subtracting the *a posteriori* mean, $\hat{\mathbf{s}}_k$, from $\mathbf{y}_k$ as in (8).
   iii) Apply SISO-BDFE on $\hat{\mathbf{y}}_k^{(B)}$ with the help of $\mathbf{C}_0$ and $\beta_{n(k-1)}(i)$ from the FSIC in the same iteration.
   iv) Calculate the *a posteriori* mean, $\hat{s}_{nk}$, and the extrinsic information, $\beta_{nk}(i)$, with the SISO-BDFE.
   v) Decrease k by 1. If $k > 1$, go back to III-ii).
   vi) If the maximum number of iteration is reached, go to Step IV). Otherwise, go back to Step II).

IV) **Hard Decision**
   Make hard decisions as $\tilde{s}_{nk} = \underset{s_{nk}\in\mathcal{S}}{\text{argmax}}\, P(s_{nk} = S_i|\mathbf{y}_k)$, for $n = 1, \cdots, N$ and $k = 1, \cdots, K$.

The system model in (5) and the proposed detector assume synchronization among the users in the sub-symbol level, even though the users employ uncoordinated transmission.

The synchronization among the users can be achieved by requiring all the users follow a reference clock from the BS, or through global position system receivers. Similar synchronization schemes are used in most practical CDMA systems.

In order to construct $\mathbf{C}_0$ and $\mathbf{C}_1$ in (5), the receiver needs to know the relative delays among the users. We estimate the timing information of different users by utilizing the unique structures of the position vectors. When a new user joins the network, the BS randomly picks a position vector from the set of unused position vectors, and assign the randomly picked vector to the new user, which is however known to both the transmitter and the receiver.

Before transmitting data, user n will first transmit a sequence of L pilot symbols, $\mathbf{a}_n = [a_{n1}, \cdots, a_{nL}]^T$, with the OOAT. The receiver can identify the relative delay by performing correlation between the received samples and the position vector as (c.f. (1))

$$\hat{p}_{n0} = \underset{p \in [0,\cdots,M-1]}{\text{argmax}} \sum_{l=1}^{L} a_{nl} \sum_{m=1}^{M} p_n(i_{nm})\, w_n^{-r_{nm}} y((l-1)M+m). \tag{9}$$

It will be shown in simulations that the receiver can obtain a reasonably accurate timing information with the above simple correlation operation.

V. MAC Layer Analysis

In the MAC layer, the data stream of a user is divided into frames. The data frames are transmitted in succession, *i.e.*, no space is inserted between two consecutive frames. This is different from most traditional MAC schemes, where random intervals are inserted between the transmission of two consecutive frames to reduce the collision among the users. The OOAT scheme, on the other hand, adds silence periods inside a frame to reduce the collision. Using on-off transmission on the symbol level, instead of the duty-cycled transmission on the frame level, renders a special structure in the received signal. This special signal structure in the MAC layer enables the joint signal detection in the PHY layer, and it is instrumental to the collision tolerance of the proposed MAC scheme.

After detection, the cyclic redundancy check or parity check can be applied to the frame to detect if there is an error in the detection. Frames with uncorrectable errors will be discarded and retransmitted. Retransmission is still employed in the CT-MAC scheme with OOAT. However, under the same offered load, the probability of retransmission in OOAT is usually much lower compared to conventional MAC schemes, where retransmission will occur whenever there is a collision or frame error at the receiver. The OOAT, on the other hand, can recover some of the collided frames with PHY layer detection, and only retransmit those frames with uncorrectable errors.

Consider an OOAT system with N active users and M sub-symbol positions per symbol period. If the duration of a MAC frame is $T_F = KT_s = KMT_0$, where K is the number of symbols per frame, then the offered data rate from all the active users is $\frac{NK}{T_F}\log_2(S) = \frac{N}{MT_0}\log_2(S)$ bps. The bandwidth of the OOAT system is inversely proportional to the duration of one sub-symbol T_0. The offered data rate normalized with respect to the bandwidth, or the normalized offered load, is then $G = \frac{N}{M}\log_2(S)$, representing the average number of bits transmitted per unit time per unit bandwidth. It should be noted that the normalized offered load is independent of the number of repetitions, R, the number of symbols per frame K, or the actual values of T_0.

