Collision-Tolerant Media Access Control for Asynchronous Users over Frequency-Selective Channels

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Abstract-In this paper, a frequency-domain cross-layer collision-tolerant (CT) media access control (MAC) scheme is proposed for the up-links of broadband wireless networks with asynchronous users. The collision tolerance is achieved with a frequency-domain on-off accumulative transmission (FD-OOAT) scheme, where the spectrum is divided into a large number of orthogonal sub-channels, and each symbol is transmitted over a small subset of the sub-channels to reduce collisions. Such a radio resource management scheme renders a special signal structure that enables *multi-user detection* (MUD) in the physical layer to resolve the collisions at the MAC layer. Most existing MUDs require precise symbol level synchronization among users. The proposed scheme, however, can operate with asynchronous users. A new theoretical framework is provided to study the impacts of time-domain user delays on system performance. Both analytical and simulation results demonstrate that the proposed FD-OOAT structure with time-domain oversampling is robust to user delays and the timing phase offset caused by the sampling clock difference between the transmitter and the receiver. It is shown that the proposed scheme can achieve significant performance gains, in terms of both the number of users supported and the normalized throughput.

Index Terms—Collision-tolerant media access control, asynchronous users, timing phase offset, and oversampling

I. INTRODUCTION

The design of reliable broadband multi-user systems faces a number of challenges, such as frequency-selective fading, the competitions for the limited spectrum resource, and the lack of precise synchronization, etc. The main objective of this work is to develop a spectrum efficient communication technique that can address all these challenges by exploiting the interactions between *physical* (PHY) layer and *media access control* (MAC) layer in a communication network.

In many conventional MAC schemes, signals collided at a receiver will be discarded and retransmitted. This results in a waste of the precious energy and spectrum resources. Various *collision-tolerant* (CT) MAC protocols have been developed to extract the salient information contained in the collided signals by resorting to cross-layer designs [1]–[10]. *Multi-packet reception* (MPR) in [1]–[4] assumes that the receiver can recover a fraction of the collided signals by signal processing in the PHY layer. Iterative *interference cancellation* (IC) methods are used to resolve multi-user collisions in a *contention-resolution diversity slotted ALOHA* (CRDSA) [5] and an *irregular repetition slotted ALOHA* (*IRSA*) scheme [6] and [7]. In the CRDSA and IRSA schemes, each packet is transmitted multiple times at random slots in a frame. A successfully detected packet can be used to iteratively subtract the interference caused by its replicas. The throughput of CRDSA and IRSA drops dramatically once the normalized offered load exceeds certain point, because the IC schemes are unable to find at least one collision-free signal at the receiver to initiate the iterative IC process under heavy loads. In addition, all above techniques rely on perfect synchronization among users, which is difficult to achieve in practical systems.

The limitations of iterative IC can be partly solved by using multi-user detection (MUD), which performs simultaneous detection of signals from two or more users collided at the receiver. MUD in the PHY layer can be combined with MAC techniques to improve the spectrum and energy efficiency in wireless networks [8]-[10]. MUD techniques are often designed with multi-dimensional signals in the PHY layer, such as *code-division multiple access* (CDMA) [8] or orthogonal frequency division multiplexing (OFDM) [9]. An on-off accumulative transmission (OOAT) scheme in [10] can support more simultaneous users than the dimension of the received signals by repeating the same signal multiple times and using silence periods between two consecutive repetitions to reduce collisions. However, the OOAT scheme in [10] works only in flat fading channels, yet broadband communications dictate an operation environment of frequency-selective fading.

In a multi-user system, two types of synchronizations are needed: the synchronization among the users, denoted as *multi*user synchronization (MUS), and the synchronization of the sampling phase between the transmitter and receiver clocks, denoted as sampling phase synchronization (SPS). The SPS is usually based on correlation between a specially designed training sequence and the received signals [10], [14] and [15]. In multi-user systems, the base station (BS) first estimates the relative delays of all the users by correlation-based SPS. The estimated timing information can either assist the detection process [10], or be fed back to the users through a down-link control channel to achieve MUS [16]. All these schemes have residual timing offsets or synchronization errors, which could cause additional multiple access interference (MAI) and/or destroy the special signal structure critical to MUD [11]. The residual SPS errors may also introduce timing phase offset that will increase inter-symbol interference (ISI) and degrade

