An Achievable Rate Region for Cooperative Multiple Access Channel

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***Abstract*— An achievable rate region for a 3-node cooperative multiple access channel (CMAC) is presented in this paper. A CMAC contains multiple users cooperatively transmitting information to a single destination node. It includes the traditional multiple access channel that assumes no communication between source nodes, and multiple input single output (MISO) system that contains ideal links between source nodes, as special cases. The achievable rate regions for both general CMAC and degraded Gaussian CMAC are derived with the assistance of a block Markov encoding scheme. The impacts of cooperation strategies between source nodes on node transmission rates are investigated by modeling node cooperation as a strategic game, and the Nash equilibrium of cooperation strategy for degraded Gaussian CMAC is identified.**

I. Introduction

A multiple access channel contains two or more users transmitting information to a common destination through a shared physical channel [1] – [3]. Examples of a multiple access channel include a set of cell phones communicating with a basestation, or a group of spatially distributed sensor nodes transmitting data to a central data collection point.

Two types of multiple access channels with different cooperation strategies have been studied extensively in the literature. The first type of multiple access channel, which is denoted as traditional multiple access channel in this paper, assumes that all source nodes (users) do not attempt to detect or relay information transmitted by other source nodes [1] – [4]. As a result, source nodes transmit independent signals without explicit form of cooperation. The second type of multiple access channel assumes that each source node is fully aware of the information to be transmitted by all other source nodes in the network [4], [5], and it can be used to model a multiple input single output (MISO) system with multiple transmission antennas and one receiving antenna. In this case, all users (antennas) can transmit cooperatively by utilizing the channel as an ordinary one user channel.

In practical communication networks with full-duplex source nodes, each source node can detect and relay signals transmitted by other users. We define cooperative multiple access channel (CMAC) as a multiple access channel that allows users to relay each other's information to destination. In this paper, we focus on the study of a 3-node CMAC with two source nodes, N_1 and N_2, and one destination node, D, as shown in Fig. 1. The two source nodes transmit their own information as well as perform cooperative communication by relaying each other's information to the destination. Thus, a CMAC can be considered as an overlay of two relay channels [6], [7], $N_1 \rightarrow N_2 \rightarrow D$, and $N_2 \rightarrow N_1 \rightarrow D$, and a traditional multiple access channel, $(N_1, N_2) \rightarrow D$.

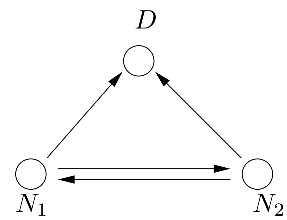


Fig. 1. A 3-node cooperative multiple access channel.

An achievable rate region for a 3-node cooperative multiple access channel is developed with the assistance of a block Markov encoding scheme. During the transmission of each information block, each source node will transmit not only its own message, but also a cooperative message derived based on distorted observation of signal transmitted by the other source node in the previous block. Achievable rate regions for both general CMAC and degraded Gaussian CMAC are developed. The results include the capacity regions of traditional multiple access channel, MISO system, and relay channel as special cases. The impact of different cooperation strategies between the two source nodes on rate regions are investigated with the help of game theory and Nash equilibrium.

II. Preliminaries

Consider a 3-node network as shown in Fig. 1. Let $x_s \in \mathcal{X}_s$ and $y_s \in \mathcal{Y}_s$ denote the signals transmitted and received by source node N_s, respectively, for $s = 1, 2$, and $y \in \mathcal{Y}$ the signal received at the destination node D. The discrete memoryless cooperative multiple access channel can then be represented as $(\mathcal{X}_1 \times \mathcal{X}_2, p(y, y_1, y_2|x_1, x_2), \mathcal{Y} \times \mathcal{Y}_1 \times \mathcal{Y}_2)$. The conditional probability density function (pdf), $p(y, y_1, y_2|x_1, x_2)$, defines the statistical properties of the transfer function from each input pair, $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$, to an output tuple, $(y, y_1, y_2) \in \mathcal{Y} \times \mathcal{Y}_1 \times \mathcal{Y}_2$.

The two source nodes perform cooperation by relaying each other's information to destination. We denote the communication links between the source nodes, *i.e.*, $N_1 \rightarrow N_2$ and $N_2 \rightarrow N_1$, as cooperative links. Traditional multiple access channel and MISO system assume a cooperative link with capacity 0 and ∞, respectively. In CMAC, the cooperative link capacity can be any value in the range of $[0, \infty)$.

The analysis of the achievable rate region is based on the concept of typical sequence. For completeness, we list the definition and properties of typical sequence as follows [8].