Due to the retransmission of frames with uncorrectable errors, the normalized throughput of the system is

$$\eta_{\text{OOAT}} = \frac{N}{M}(1 - \text{FER}_{\text{OOAT}})\log_2(S), \tag{10}$$

where FER_{OOAT} is the *frame-error rate* (FER) of the OOAT scheme.

We next compare the performance of the OOAT scheme to the well known slotted-ALOHA MAC protocol through theoretical analysis. The performance of the OOAT scheme is also compared to more advanced MAC protocols, such as CRDSA [7] and IRSA [9], with simulations in Section VI.

For the slotted-ALOHA, we follow a similar model as in [9], where a MAC frame of duration T_F is divided into M slots of duration T_F/M each, and each of the N active users performs a single transmission attempt by transmitting a packet in one of the M slots. It is assumed that one packet contains K symbols with a symbol period T_0, such that the normalized offered load is $G = \left(\frac{NK}{T_F}/\frac{1}{T_0}\right)\log_2(S) = \frac{N}{M}\log_2(S)$ bps/Hz. The parameters are chosen such that OOAT and slotted-ALOHA have the same normalized offered load. The normalized offered load is independent of the actual values of T_F, T_0, or K, which means the two systems can choose different frame duration, symbol period, or bandwidth, yet still maintain the same normalized offered load.

It should be noted that the slotted-ALOHA has a transmission duty cycle of $\frac{1}{M}$, yet the OOAT has a transmission duty cycle of $\frac{R}{M}$. For fair comparison, the energy per transmitted bit of slotted-ALOHA is R times as high as that of OOAT, such that the energy per raw information bit of the two systems are the same.

In the slotted-ALOHA, each user chooses one of the M slots to transmit its packet with a probability of $P_t = \frac{1}{M}$. The packet from a user is successfully transmitted if the following two conditions are met: 1) there is one and only one packet transmitted at the slot chosen by the user; and 2) the packet at the receiver can be successfully detected. Therefore, the throughput of the slotted-ALOHA with N users is

$$\eta_{\text{SA}} = \frac{N}{M}\left(1 - \frac{1}{M}\right)^{N-1}(1 - \text{FER}_{\text{SA}})\log_2(S). \tag{11}$$

where FER_{SA} is the *non-collision* FER of the uncollided packets in the slotted-ALOHA system. Under the same system configuration, $\text{FER}_{\text{SA}} \leq \text{FER}_{\text{OOAT}}$ because the OOAT system performs detection over the colliding signals at the receiver. In the mean time, with the detection algorithm described in Section IV, the difference between FER_{SA} and FER_{OOAT} is usually very small. Our simulation results indicate that FER_{SA} and FER_{OOAT} are on the same order of magnitude.

Comparing (10) to (11), we can see that $\eta_{\text{OOAT}} \geq \eta_{\text{SA}}$ if the following condition is met

$$\left(1 - \frac{1}{M}\right)^{N-1} \leq \frac{1 - \text{FER}_{\text{OOAT}}}{1 - \text{FER}_{\text{SA}}}. \tag{12}$$

Table 1 The maximum value of $\mathrm{FER}_{\mathrm{OOAT}}$ such that $\eta_{\mathrm{OOAT}} > \eta_{\mathrm{SA}}$ ($N = 10$)

M	10	20	30	40	50
$\mathrm{FER}_{\mathrm{SA}} = 10^{-3}$	0.88	0.64	0.50	0.40	0.33
$\mathrm{FER}_{\mathrm{SA}} = 10^{-2}$	0.88	0.65	0.50	0.40	0.34
$\mathrm{FER}_{\mathrm{SA}} = 10^{-1}$	0.89	0.68	0.54	0.46	0.40

The above inequality is met for almost all the practical system configurations. Based on (12), Table 1 lists the maximum values of $\mathrm{FER}_{\mathrm{OOAT}}$ for various values of $\mathrm{FER}_{\mathrm{SA}}$ and M, for a system with $N = 10$ users. In practical systems, we usually have M on the order of tens. It can be seen from Table 1 that even when $\mathrm{FER}_{\mathrm{SA}} = 10^{-3}$ and $M = 50$, we always have $\eta_{\mathrm{OOAT}} > \eta_{\mathrm{SA}}$ as long as $\mathrm{FER}_{\mathrm{OOAT}} < 0.3331$, which is 333 times higher than $\mathrm{FER}_{\mathrm{SA}}$. Decreasing M from 50 to 10 further looses the requirement to $\mathrm{FER}_{\mathrm{OOAT}} < 0.8785$. Our simulation results indicate that $\mathrm{FER}_{\mathrm{SA}}$ and $\mathrm{FER}_{\mathrm{OOAT}}$ are usually on the same order of magnitude, such that $\frac{1-\mathrm{FER}_{\mathrm{OOAT}}}{1-\mathrm{FER}_{\mathrm{SA}}} \approx 1$. Thus $\eta_{\mathrm{OOAT}} > \eta_{\mathrm{SA}}$ can be achieved for almost all the practical system configurations. The above analysis is corroborated by simulation results presented in the next section.