The material in this paper was presented in part at the the IEEE International Conference on Communications 2012. G. Wang and J. Wu are with the Department of Electrical Engineering, University of Arkansas, Fayetteville, AR 72701 USA. G. Zhou is with Litepoint Corp., Sunnyville, CA 94058 USA. G. Y. Li is with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atalanta, GA 30332 USA (e-mails: {wuj, wangg}@uark.edu, guoqing.zhou@litepoint.com, liye@ece.gatech.edu). This work was supported in part by the National Science Foundation under Grants ECCS-0917041, ECCS-1202075, and the Arkansas NASA EPSCoR Research Infrastructure Development grant.

signal-to-noise ratio (SNR) at the receiver [12], [13].

In this paper, we propose a new cross-layer CT-MAC scheme that can support a large number of simultaneous users operating in frequency-selective fading, require neither MUS nor SPS, and is also robust to timing phase offsets. Most existing CT-MAC schemes in the literature are developed for flat fading channels [5]–[7], [10]. For example, time dispersion caused by frequency-selective fading will destroy the special signal structure that is critical to the original time-domain OOAT scheme [10]. We address this problem by developing a new *frequency-domain OOAT* (FD-OOAT), where a frequency-selective channel is divided into multiple orthogonal sub-channels in the frequency domain with the help of OFDM. Different from conventional OFDM, each symbol is transmitted over several sub-channels with a certain onoff pattern in our scheme. The frequency-domain repetition increases the degree-of-freedom (DoF) of the signals at the receiver, thus enables the collision tolerance of the system. With the FD-OOAT, the relative transmission delays among the users in the time-domain are manifested as phase shifts in the frequency domain, and our theoretical analysis shows that they have negligible impacts on system performance. Therefore, FD-OOAT does not require precise MUS or SPS, yet synchronization is critical to most existing CT-MAC systems. More importantly, the frequency-domain operations allow us to minimize the number of users colliding on each sub-channel by using simple on-off patterns that are radically different from those used by the original time-domain OOAT schemes [10].

Another important contribution of this work is the development of a new theoretical framework that quantifies the impacts of timing phase offset on system performance in multi-user multi-carrier systems. New analytical expressions of the frequency-domain channel coefficients are developed as functions of the timing phase offsets. Both theoretical and simulation results demonstrate that time-domain oversampling can effectively remove the effects of timing phase offset for multi-carrier systems. Therefore, the proposed scheme can operate in an asynchronous environment without incurring additional interference or SNR degradation. The collisions in FD-OOAT are resolved by using optimum and sub-optimum MUDs, which do not require precise synchronization as most existing MUD schemes. An analytical performance bound is derived to quantify the performance of the proposed scheme.

The rest of this paper is organized as follows. The FD-OOAT scheme with time-domain oversampling is presented in Section II. The optimum and sub-optimum detection methods that can resolve collisions and collect the diversity gains are described in Section III. In Section IV, theoretical studies are performed to quantify the impacts of multipath diversity gain and timing phase offset. Simulation results are given in Section V, and Section VI concludes the paper.

II. FREQUENCY-DOMAIN OOAT WITH TIME-DOMAIN OVERSAMPLING

The model of the proposed FD-OOAT scheme with timedomain oversampling are presented in this section.

A. Proposed System Structure

Consider a wireless network with N users transmitting to the same receiver through a shared channel. Each MAC frame contains K symbols. To achieve collision tolerance in the MAC layer, users employ the FD-OOAT in the PHY layer as shown in Fig. 1.

The entire available bandwidth, B, is divided into KMsub-channels, denoted as sub-channels $0, 1, \dots, KM - 1$ in order, with a bandwidth $B_0 = \frac{B}{KM}$ each. Each symbol uses M sub-channels uniformly spread over the entire frequency band, that is, the M sub-channels with indices, $\{mK +$ $k\}_{m=0}^{M-1}$, are assigned for the k-th symbol in the frame, for $k = 0, \cdots, K - 1$. During each transmission, only R subchannels from the M ones for each symbol are occupied. The indicator vector of the occupied sub-channels for the *n*th user can be represented by a binary vector of length M, $\mathbf{p}_n = [p_n[0], \cdots, p_n[M-1]]^T \in \mathcal{B}^{M \times 1}$, where $\mathcal{B} = \{0, 1\}$, with $p_n[m] = 1$ if the k-th symbol is transmitted at the $\{mK+k\}$ -th sub-channel, and $p_n[m] = 0$ otherwise. Symbols of the same user use the same transmission pattern \mathbf{p}_n . With such a scheme, each symbol is repeated over R sub-channels (accumulative transmission), and the utilization of the subchannels are determined by an on-off transmission pattern \mathbf{p}_n . In the example shown in Fig. 1, there are N = 5 users, M = 12 available sub-channels per symbol, and R = 4 out of the 12 available sub-channels are occupied. It is assumed that all users use the same carrier frequency, thus the same set of sub-channels. As a result, signals from different users are aligned in the frequency domain as shown in Fig. 1.