Definition 1: Consider a group of p length-n random sequences, $(\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_p)$, drawn according to the pdf, $\prod_{k=1}^{n} p(x_{1k}, x_{2k}, \cdots, x_{pk})$, where $x_{mk} \in \mathcal{X}_m$ is the kth element of the length-n sequence $\mathbf{x}_m$, for $m = 1, \cdots, p$. Define the jointly ϵ-typical set of $(\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_p)$ as

$$A_\epsilon(X_1, \cdots, X_p) = \Big\{(\mathbf{x}_1, \cdots, \mathbf{x}_p) \in \mathcal{X}_1^n \times \cdots \times \mathcal{X}_p^n : \left| -\frac{1}{n}\log p(\mathbf{s}) - H(\mathbf{S}) \right| \leq \epsilon, \forall \mathbf{S} \subseteq \{X_1, \cdots, X_p\} \Big\}, \quad (1)$$

where $H(\mathbf{S})$ is the entropy of the random variable vector $\mathbf{S}$.

Lemma 1: For any $\epsilon > 0$ and $\mathbf{S} \subseteq \{X_1, \cdots, X_p\}$, there exists an integer n such that $A_\epsilon(\mathbf{S})$ satisfies

$$P\{A_\epsilon(\mathbf{S})\} \geq 1 - \epsilon, \quad \forall \mathbf{S} \subseteq \{X_1, \cdots, X_p\}. \quad (2a)$$

$$\mathbf{s} \in A_\epsilon(\mathbf{S}) \Rightarrow \left| -\frac{1}{n}\log p(\mathbf{s}) - H(\mathbf{S}) \right| \leq \epsilon, \quad (2b)$$

$$(1-\epsilon)2^{n(H(\mathbf{S})-\epsilon)} \leq |A_\epsilon(\mathbf{S})| \leq 2^{n(H(\mathbf{S})+\epsilon)}, \quad (2c)$$

Lemma 2: Let $(\mathbf{U}, \mathbf{V}, \mathbf{W}) \sim \prod_{k=1}^{n} p(u_k, v_k, w_k)$ and $(\mathbf{U}', \mathbf{V}', \mathbf{W}) \sim \prod_{k=1}^{n} p(u_k|w_k)p(v_k|w_k)p(w_k)$. Then for the n that yields $P\{A_\epsilon(U, V, W)\} \geq 1 - \epsilon$,

$$(1-\epsilon)2^{-n(I[U;V|W]+\epsilon]} \leq P\{(\mathbf{U}', \mathbf{V}', \mathbf{W}) \in A_\epsilon(U, V, W)\} \leq 2^{-n[I(U;V|W)-\epsilon]}.$$

III. An Achievable Rate Region

Based on the definitions and preliminaries presented in Section II, an achievable rate region for a general cooperative multiple access channel is developed in this section.

To facilitate analysis, define auxiliary random variables (RVs), C_s and C_0, which are used to represent respectively the self information and cooperative information to be transmitted at source node N_s, for $s = 1, 2$. The actual signal transmitted at N_s is denoted as X_s, which is a function of C_s and C_0, and the signals received at nodes N_s and D are represented as Y_s and Y, respectively. With the above book keeping notations, the achievable rate region for the cooperative multiple access channel is presented as follows.

Theorem 1: For a 3-node cooperative multiple access channel, the achievable rate region is the union of (R_1, R_2) satisfying the following inequalities

$$R_1 < \min\{I(X_1; Y_2|X_2), I(X_1; Y|C_2)\}, \quad (3a)$$

$$R_2 < \min\{I(X_2; Y_1|X_1), I(X_2; Y|C_1)\}, \quad (3b)$$

$$R_1 + R_2 < I(X_1, X_2; Y). \quad (3c)$$

The achievability of the rate region will be proved by showing that there exists a random code that can achieve the conditions as shown in (3). The details are presented in the following two subsections.

A. Encoding and Decoding

We present in this subsection a random coding scheme that can achieve the rate region described in Theorem 1.

1) Random Codebook. First generate 2^{nR_0} independent identically distributed (i.i.d.) n-sequences, $\mathbf{c}_0 = [c_{01}, \cdots, c_{0n}]$, each drawn according to the joint pdf, $p(\mathbf{c}_0) = \prod_{k=1}^{n} p(c_{0k})$. Index them as $\mathbf{c}_0(m)$, for $m \in \mathcal{M}_0 = \{1, 2, \cdots, 2^{nR_0}\}$.

For each $\mathbf{c}_0(m)$, generate 2^{nR_s} conditionally independent n-sequences, $\mathbf{x}_s = [x_{s1}, \cdots, x_{sn}]$, according to the conditional pdf, $p(\mathbf{x}_s|\mathbf{c}_0(m)) = \prod_{k=1}^{n} p(x_{sk}|c_{0k}(m))$, for $s = 1$ and 2, respectively. Index the sequences $\mathbf{x}_s$ as $\mathbf{x}_s(m_s|m)$, for $m_s \in \mathcal{M}_s = \{1, 2, \cdots, 2^{nR_s}\}$. This results in a random codebook, $\mathcal{C} = \{\mathbf{c}_0(m), \mathbf{x}_1(m_1|m), \mathbf{x}_2(m_2|m)\}$.