VI. Simulation Results

Simulation results are presented in this section to demonstrate the performance of the proposed OOAT scheme.

We first investigate the BER performance of the OOAT with the sub-optimum iterative detection method. In the simulation, there are $M = 12$ sub-symbol positions and $R = 4$ repetitions per symbol period. The results are shown in Fig. 3. When $N = 1$, there is no collision, and the results serve as a lower bound for the best possible performance of the system. The iterative detection method was performed with 4 iterations. For comparison, the result from the system with the optimum trellis-based detection and $N = 4$ is also shown in the figure. It is clear from the figure that the sub-optimum iterative detection can achieve a performance that is almost identical to its optimum counterpart. In addition, the performance of the sub-optimum iterative detection method is reasonably close to the lower bound for a large range of N. As a result, the system is highly tolerant to collisions at the receiver.

Fig. 4 demonstrates the impact of the number of iterations on the FER performance. There are 100 symbols per frame, and all the parameters are the same as Fig. 3. The biggest improvement in the FER performance is achieved from the 1st iteration to the 2nd one. The performance improvement gradually diminishes as the number of iterations increases. The performance difference between the 3rd and 4th iterations is almost negligible. In addition, the benefit of iterations is more apparent at high E_b/N_0. When $E_b/N_0 \leq 10$ dB, it is sufficient to use 2 iterations.

Fig. 5 shows the normalized throughput as a function of the offered load, G, for the OOAT, slotted-ALOHA, CRDSA [7], and IRSA [8] systems, respectively. The result for CSA system [10] is only slightly better than IRSA [8] and is not shown here. The results for the OOAT and slotted-ALOHA systems are obtained at $E_b/N_0 = 20$ dB, whereas the simulations for the CRDSA and IRSA are performed in a noise-free channel, which gives the best possible performances for the two systems. The normalized throughputs of CRDSA, IRSA,

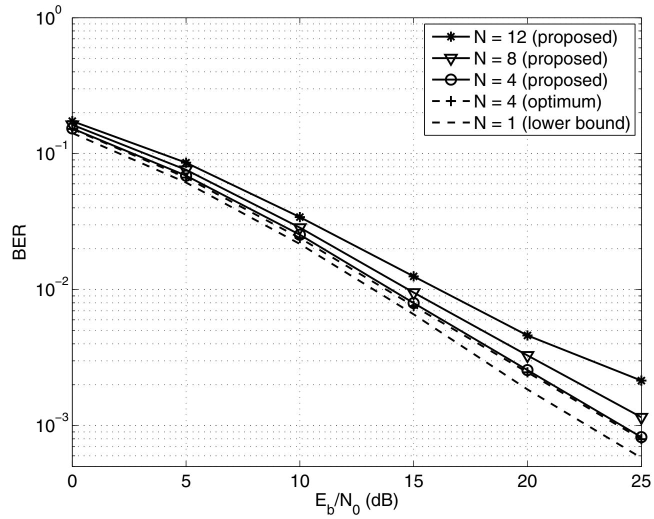

Fig. 3. BER performance of a system with $M = 12$ sub-symbols/symbol, $R = 4$ repetitions, and the sub-optimum iterative detection method.