Based on the above description, the signal transmitted on the (mK + k)-th sub-channel of the *n*-th user is $d_n[mK + k] = p_n[m]s_{nk}$, where s_{nk} is the *k*-th symbol from user *n*. Consequently, the signal vector of the *n*-th user can be expressed as $\mathbf{d}_n = [d_n[0], d_n[1], \cdots, d_n[KM - 1]]^T \in \mathcal{S}^{L \times 1}_+$, where L = KM, $\mathcal{S}_+ = \{\mathcal{S}, 0\}$, and \mathcal{S} is the modulation constellation set with a cardinality $S = |\mathcal{S}|$.

The signal vector, \mathbf{d}_n , is converted to the time domain by applying an *L*-point *inverse discrete Fourier transform* (IDFT)

$$\mathbf{x}_n = \mathbf{F}_{_L}^H \cdot \mathbf{d}_n,\tag{1}$$

where $\mathbf{x}_n = [x_n[0], x_n[1], \cdots, x_n[L-1]]^T$ is the time-domain signal vector, \mathbf{A}^H is the matrix Hermitian operator, and $\mathbf{F}_L \in \mathcal{C}^{L \times L}$ is the *L*-point discrete Fourier transform (DFT) matrix with the (r + 1, c + 1)-th element being $[\mathbf{F}_L]_{r,c} = \frac{1}{\sqrt{L}} \exp\left(-j2\pi\frac{r \cdot c}{L}\right)$, for $r, c = 0, 1, \cdots, L - 1$. The space between two consecutive time-domain samples is $T_1 = \frac{1}{B}$.

Before transmission, a length- l_{cp} cyclic prefix (CP) is added to the time-domain signal \mathbf{x}_n to avoid interference between consecutively transmitted frames. The time-domain signals pass through a transmit filter, $\varphi_1(t)$, and then transmitted over a quasi-static frequency-selective fading channel with impulse response $g_n(t)$. In a quasi-static channel, the fading is constant inside a frame, and varies independently from frame to frame. At the receiver, the received signals pass through a receive filter, $\varphi_2(t)$. Define the composite impulse response (CIR) of



Fig. 1. A frequency-domain OOAT system with N = 5 users, R = 4 sub-channels occupied out of M = 12 sub-channels for each symbol.

the channel as

$$h_{nc}(t) = \varphi_1(t) \odot g_n(t) \odot \varphi_2(t), \qquad (2)$$

where \odot is the convolution operator. The CIR, $h_{nc}(t)$, includes the effects of the physical channel and the transmit and receive filters. The transmit receive filters are used to limit the bandwidth of the transmitted signal. For systems with rootraised-cosine (RRC) filters as the transmit and receive filters, the bandwidth of $h_{nc}(t)$ is $\frac{1+\alpha}{T_1}$, where α is the roll-off factor of the filter, and T_1 is the space between two consecutive time domain samples at the transmitter.

The output of the receive filter is

$$y_c(t) = \sum_{n=1}^{N} \sum_{l=-\infty}^{+\infty} \sqrt{\frac{E_s}{R}} x_n[l] h_{nc}(t - lT_1 - \tau_n) + z_c(t), \quad (3)$$

where E_s is the energy per symbol, τ_n is the relative delay of the *n*-th user, $x_n[l]$ is the *l*-th time-domain sample from the *n*-th user with a sample period T_1 , $z_c(t) = p_2(t) \odot v_c(t)$ is the noise component at the output of the receive filter, with $v_c(t)$ being the *additive white Gaussian noise* (AWGN) with one-sided power spectral density N_0 . The relative delay, τ_n , introduces mis-match between the receive filter and transmit filter, and the effects are captured as a time shift in the CIR $h_{nc}(t)$. It should be noted that the effects of frequency offsets are not considered in (3). In case of non-zero frequency offsets, they can be estimated at the receiver then compensated at the transmitter with the help of a feedback channel.