Randomly partition the set of index pairs, $\mathcal{M}_{12} = \mathcal{M}_1 \times \mathcal{M}_2$, into 2^{nR_0} subsets as, $\mathcal{M}_{12} = \{\mathcal{S}_1, \mathcal{S}_2, \cdots, \mathcal{S}_{2^{nR_0}}\}$, such that $\forall (u, v) \in \mathcal{M}_{12}$ and $\forall m \in \mathcal{M}_0$, $P\{(u, v) \in \mathcal{S}_m\} = 2^{-nR_0}$.

2) Encoding. The information are transmitted in blocks. For a duration of B blocks, the information to be transmitted by source node N_s is denoted as $[m_s^{(1)}, m_s^{(2)}, \cdots, m_s^{(B)}] \in \mathcal{M}_s^B$.

At the end of block $i-1$, assume that node N_1 knows $m_2^{(i-1)}$, N_2 knows $m_1^{(i-1)}$, and the destination node D knows $\left(m_1^{(i-2)}, m_2^{(i-2)}\right)$. As a result, the index pair, $\left(m_1^{(i-1)}, m_2^{(i-1)}\right)$, is known at both s_1 and s_2 at the end of block $i-1$. Let $\mathcal{S}_{m^{(i-1)}}$ denote the partition containing $\left(m_1^{(i-1)}, m_2^{(i-1)}\right)$. Based on the index $m^{(i-1)}$, which is known at both source nodes, select $\mathbf{c}_0\left(m^{(i-1)}\right)$.

At source node N_s, select and transmit the n-sequence, $\mathbf{x}_s\left(m_s^{(i)}|m^{(i-1)}\right)$, where $m_s^{(i)}$ is the new information to be transmitted by node N_s at block i, and $m^{(i-1)}$ is the cooperative information.

3) Decoding. At the end of block i, the received signal at source node N_s is $\mathbf{y}_s^{(i)}$. Source node N_1 declares $\hat{m}_2^{(i)} = v$ if there is one and only one $v \in \mathcal{M}_2$ such that $\left\{\mathbf{c}_0\left(m^{(i-1)}\right), \mathbf{x}_1\left(m_1^{(i)}|m^{(i-1)}\right), \mathbf{x}_2\left(v|m^{(i-1)}\right), \mathbf{y}_1^{(i)}\right\} \in A_\epsilon(C_0, X_1, X_2, Y_1)$. Similarly, source node N_2 declares $\hat{m}_1(i) = u$ if there is one and only one $u \in \mathcal{M}_1$ such that $\left\{\mathbf{c}_0\left(m^{(i-1)}\right), \mathbf{x}_1\left(u|m^{(i-1)}\right), \mathbf{x}_2\left(m_2^{(i)}|m^{(i-1)}\right), \mathbf{y}_2^{(i)}\right\} \in A_\epsilon(C_0, X_1, X_2, Y_2)$.

The decoding at destination node consists of three steps. First, the destination node declares $\hat{m}^{(i-1)} = w$ if there is one and only one $w \in \mathcal{M}$ such that $\left\{\mathbf{c}_0(w), \mathbf{y}^{(i)}\right\} \in A_\epsilon(C_0, Y)$, where $\mathbf{y}^{(i)}$ is the received signal at the destination node at the end of block i. Second, the destination node calculates its ambiguity set, $\mathcal{L}(i)$, which contains all the index pairs, $(u, v) \in \mathcal{M}_{12}$, such that $\left\{\mathbf{c}_0\left(m^{(i-1)}\right), \mathbf{x}_1\left(u|m^{(i-1)}\right), \mathbf{x}_2\left(v|m^{(i-1)}\right), \mathbf{y}^{(i)}\right\} \in A_\epsilon(C_0, X_1, X_2, Y)$. Third, the receiver declares $\left(\hat{m}_1^{(i-1)}, \hat{m}_2^{(i-1)}\right) = (u, v)$ if there is one and only one pair of $(u, v) \in \mathcal{M}_{12}$ satisfying $(u, v) \in \mathcal{L}(i-1) \cap \mathcal{S}_{m^{(i-1)}}$.

B. Probability of Error

The average error probability of the above random coding scheme is analyzed in this subsection. To facilitate the error probability analysis, define the following events.

$E_{0uvw}(i)$: $\left\{\mathbf{c}_0(w), \mathbf{x}_1(u|w), \mathbf{x}_2(v|w), \mathbf{y}_1^{(i)}, \mathbf{y}_2^{(i)}, \mathbf{y}^{(i)}\right\}$ is jointly ϵ-typical.