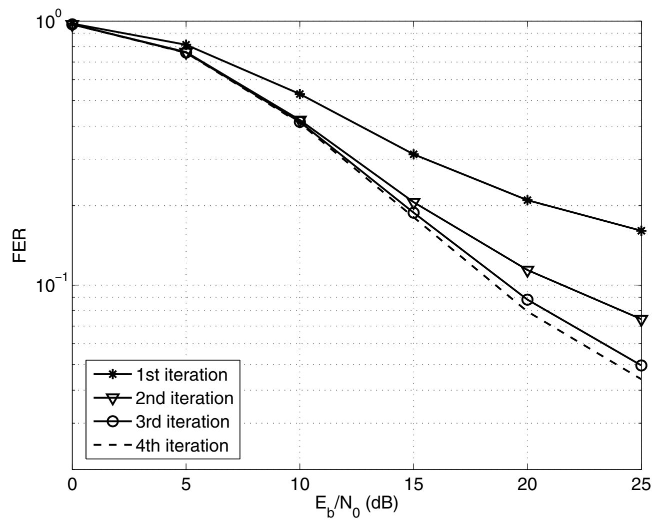

Fig. 4. FER performance of a system with $M = 12$ sub-symbols/symbol, $R = 4$ repetitions, $N = 12$ users, and the sub-optimum detection method.

slotted-ALOHA, and OOAT peak at $G = 0.6$, 0.8, 1, and 1.4, respectively, with the corresponding peak throughputs being 0.54, 0.77, 0.37, and 1.07, respectively. Among the four systems, only the OOAT system can effectively support a normalized offered load larger than 1, which means it can support $N > M$ simultaneous users. Therefore, the OOAT can support 133.3%, 75.0%, and 40.0% more users than CRDSA, IRSA, and slotted-ALOHA, respectively, with a peak throughput exceeding them by 98.5%, 39.0%, and 189.2%, respectively.

The next example investigates the impact of uneven power distribution among the users on the system performance. Since multiuser detection is employed at the OOAT receiver, the performance of the system will be negatively affected by the near-far effect, *i.e.*, signals from users that are far from the BS might be buried by the strong signals from users that are close to the BS if no power control is employed. The transmission power from all the users are assumed to be

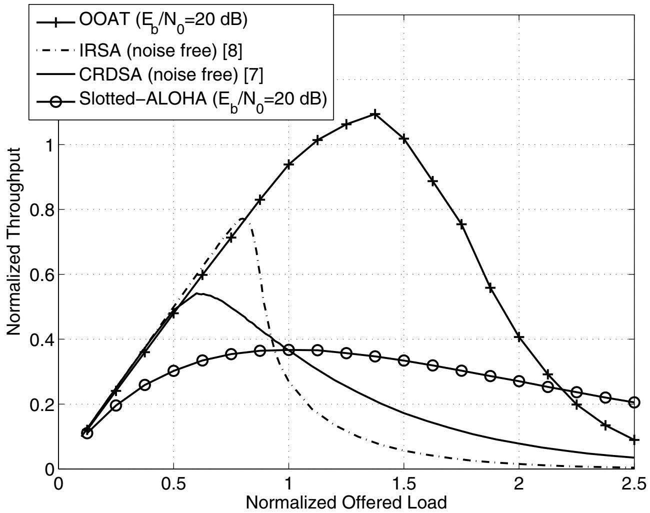

Fig. 5. Normalized throughput v.s. normalized offered load.

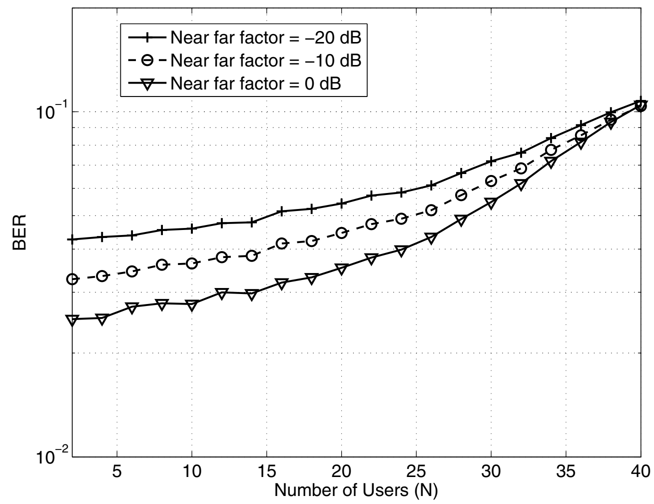

Fig. 6. Impacts of the near-far effect ($M = 12$ sub-symbols/symbol, $R = 4$ repetitions, $E_b/N_0 = 20$ dB).

uniformly distributed between $E_s[1-\omega, 1+\omega]$, and the near-far factor is defined as $\zeta = 10\log_{10}\frac{1-\omega}{1+\omega}$ dB [23]. Fig. 6 shows the BER as a function of N under various values of the near-far factor. There are $M = 12$ sub-symbol positions and $R = 4$ repetitions per symbol period, and E_b/N_0 = 10 dB. The near-far effect has relatively small impacts on the BER performance. For example, when $N = 20$, the BER degrades from 3.4×10^{-2} at $\zeta = 0$ dB to 5.5×10^{-2} at $\zeta = -20$ dB. Such a small change on BER has almost negligible effects on the normalized throughput. The impacts of the near far effect gradually diminish as N increases.