The output of the receive filter is sampled at the time instant $t = iT_2$, where $T_2 = T_1/u$ is the sampling period at the receiver, with the oversampling factor, u, being an integer. Denote the relative delays among the users as $\tau_n = l_n T_2 + \tau_{n0}$, where l_n represents the mis-alignment among the users in terms of receive samples, and $\tau_{n0} \in [0, T_2]$ is the timing phase offset between the sampling clocks at the transmitter and receiver. The discrete-time samples are

$$y_{T}[i] = \sum_{n=1}^{N} \sum_{l=0}^{ul_{c}-1} \sqrt{\frac{E_{s}}{R}} x_{nT}[i-l-l_{n}]h_{nT}[l] + z_{T}[i], \quad (4)$$

where $y_T[i] = y_c(iT_2)$ and $z_T[i] = z_c(iT_2)$ are the T_2 -spaced samples of the received signals and noise components, respectively, $h_{nT}[l] = h_{nc}(lT_2 - \tau_{n0})$ is the sampled version of

the continuous-time CIR $h_{nc}(t)$, and $x_{nT}[i]$ is the oversampled version of $x_n[i]$ as $x_{nT}[i] = x_n[i/u]$, if i/u is an integer, and 0 otherwise. It is assumed that the length of the CIR, ul_c , is an integer multiple of u, with l_c being the length of the CIR without oversampling, which can be always met by appending zeros to the CIR. The timing phase offset τ_{n0} is incorporated in the discrete-time CIR $h_{nT}[l]$. We will study in Section IV the impacts of τ_{n0} on the statistical properties of the channel coefficients and the system performance.

With the discrete-time system model given in (4), the length of the CP should satisfy $l_{cp} \geq l_c + l_d/u - 1$, where $l_d = \max\{l_n\}$ is the maximum relative transmission delay among the users. It should be noted that the proposed method can work for arbitrary value of l_d , and a larger l_d means a longer CP. To achieve better spectral and energy efficiency, it is assumed in the simulations that $l_d \in [0, uK)$.

Due to the time span of the transmit and receive filters, the CIR coefficients, $\{h_{nT}[l]\}_{l=0}^{uL_c-1}$, are correlated, even though the underlying channel might undergo uncorrelated scattering. The correlation coefficient, $c_n[l_1, l_2] = \mathbb{E} \left[h_{nT}[l_1]h_{nT}^*[l_2]\right]$, can be calculated as in [18, eqn. (17)].

After the removal of the CP, the received symbols can be written in a matrix form as

$$\mathbf{y}_{T} = \sqrt{\frac{E_{s}}{R}} \sum_{n=1}^{N} \mathbf{H}_{nT} \cdot \mathbf{x}_{n} + \mathbf{z}_{T}, \qquad (5)$$

where $\mathbf{y}_T = [y_T[0], \cdots, y_T[uL - 1]]^T \in \mathcal{C}^{uL \times 1}, \mathbf{z}_T = [z_T[0], \cdots, z_T[uL - 1]]^T \in \mathcal{C}^{uL \times 1}, \mathbf{H}_{nT} = [\mathbf{h}_{n,1}, \mathbf{h}_{n,u+1}, \cdots, \mathbf{h}_{n,(L-1)u+1}] \in \mathcal{C}^{uL \times L}$, with $\mathbf{h}_{n,k} \in \mathcal{C}^{uL \times 1}$ being the k-th column of a circulant matrix $\mathbf{H}_n \in \mathcal{C}^{uL \times uL}$. The first column of $\mathbf{H}_n \in \mathcal{C}^{uL \times uL}$ is $\mathbf{h}_{n,1} = [\mathbf{0}_{n}^T, h_{nT}[0], h_{nT}[1], \cdots, h_{nT}[ul_c - 1], \mathbf{0}_{uL - l_n - ul_c}^T,]^T$, and $\mathbf{0}_a$ is a length-a all-zero vector. With the equivalent discrete-time CIR representation, mis-alignments among users are represented in the form of time shifts in the columns of the circulant channel matrix \mathbf{H}_n , and the timing phase offsets are incorporated in the discrete-time CIR $h_{nT}[l]$.