$E_{1v}(i)$: $\left\{\mathbf{c}_0\left(m^{(i-1)}\right), \mathbf{x}_1\left(m_1^{(i)}|m^{(i-1)}\right), \mathbf{x}_2\left(v|m^{(i-1)}\right), \mathbf{y}_1^{(i)}\right\}$ is jointly ϵ-typical.

$E_{2u}(i)$: $\left\{\mathbf{c}_0\left(m^{(i-1)}\right), \mathbf{x}_1\left(u|m^{(i-1)}\right), \mathbf{x}_2\left(m_2^{(i)}|m^{(i-1)}\right), \mathbf{y}_2^{(i)}\right\}$ is jointly ϵ-typical.

$E_{3w}(i)$: $\left\{\mathbf{c}_0(w), \mathbf{y}^{(i)}\right\}$ is jointly ϵ-typical.

$E_{4uv}(i)$: $(u,v) \in \mathcal{L}(i-1) \cap \mathcal{S}_{m^{(i-1)}}$.

Without loss of generality, assume that the index pair, $(1,1)$, is transmitted during blocks $i-1$ and i, and $(1,1) \in \mathcal{S}_1$. Based on the encoding and decoding scheme described earlier, define the error event F_i for decoding error at block i as

$$F_i = E_{0111}^c(i) \cup \left(\bigcup_{v\neq 1} E_{1v}(i)\right) \cup \left(\bigcup_{u\neq 1} E_{2u}(i)\right) \cup \left(\bigcup_{w\neq 1} E_{3w}(i)\right) \cup \left(E_{411}^c(i) \bigcup_{u\neq 1} E_{4u1}(i) \bigcup_{v\neq 1} E_{41v}(i) \bigcup_{\substack{u\neq 1\\ v\neq 1}} E_{4uv}(i)\right).$$

where E^c denotes the complement of event E.

From Lemmas 1 and 2, it can be shown that, $\forall\epsilon$ and for n sufficiently large,

$$\begin{aligned}
P\left\{E_{0111}^c(i)|F_{i-1}^c\right\} &< \frac{\epsilon}{8B} \\
P\left\{\bigcup_{v\neq 1} E_{1v}(i)|F_{i-1}^c\right\} &< \frac{\epsilon}{8B}, \text{ if } R_2 < I(X_2;Y_1|X_1) \\
P\left\{\bigcup_{2\neq 1} E_{2u}(i)|F_{i-1}^c\right\} &< \frac{\epsilon}{8B}, \text{ if } R_1 < I(X_1;Y_2|X_2) \\
P\left\{\bigcup_{w\neq 1} E_{3w}(i)|F_{i-1}^c\right\} &< \frac{\epsilon}{8B}, \text{ if } R_0 < I(C_0;Y)
\end{aligned}$$

where $I(X_s;Y_t|C_0,X_t) = I(X_s;Y_t|X_t)$ is used for the second and third inequalities. The analysis is similar to that in [7], and details are omitted here for brevity.

Next we evaluate the error probabilities related to the event of $E_{4uv}(i)$. To simplify notation, define $A_1 \triangleq E_{411}^c(i)|F_{i-1}^c$, $A_2 \triangleq \bigcup_{u\neq 1} E_{4u1}(i)|F_{i-1}^c$, $A_3 \triangleq \bigcup_{v\neq 1} E_{41v}(i)|F_{i-1}^c$, and $A_4 \triangleq \bigcup_{\substack{u\neq 1\\ v\neq 1}} E_{4uv}(i)|F_{i-1}^c$.

1) $P\{A_1\}$. From Lemma 1, we have $P\{(1,1) \notin \mathcal{L}(i-1)\} < \frac{\epsilon}{16B}$, and $P\{(1,1) \notin \mathcal{S}_{m^{(i-1)}}\} = P\{E_{31}^c(i)\} < \frac{\epsilon}{16B}$, for n sufficiently large. Thus

$$P\{A_1\} < P\{(1,1) \notin \mathcal{L}(i-1)\} + P\{(1,1) \notin \mathcal{S}_{m^{(i-1)}}\} < \frac{\epsilon}{8B}.$$

2) $P\{A_2\}$ and $P\{A_3\}$. $P\{A_2\}$ can be expressed as

$$P\{A_2\} \leq \sum_{u=2}^{2^{nR_1}} P\{(u,1) \in \mathcal{L}(i-1)\} P\{(u,1) \in \mathcal{S}_{m^{(i-1)}}\}.$$