The impacts of timing estimation on the system performance is shown in Fig. 7. There are $N = 12$ users in the system, and $R = 4$ repetitions per symbol period. The E_b/N_0 is 20 dB. As expected, the performance improves as the number of pilot frames increases. The timing estimation in (9) treats the signals from all but the desired user as interference. Therefore there will be timing estimation error even at high E_b/N_0 due to the multiuser interference, especially when the number of pilot frames is small. This explains the gap among the different curves at high E_b/N_0. For the OOAT system, the performance of the system with the estimated timing information and 20 pilot frames is very close to that with ideal timing information.

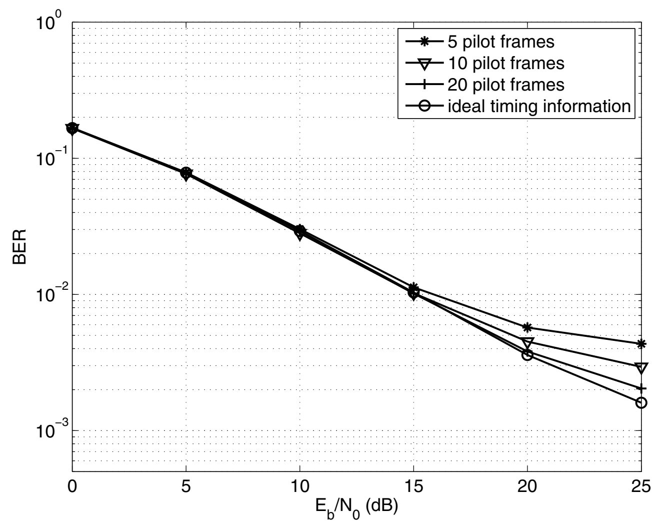

Fig. 7. BER performance of a system with estimated timing information ($N = 12$ users, $M = 12$ sub-symbols/symbol, $R = 4$ repetitions).

VII. Conclusion

A new OOAT scheme has been proposed for the cross-layer CT-MAC. The collision tolerance in the MAC layer is enabled through PHY layer operations, which include a simple OOAT scheme at the wireless users, and an iterative detector at the BS. In the MAC layer, the on-off transmission inserts silence periods inside a frame, instead of random intervals between frames in conventional MAC transmissions. Such a mechanism reduces the collision probability at the receiver and renders a special structure of the signals received at the receiver. The algebraic properties of the random position vectors have been analyzed, which provide theoretical guidelines on the choice of the OOAT system parameters. Both analytical and simulation results have shown that the OOAT scheme can reliably operate at the presence of severe signal collision. Compared to the other MAC schemes under the same offered load, the CT-MAC with OOAT supports more users, and increases the normalized throughput considerably.

Appendix

Before proceeding to the proof of Theorems 1 and 2, we need the following results about cyclic sequences.

Lemma 1: If the cyclic order of $\mathbf{p} \in \mathcal{P}_M^R$ is k, then $C(\mathbf{q}) = k$, $\forall \mathbf{q} \in \mathcal{O}(\mathbf{p})$.

Proof: Since $\mathbf{q} \in \mathcal{O}(\mathbf{p})$, $\exists 0 \le m < M$ such that $\mathbf{q} = (\mathbf{p})_m$. Then, $(\mathbf{q})_k = ((\mathbf{p})_m)_k = (\mathbf{p})_{m+k} = ((\mathbf{p})_k)_m = (\mathbf{p})_m = \mathbf{q}$.