Due to the time span of the receive filter and the oversampling operation, the time-domain noise vector is also correlated. The vector, \mathbf{z}_{T} , is zero-mean complex Gaussian distributed with a covariance matrix $\mathbf{R}_{\mathbf{z}_{T}} = \mathbb{E}(\mathbf{z}_{T}\mathbf{z}_{T}^{H}) =$ $N_{0}\mathbf{R}_{\varphi} \in \mathcal{C}^{uL \times uL}$, where the (m, n)-th element of \mathbf{R}_{φ} is $\int_{-\infty}^{+\infty} \varphi_{2}((m-n)T_{2}+\tau)\varphi_{2}^{*}(\tau)d\tau$ [19, Lemma 2].

The *uL*-point DFT is applied to the vector \mathbf{y}_T to convert the signal to the frequency domain as

$$\mathbf{y}_{F} = \sqrt{\frac{E_{s}}{R}} \sum_{n=1}^{N} \mathbf{H}_{nF} \cdot \mathbf{d}_{n} + \mathbf{z}_{F}, \qquad (6)$$

where $\mathbf{y}_F = \mathbf{F}_{uL} \mathbf{y}_T$ and $\mathbf{z}_F = \mathbf{F}_{uL} \mathbf{z}_T$ are the frequencydomain signal vector and noise vector, respectively, and $\mathbf{H}_{nF} = \mathbf{F}_{uL} \mathbf{H}_{nT} \mathbf{F}_{uL}^H \in \mathcal{C}^{uL \times L}$ is the frequency-domain channel matrix, with $\mathbf{F}_{uL} \in \mathcal{C}^{uL \times uL}$ being the *uL*-point DFT matrix. Due to the correlation among the noise samples in the time domain, they are still correlated in the frequency domain. The covariance matrix of \mathbf{z}_F is $\mathbf{R}_{\mathbf{z}_F} = N_0 \mathbf{F}_{uL} \mathbf{R}_{\varphi} \mathbf{F}_{uL}^H$. It should be noted that due to the on-off transmission, only RK out of the L = MK elements in \mathbf{d}_n are non-zero.

From (6), signals from different users are aligned in the frequency domain, even though they are asynchronous in the time domain. The matrix \mathbf{H}_{nF} can be partitioned into a stack of u sub-matrices as $\mathbf{H}_{nF} = [\mathbf{G}_{n0}^T, \cdots, \mathbf{G}_{n(u-1)}^T]^T$, where $\mathbf{G}_{nv} \in \mathcal{C}^{L \times L}$. The matrix, \mathbf{G}_{nv} , is a diagonal matrix, with the (m + 1)-th diagonal element being [19, Corollary 1]

$$G_{nv}[m] = \frac{e^{-j2\pi \frac{l_n \cdot (vL+m)}{uL}}}{\sqrt{u}} \sum_{l=0}^{ul_c-1} h_{nT}[l] e^{-j2\pi \frac{(vL+m) \cdot l}{uL}}.$$
 (7)

In (7), the delay, l_n , in the time-domain is manifested as a phase shift, $e^{-j2\pi \frac{l_n \cdot (vL+m)}{uL}}$, in the frequency domain.

With the model given in (6) and (7), each $d_n[m]$ is equivalently transmitted over u sub-channels with coefficients $\{\overline{G}_{nv}[m]\}_{v=0}^{u-1}$. Due to oversampling, u sub-channels at the receiver occupy the same bandwidth as one sub-channel at the transmitter. However, the relative alignment of the subchannels from different users remain unchanged. Consider the example in Fig. 1, with the block diagonal structure of \mathbf{H}_{nF} , the sub-channels at the receiver side can be obtained by duplicating the diagram in Fig. 1 u times in the frequency domain, then reduce the bandwidth of each sub-channel by a factor of u. Each modulated symbol, s_{nk} , is equivalently transmitted over uR sub-channels in the frequency domain. Therefore, frequency diversity is achieved with the proposed FD-OOAT scheme. The uR sub-channels spread over the entire frequency band to maximize the frequency diversity. We will quantify the frequency diversity order by resorting to an analytical performance bound in Section IV.