From Lemma 2, for $u \neq 1$, $P\{(u,1) \in \mathcal{L}(i-1)\} < 2^{-n[I(X_1;Y|C_0,X_2)-\epsilon]}$. Based on the random partition of $\mathcal{M}_{12}$, we have $P\{(u,1) \in \mathcal{S}_{m^{(i-1)}}\} = 2^{-nR_0}$. Thus,

$$P\{A_2\} < 2^{-n[I(X_1;Y|C_2)-R_1-\epsilon]}, \tag{4}$$

where C_2 is defined as a RV with entropy satisfying $H(C_2) = H(X_2|C_0)$, and $R_0 < I(C_0;Y)$ is used in the analysis. Therefore, if $R_1 < I(X_1;Y|C_2)$, then $P\{A_2\} < \frac{\epsilon}{8B}$ for n sufficiently large. Similarly, if $R_2 < I(X_2;Y|C_1)$, with $H(C_1) = H(X_1|C_0)$, $P\{A_3\} < \frac{\epsilon}{8B}$ for n sufficiently large.

3) $P\{A_4\}$. The probability can be expressed as

$$P\{A_4\} \leq \sum_{u=2}^{2^{nR_1}} \sum_{v=2}^{2^{nR_2}} P\{(u,v) \in \mathcal{L}(i-1)\} P\{(u,v) \in \mathcal{S}_{m^{(i-1)}}\}.$$

From Lemma 2, $P\{(u,v) \in \mathcal{L}(i-1)\} < 2^{-n[I(X_1,X_2;Y|C_0)-\epsilon]}$. Thus

$$P\{A_4\} < 2^{-n[I(X_1,X_2;Y)-(R_1+R_2)-\epsilon]}, \tag{5}$$

where $R_0 < I(C_0;Y)$ and $I(C_0,X_1,X_2;Y) = I(X_1,X_2;Y)$ are used in the analysis. Therefore, $P\{A_4\} < \frac{\epsilon}{8B}$ with $R_1 + R_2 < I(X_1,X_2;Y)$ and n sufficiently large.

From the analysis above, with the rates inequalities given in (3), $\forall\epsilon$ and for sufficiently large n, the conditional error probability for the i-th block satisfies $P\{F_i|F_{i-1}^c\} < \frac{\epsilon}{B}$.

For B consecutive blocks, the error probability is given by

$$\begin{aligned}
P_e &\leq P\left\{\bigcup_{i=1}^{B} F_i\right\} = \sum_{i=1}^{B} P\left\{F_i \cap F_{i-1}^c \cap \cdots \cap F_1^c\right\} \\
&\leq \sum_{i=1}^{B} P\left\{F_i \cap F_{i-1}^c\right\} \leq \sum_{i=1}^{B} P\left\{F_i|F_{i-1}^c\right\} < \epsilon.
\end{aligned}$$

This completes the proof of Theorem 1.

IV. The Degraded Gaussian CMAC

An achievable rate region for a degraded Gaussian CMAC is presented in this section. The signals at a 3-node Gaussian cooperative multiple access channel can be represented by

$$y_s = x_s + z_s, \qquad \text{for } s = 1,2, \tag{6a}$$

$$y = x_1 + x_2 + z, \tag{6b}$$

where x_s, y_s, and z_s are the transmitted signal, received signal, and noise at source node N_s, respectively, y is the received signal at destination node, and z is the noise at destination node. The noise components, z_1, z_2, z, are zero-mean Gaussian distributed with variance σ_1^2, σ_2^2, and σ^2, respectively. A degraded Gaussian cooperative multiple access channel is defined as one with $\sigma^2 \geq \max\{\sigma_1^2, \sigma_2^2\}$.

A. Achievable Rate Region

Theorem 2: For a degraded Gaussian cooperative multiple access channel, the following rate region is achievable

$$R_s \leq r_s \triangleq \min\left\{\frac{1}{2}\log\left(1+\frac{\bar{\alpha}_s P_s}{\sigma_t^2}\right), \frac{1}{2}\log\left(1+\frac{P_s+\alpha_t P_t+2\sqrt{\alpha_1\alpha_2 P_1 P_2}}{\sigma^2}\right)\right\} \tag{7a}$$

$$R_1 + R_2 \leq r_{12} \triangleq \frac{1}{2}\log\left(1+\frac{P_1+P_2+2\sqrt{\alpha_1\alpha_2 P_1 P_2}}{\sigma^2}\right) \tag{7b}$$

where $(s,t) \in \{(1,2),(2,1)\}$, P_s is the transmission power of node N_s, $\bar{\alpha}_s = 1-\alpha_s$, and the union of regions is defined over all $(\alpha_1,\alpha_2)\in[0,1]^2$.