Assume there exists $k' < k$ such that $(\mathbf{q})_{k'} = \mathbf{q}$. Since $\mathbf{p} \in \mathcal{O}(\mathbf{q})$, then $(\mathbf{p})_{k'} = \mathbf{p}$. This is contradictory to the fact that k is the cyclic order of $\mathbf{p}$. Thus the assumption is not true, and k is the smallest integer satisfying $(\mathbf{q})_k = \mathbf{q}$. ■

Proposition 1: If $\mathbf{p} \in \mathcal{P}_M^R$ and $0 < R < M$, then the cyclic-order of $\mathbf{p}$ must be in the form of $k = M/c$, where c

is a common divisor of M and R.

Proof: Assume $\mathbf{p} = (\mathbf{p})_k$, and $M = kc + u$, where c and u are integers, with $0 \leq u < k$. Based on Lemma 1, if the cyclic order of $\mathbf{p}$ is k, then all the vectors in $\mathcal{O}(\mathbf{p})$ have a cyclic order k. Form the M elements in $\mathbf{p}$ into a circle. Randomly pick an element, p_m, on the circle as the starting element of a vector $\mathbf{q}$, *i.e.* $q_1 = p_m$. Since $\mathbf{q} = (\mathbf{q})_k$, we have $q_1 = q_{k+1} = \cdots = q_{ck+1} = q_{k-u}$. The relationship $q_1 = q_{k-u}$ is true for any element of $\mathbf{p}$, thus $\mathbf{q} = (\mathbf{q})_{k-u}$. Therefore $u = 0$ because $C(\mathbf{q}) = k$, or $M = kc$, and c is a divisor of M.

Rewrite $\mathbf{p}$ as $\mathbf{p} = [\mathbf{p}_0, \cdots, \mathbf{p}_{c-1}]$, where $\mathbf{p}_m = [p(mk+1), \cdots, p(mk+k)] \in \mathcal{B}^k$, for $m = 0, \cdots, c-1$, are length-k sequences. Since $(\mathbf{p})_k = \mathbf{p}$, we must have $\mathbf{p}_0 = \cdots = \mathbf{p}_{c-1}$. Therefore, $R = w(\mathbf{p}) = c \cdot w(\mathbf{p}_0)$, so c is a divisor of R. ■

The following corollary follows immediately from Proposition 1.

Corollary 1: If $\mathbf{p} \in \mathcal{Q}_M^R$ and $\mathbf{p} = (\mathbf{p})_k$, then $\mathbf{p}$ can be represented as $\mathbf{p} = [\bar{\mathbf{p}}, \cdots, \bar{\mathbf{p}}]$, where $\bar{\mathbf{p}}$ is a length-k binary vector with weight R/c, and $c = M/k$.

With the above properties, now we are ready to prove Theorem 1.

Proof of Theorem 1: By shifting a sequence, $\mathbf{p} \in \mathcal{P}_M^R$, to the right by 1 position for M consecutive times, we can get M sequences, and some of the M sequences might be duplicates if the cyclic order of the sequence is less than M. Therefore we can get a total number of $P(M,R)M$ sequences by right shifting all the sequences in $\mathcal{Q}_M^R$ by 1 position for M consecutive times. $\binom{M}{R}$ sequences out of the $P(M,R)M$ sequences are unique, and the remaining $P(M,R)M - \binom{M}{R}$ are duplicates of one of the unique sequences.

If the cyclic order of $\mathbf{p}$ is k, then k out of the M sequences obtained by cyclic shifting $\mathbf{p}$ are unique (part of the $\binom{M}{R}$ sequences), and the remaining $M-k$ sequences are duplicates of one of the k unique sequences.

It has been shown that: 1) the cyclic order of the sequences in $\mathcal{Q}_M^R$ must be in the form of $\frac{M}{c}$, $c \in \mathcal{D}_M^R$ (Proposition 1); 2) by definition there are totally $V_M^R(c)$ unique sequences in $\mathcal{Q}_M^R$ with a cyclic order $\frac{M}{c}$. Then the total number of duplicate sequences from sequences with cyclic order $\frac{M}{c}$ is $V_M^R(c)\left(M - \frac{M}{c}\right)$. The total number of duplicate sequences is thus $\sum_{c \in \mathcal{D}_M^R} V_M^R(c)\left(M - \frac{M}{c}\right)$, which equals to $P(M,R)M - \binom{M}{R}$, and (3) follows immediately. ■

The proof of Theorem 2 requires the following Lemmas.