Proof: Select a random coding scheme such that the information transmitted at source node N_s is

$$X_s = \sqrt{\alpha_s P_s} C_0 + \sqrt{\bar{\alpha}_s P_s} C_1, \quad u = 1, 2, \tag{8}$$

where C_s and C_0 are zero mean unit variance Gaussian RV, with C_0 being the cooperative component and C_s the new information to be transmitted by node N_s. The variable, $\alpha_s \in [0,1]$, is the cooperative coefficient defined as the percentage of transmission power allocated by node N_s for cooperation. Substituting (8) into (3) and (6) leads to (7). ■

The rate region given in (7) captures the effects of cooperation between the source nodes in terms of the cooperation coefficients, (α_1,α_2). Setting $\alpha_s = 0$ corresponds to the case of no cooperation, and (7) degrades to the capacity region of traditional multiple access channel [3, Eqns. (15.145) and (15.149)]; setting $\alpha_s = 1$ corresponds to the case of full cooperation, and (7) degrades to the capacity of MISO system if $\frac{P_s}{\sigma_t^2} = \infty$ [4, Eqn. (1)]. For a general CMAC, the values of (α_1,α_2) can be chosen by following different system design criteria. For example, the nodes can either compete with each other to maximize their respective transmission rates, or collaborate with each other to maximize the total transmission rate. We denote the first case as competing mode and the second case as collaborative mode.

B. Competing Source Nodes

The competing operation mode can be used to model a network with nodes competing for common resources, *e.g.*, two wireless terminals connecting to the Internet through the same access point. In this case, node N_s will choose the value of α_s that can maximize its own transmission rate based on the knowledge of the value of α_t selected by node N_t. This can be modeled as a strategic non-cooperative game [9] with the available strategies for player (node) N_s being $0\le\alpha_s\le 1$. In this paper, we define the payoff function of the game as the transmission rate boundary r_s given in (7).

In this strategic game, for a given value of α_t, node N_s will choose α_s such that r_s is maximized, and such operation is summarized in the following Corollary.

Corollary 1: Given α_t from node N_t, node N_s will maximize its own transmission rate boundary, r_s, by setting $\alpha_s = f_s(\alpha_t)$ defined as follows

$$f_s(\alpha_t) \triangleq \begin{cases} 0, & \text{if } \frac{P_s}{\sigma_t^2} \le \frac{\alpha_t P_t}{\sigma^2-\sigma_t^2}, \\ 1 - \frac{\left(\sqrt{\frac{P_s}{\sigma_t^2} - \frac{\alpha_t P_t}{\sigma^2}} + \sqrt{\frac{\alpha_t P_t}{\sigma_t^2} - \frac{\alpha_t P_t}{\sigma^2}}\right)^2}{\frac{P_s}{\sigma_t^2}\frac{\sigma^2}{\sigma_t^2}}, & \text{if } \frac{P_s}{\sigma_t^2} > \frac{\alpha_t P_t}{\sigma^2-\sigma_t^2}. \end{cases} \tag{9}$$

The corresponding rate boundary is

$$r_s(\alpha_t) = \frac{1}{2}\log\left[1 + \frac{P_s - f_s(\alpha_t)P_s}{\sigma_t^2}\right]. \tag{10}$$

Proof: The results are derived based on the expression in (7a). To simplify notation, define $\xi_1 \triangleq \frac{\bar{\alpha}_s P_s}{\sigma_t^2}$, and $\xi_2 \triangleq \frac{P_s+\alpha_t P_t + 2\sqrt{\alpha_1\alpha_2 P_1 P_2}}{\sigma^2}$. Since $\alpha_s \in [0,1]$, we have $\xi_1 \in \left[0, \frac{P_s}{\sigma_t^2}\right]$, and $\xi_2 \in \left[\frac{P_s+\alpha_t P_t}{\sigma^2}, \frac{P_s+\alpha_t P_t+2\sqrt{\alpha_t P_1 P_2}}{\sigma^2}\right]$.

If $\frac{P_s}{\sigma_t^2} \le \frac{\alpha_t P_t}{\sigma^2-\sigma_t^2}$, then it can be shown that $\frac{P_s}{\sigma_t^2} \le \frac{P_s+\alpha_t P_t}{\sigma^2}$. It's obvious that $\xi_1\le\xi_2$ always holds, and this leads to $r_s = \frac{1}{2}\log\left(1+\frac{\bar{\alpha}_s P_s}{\sigma_t^2}\right)$, which can be maximized by setting $\alpha_s = 0$.

If $\frac{P_s}{\sigma_t^2} > \frac{\alpha_t P_t}{\sigma^2-\sigma_t^2}$, then $\frac{P_s}{\sigma_t^2} > \frac{P_s+\alpha_t P_t}{\sigma^2}$. In this case, the ranges of ξ_1 and ξ_2 overlap, and the value of α_s that maximizes r_s can be obtained by solving $\xi_1 = \xi_2$, which is a second order linear equation. The details are omitted here for brevity, and the solution is given in (9).