Lemma 2: Consider $\mathbf{p} = [\bar{\mathbf{p}}, \cdots, \bar{\mathbf{p}}] \in \mathcal{Q}_M^R$, and $\mathbf{q} = [\bar{\mathbf{q}}, \cdots, \bar{\mathbf{q}}] \in \mathcal{Q}_M^R$, where $\bar{\mathbf{p}}$ and $\bar{\mathbf{q}}$ are length-k binary vectors with weight R/c, where $k = M/c$ and $c \in \mathcal{D}_M^R$. Then the following statements are equivalent. 1) $\mathbf{p}$ and $\mathbf{q}$ are cyclic-exclusive; 2) $\bar{\mathbf{p}}$ and $\bar{\mathbf{q}}$ are cyclic-exclusive.

Proof: Proof by contradiction. If $\mathbf{p}$ and $\mathbf{q}$ are cyclic-exclusive, assume $\bar{\mathbf{p}} = (\bar{\mathbf{q}})_j$ for some integer j. Then $\mathbf{p} = [(\bar{\mathbf{q}})_j, \cdots, (\bar{\mathbf{q}})_j] = (\mathbf{q})_j$, which contradicts with the fact that $\mathbf{p}$ and $\mathbf{q}$ are cyclic-exclusive. Therefore $\bar{\mathbf{p}}$ and $\bar{\mathbf{q}}$ are cyclic-exclusive.

On the other hand, if $\bar{\mathbf{p}}$ and $\bar{\mathbf{q}}$ are cyclic-exclusive, assume $\mathbf{p} = (\mathbf{q})_j$ for some integer j. Then $(\mathbf{q})_j = [(\bar{\mathbf{q}})_j, \cdots, (\bar{\mathbf{q}})_j] = [\bar{\mathbf{p}}, \cdots, \bar{\mathbf{p}}]$. Thus $\bar{\mathbf{p}} = (\bar{\mathbf{q}})_j$, and this contradicts with the fact that $\bar{\mathbf{p}}$ and $\bar{\mathbf{q}}$ are cyclic-exclusive. Therefore $\mathbf{p}$ and $\mathbf{q}$ are cyclic-exclusive. ■

Lemma 3: Consider $\mathbf{p} = [\bar{\mathbf{p}}, \cdots, \bar{\mathbf{p}}] \in \mathcal{Q}_M^R$. If $C(\bar{\mathbf{p}}) = k$, then $C(\mathbf{p}) = k$.

Proof: If $C(\bar{\mathbf{p}}) = k$, it is straightforward to show that $(\mathbf{p})_k = \mathbf{p}$, thus $C(\mathbf{p}) \leq k$. If $\exists j < k$ such that $\mathbf{p} = (\mathbf{p})_j = [(\bar{\mathbf{p}})_j, \cdots, (\bar{\mathbf{p}})_j]$, then $(\bar{\mathbf{p}})_j = \bar{\mathbf{p}}$, which contradicts $C(\bar{\mathbf{p}}) = k$. Thus $C(\mathbf{p}) = k$. ■

Now we are ready to prove Theorem 2.

Proof of Theorem 2: Given $c \in \mathcal{D}_M^R$, define $\mathcal{A}_M^R(c) = \left\{\mathbf{q} | \mathbf{q} = \mathbf{1}_c \otimes \bar{\mathbf{q}}, \bar{\mathbf{q}} \in \mathcal{Q}_{M/c}^{R/c}\right\}$, where $\mathbf{1}_c$ is a length-c all-one row vector, and $\otimes$ is the Kronecker product. The set $\mathcal{A}_M^R(c)$ contains all the sequences obtained by repeating $\bar{\mathbf{q}} \in \mathcal{Q}_{M/c}^{R/c}$ c times. We will prove that the cardinality of $\mathcal{A}_M^R(c)$ is $\left|\mathcal{A}_M^R(c)\right| = P\left(\frac{M}{c}, \frac{R}{c}\right) = \sum_{u \in \mathcal{D}_{M/c}^{R/c}} V_M^R(uc)$.

1) By definition, there are $P\left(\frac{M}{c}, \frac{R}{c}\right) = |\mathcal{Q}_{M/c}^{R/c}|$ unique subsequences $\{\bar{\mathbf{q}}_m\}_m$ that are mutually cyclic-exclusive. Each $\bar{\mathbf{q}}_m$ uniquely determines one $\mathbf{q}_m$ in $\mathcal{A}_M^R(c)$. From Lemma 2, for $m \neq n$, the sequences $\mathbf{q}_m, \mathbf{q}_n \in \mathcal{A}_M^R(c)$ are cyclic-exclusive. Therefore, $|\mathcal{A}_M^R(c)| = P\left(\frac{M}{c}, \frac{R}{c}\right)$.