Eqn. (10) can be obtained by substituting (9) into (7a). ■

The two source nodes will choose their respective strategies following (9) until it reaches a steady state. The steady state strategy for such game is the well known Nash equilibrium (NE) [10], which is the strategy with the property that no player can benefit from unilateral deviating from its strategy. If $(\alpha_1^*,\alpha_2^*)\in[0,1]^2$ is an NE, then it satisfies

$$r_s\left(\alpha_s^*,\alpha_t^*\right) \ge r_s\left(\alpha_s,\alpha_t^*\right), \ \forall \alpha_s\in[0,1], \ (s,t)\in\{(1,2),(2,1)\}.$$

From Corollary 1, the Nash Equilibrium of the game, if exist, must satisfy the following solution

$$(\alpha_1^*,\alpha_2^*) = \mathbf{f}\left(\alpha_1^*,\alpha_2^*\right) \triangleq \left[f_1(\alpha_2^*), f_2(\alpha_1^*)\right]. \tag{11}$$

Therefore, the NE is a fixed point of the function $\mathbf{f}(\mathbf{x})$, where $\mathbf{x} = [x_1,x_2]$. The existence of NE in this game depends on the properties of the function $\mathbf{f}(\mathbf{x})$, which are summarized in the following two Lemmas.

Lemma 3: $\mathbf{f}(\mathbf{x})$ is surjective over $[0,1]^2$.

Proof: To simplify notation, define $a = \sqrt{\frac{P_s}{\sigma_t^2}/\frac{xP_t}{\sigma^2} - 1}$, $b = \sqrt{\frac{\sigma^2}{\sigma_t^2} - 1}$, then the value of $f_s(x)$ can be simplified to $f_s(x) = \frac{(a+b)^2}{(a^2+1)(b^2+1)}$, if $\frac{P_s}{\sigma_t^2} > \frac{xP_t}{\sigma^2-\sigma_t^2}$. It can be shown that $f_s(x)\in[0,1]$ by expanding its numerator and denominator. Thus the range and domain of $f_p(x)$ is both $[0,1]$. ■

Lemma 4: $\mathbf{f}(\mathbf{x})$ is continuous over $[0,1]^2$.

Proof: It's trivial that $f_s(x)$ is continuous when $\frac{P_s}{\sigma_t^2} \ne \frac{xP_t}{\sigma^2-\sigma_t^2}$. Define $x_0 \triangleq \frac{P_s}{\sigma_t^2}\frac{\sigma^2-\sigma_t^2}{P_t}$. From (11), we have

$$\lim_{x\to x_0^+} f_s(x) = \frac{\left[\sqrt{\frac{P_s}{\sigma_t^2} - \frac{P_s}{\sigma_t^2}\frac{\sigma^2-\sigma_t^2}{P_t}\frac{P_t}{\sigma^2}} + \sqrt{\frac{P_s}{\sigma_t^2}\frac{\sigma^2-\sigma_t^2}{P_t}\left(\frac{P_t}{\sigma_t^2} - \frac{P_t}{\sigma^2}\right)}\right]^2}{\frac{P_s}{\sigma_t^2}\frac{\sigma^2}{\sigma_t^2}},$$

which simplifies to 0, or $f_s(x_0)$. Therefore $f_s(x)$ is continuous over $[0,1]$, and it can be shown that $\mathbf{f}(\mathbf{x})$ is continuous over $[0,1]^2$. ■

The existence of the NE is stated as follows.

Proposition 1: For a degraded Gaussian cooperative multiple access channel with two competing source nodes, there exists at least one Nash equilibrium.

Proof: The existence of NE is equivalent to the existence of fixed point for the function $\mathbf{f}(\mathbf{x})$. Since $\mathbf{f}(\mathbf{x})$ is surjective and continuous on $[0,1]^2$, there must be at least one fixed point based on the fixed point theorem [11]. ■

The NE can be obtained by iteratively updating the values of α_1 and α_2 following (9) by the two users. It should be noted that in most system configurations the maximum transmission rate given in (10) can not be simultaneously achieved by the two source nodes due to the limit on the total rate $R_1 + R_2$ as given in (7b), and details are illustrated with numerical examples in Section V.

C. Collaborative Source Nodes

The collaborative operation mode can be used to model a network with source nodes belonging to the same operator and sharing common interests, *e.g.*, nodes in a wireless sensor network transmit data to a central data collection point. Given certain system configuration, the two source nodes can collaborate with each other to select the values of (α_1, α_2) such that the total transmission rate, $R_1 + R_2$, is maximized, *i.e.*

$$(\alpha_1^+, \alpha_2^+) = \underset{(\alpha_1,\alpha_2)\in[0,1]^2}{\text{argmax}} \left\{ \min\left[r_{12}, r_1 + r_2\right]\right\},$$

where r_s and r_{12} are defined in (7). The solution of (12) eludes a closed-form expression. It can be evaluated numerically and the results are discussed in Section V.

V. Numerical Examples

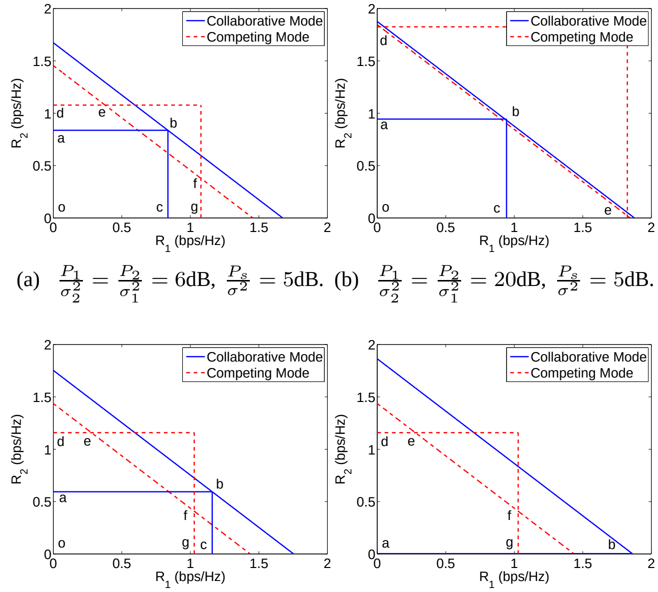
Numerical examples are given in this section to demonstrate the achievable rate region for degraded Gaussian cooperative multiple access channel under various system configurations.

First we investigate the case with symmetric cooperative links, *i.e.*, $\frac{P_1}{\sigma_2^2} = \frac{P_2}{\sigma_1^2}$. Fig. 2(a) shows the rate regions when $\frac{P_s}{\sigma_t^2} = 6$dB. In this figure, the rate regions for network in collaborative mode and competing mode are defined by the polygons, 'a-b-c-o' and 'd-e-f-g-o', respectively. Network in collaborative mode has a larger total data rate (point b), yet network in competing mode can achieve a larger unbalanced individual transmission rate (section 'd-e' or 'f-g'). It should be noted that in competing mode the maximum data rates for the two source nodes cannot be achieved simultaneously. The difference in total transmission rate between the two modes can be reduced by increasing $\frac{P_s}{\sigma_t^2}$ as shown in Fig. 2(b), where $\frac{P_s}{\sigma_t^2} = 20$dB and competing mode can achieve a total transmission rate similar to collaborative mode.

The case with asymmetric cooperative links, $\frac{P_1}{\sigma_2^2} \neq \frac{P_2}{\sigma_1^2}$, is investigated in Fig. 3. The results in Fig. 3 indicate that the difference between $\frac{P_1}{\sigma_2^2}$ and $\frac{P_2}{\sigma_1^2}$ has little impact on the rate region of network operating in competing mode. On the other hand, the rate region for network in collaborative mode changes dramatically with the increase of the difference between the two cooperative links. When $\frac{P_1}{\sigma_2^2} = 20$dB, the rate region under collaborative mode collapses to a single line 'a-b', which corresponds to $R_2 = 0$. Thus source node N_2 stops transmission of its own information and degrades to a pure relay node. In this case, the cooperative multiple access channel degrades to a relay channel as described in [7].

VI. Conclusions

An achievable rate region for a 3-node cooperative multiple access channel was presented in this paper. The results include the traditional multiple access channel, MISO system, and relay channel as special cases. With the help of game theory and Nash equilibrium, the impact of node cooperation strategies on the achievable rate regions of degraded Gaussian cooperative multiple access channel was investigated under competing mode and collaborative mode. Numerical examples show that, for collaborative mode, if the signal to noise ration (SNR) of one cooperative link is significantly higher than the SNR of the other cooperative link, the source node corresponding to the transmitter of the weaker cooperative link will degrade to a pure relay node. The asymmetry between the two cooperative links has little impact on the rate region of network in competing mode.



(a) $\frac{P_1}{\sigma_2^2} = \frac{P_2}{\sigma_1^2} = 6$dB, $\frac{P_s}{\sigma^2} = 5$dB. (b) $\frac{P_1}{\sigma_2^2} = \frac{P_2}{\sigma_1^2} = 20$dB, $\frac{P_s}{\sigma^2} = 5$dB.

(a) $\frac{P_1}{\sigma_2^2} = 10$dB. (b) $\frac{P_1}{\sigma_2^2} = 20$dB.

Fig. 3. Rate regions for networks with asymmetric cooperative links ($\frac{P_2}{\sigma_1^2} = 6$dB, $\frac{P_s}{\sigma^2} = 5$dB.)

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