2) Given $\bar{\mathbf{q}}_m \in \mathcal{Q}_{M/c}^{R/c}$, from Proposition 1, the cyclic order of $\bar{\mathbf{q}}_m$ can only take the values in the form of $j = M/(cu)$, where $u \in \mathcal{D}_{M/c}^{R/c}$. Given the one-to-one relationship between $\bar{\mathbf{q}}_m$ and $\mathbf{q}_m$, the cyclic order $\mathbf{q}_m \in \mathcal{A}_M^R(c)$ must be in the form of $M/(cu)$ from Lemma 3. By definition, there are $V_M^R(cu)$ cyclic-exclusive sequences in $\mathcal{A}_M^R(c) \subseteq \mathcal{Q}_M^R(c)$ with a cyclic order $M/(cu)$. Therefore, the total number of sequences in $\mathcal{A}_M^R(c)$ is $|\mathcal{A}_M^R(c)| = \sum_{u \in \mathcal{D}_{M/c}^{R/c}} V_M^R(uc)$. ■

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Jingxian Wu (S'02-M'06) received the B.S. (EE) degree from the Beijing University of Aeronautics and Astronautics, Beijing, China, in 1998, the M.S. (EE) degree from Tsinghua University, Beijing, China, in 2001, and the Ph.D. (EE) degree from the University of Missouri at Columbia, MO, USA, in 2005.

He is currently an Assistant Professor with the Department of Electrical Engineering, University of Arkansas, Fayetteville. His research interests mainly focus on wireless communications and wireless networks, including ultra-low power communications, energy efficient communications, cognitive radio, and cross-layer optimization, etc. He is an Editor of the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, and served as an Associate Editor of the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY from 2007 to 2011. He served as a Cochair for the 2012 Wireless Communication Symposium of the IEEE International Conference on Communication, and a Cochair for the 2009 Wireless Communication Symposium of the IEEE Global Telecommunications Conference. Since 2006, he has served as a Technical Program Committee Member for a number of international conferences, including the IEEE Global Telecommunications Conference, the IEEE Wireless Communications and Networking Conference, the IEEE Vehicular Technology Conference, and the IEEE International Conference on Communications.

Geoffrey Ye Li received his B.S.E. and M.S.E. degrees in 1983 and 1986, respectively, from the Department of Wireless Engineering, Nanjing Institute of Technology, Nanjing, China, and his Ph.D. degree in 1994 from the Department of Electrical Engineering, Auburn University, Alabama.

He was a Teaching Assistant and then a Lecturer with Southeast University, Nanjing, China, from 1986 to 1991, a Research and Teaching Assistant with Auburn University, Alabama, from 1991 to 1994, and a Post-Doctoral Research Associate with the University of Maryland at College Park, Maryland, from 1994 to 1996. He was with AT&T Labs - Research at Red Bank, New Jersey, as a Senior and then a Principal Technical Staff Member from 1996 to 2000. Since 2000, he has been with the School of Electrical and Computer Engineering at Georgia Institute of Technology as an Associate and then a Full Professor. He is also holding the Cheung Kong Scholar title at the University of Electronic Science and Technology of China since March 2006.

His general research interests include statistical signal processing and telecommunications, with emphasis on cross-layer optimization for spectral- and energy-efficient networks, cognitive radios, and practical techniques in LTE systems. In these areas, he has published over 100 referred journal papers and two books in addition to many conference papers. He also has over 20 granted patents. His publications have been cited over 10,000 times and he is listed as a highly cited researcher by Thomson Reuters. He once served or is currently serving as an editor, a member of editorial board, and a guest editor for over 10 technical journals. He organized and chaired many international conferences, including technical program vice-chair of IEEE ICC'03 and co-chair of IEEE SPARC'11. He has been awarded an IEEE Fellow for his contributions to signal processing for wireless communications since 2006, selected as a Distinguished Lecturer for 2009 - 2010 by IEEE Communications Society, and won 2010 IEEE Communications Society Stephen O. Rice Prize Paper Award in the field of communications theory.