B. Collision Tolerance

With the frequency-domain system representation in (6), the received information at the *m*-th sub-channel at the BS is the superposition of a set of signals, $\{d_n[m]\}_{n=1}^N$. The value of $d_n[m]$ is 0 if $p_n[i_m] = 0$. Therefore, only a subset of the users collide at the *m*-th sub-channel. The collision order at the *m*-th sub-channel is $N_c[m] = \sum_{n=1}^N p_n[i_m]$. The collision order of the network is defined as $N_c = \max_m N_c[m]$. We have $N_c = 2$ for the system shown in Fig. 1. For a system with N users, R repetitions, and M sub-channels per symbol, there are NR repetitions transmitted over M sub-channels, thus it can be shown that the minimum collision order is $N_c = \left\lceil \frac{NR}{M} \right\rceil$, with $\lceil a \rceil$ being the smallest integer no less than a.

There are many different ways to construct the position vectors to achieve the minimum collision order. Here we present one simple construction scheme through cyclic shifting.

Definition 1: Given M and R, define the position vector of the first user as $\mathbf{p}_1 = [\mathbf{1}_R^T, \mathbf{0}_{M-R}^T]^T$, where $\mathbf{1}_n$ and $\mathbf{0}_n$ are length-n all-one and all-zero vectors, respectively. The position vector of the n-th user can then be obtained by cyclically shifting \mathbf{p}_1 to the right by (n-1)R positions, for $n = 2, \dots, N$.

Lemma 1: Consider an FD-OOAT system with N users, R repetitions, and M sub-channels per symbol. If the position

vectors are constructed as described in Definition 1, then the collision order of the system is $N_c = \left\lceil \frac{NR}{M} \right\rceil$.

Proof: Without loss of generality, consider sub-channel with index 0. Based on the cyclic shifting construction method, user n will transmit on sub-channel 0 if and only if there exists a non-negative integer q such that $(n-1)R \le qM \le (n-1)R + R - 1 < nR$. Since q is an integer, the above inequality can be alternatively written as

$$\lceil \frac{(n-1)R}{M} \rceil \le q < \lceil \frac{nR}{M} \rceil.$$
(8)

For a system with N users, we thus have $\max(q) \leq \lceil \frac{NR}{M} \rceil - 1 < \lceil \frac{NR}{M} \rceil$. On the other hand, $\lceil \frac{NR}{M} \rceil - 1 \leq \lceil \frac{(N-1)R}{M} \rceil \leq \max(q)$. Therefore $\max(q) = \lceil \frac{NR}{M} \rceil - 1$. The minimum value of q is 0. Therefore there are at most $\lceil \frac{NR}{M} \rceil$ values of q satisfying the inequality. Each value of q uniquely determines an n, thus there are at most $\lceil \frac{NR}{M} \rceil$ users transmitting at subchannel 0. The collision orders on the other sub-channels can be bounded in a similar manner.

It should be noted that the construction described in Definition 1 is not unique. We can get a set of position vectors that achieve the minimum collision order by performing the same permutations on all the position vectors obtained from Definition 1. Since all users permute their position vectors following the same pattern, the relative collision relationship among the N users remains unchanged.

The oversampled FD-OOAT scheme contributes to the performance improvement of the wireless network from the following perspectives. First, the on-off transmission will reduce the collision order. Second, the transmission of R identical sub-symbols with oversampling results in a uR-dimensional received signal in the frequency domain, which can be used for the detection of the N_c -dimensional signal in the space domain. Third, frequency diversity is achieved by transmitting the k-th symbol in uR sub-channels. Fourth, the relative delays among the users in the time domain are represented as phase shifts in the frequency domain, thus the user mis-alignment does not affect the collision order in the frequency domain.

III. COLLISION RESOLUTION WITH OPTIMUM AND SUB-OPTIMUM DETECTIONS

In this section, optimum and sub-optimum detectors are developed for the oversampled FD-OOAT system to resolve the collisions among the users and to collect the inherent frequency diversity. The detectors do not require precise synchronizations among the users. The complexity of the receiver is also studied.

A. Muti-user Detection

Since the time-domain mis-alignment among the users does not affect the user alignment in the frequency domain as shown in Fig. 1, the k-th symbol from one user will only interfere the k-th symbols from the other users. This is different from the time-domain OOAT [10], where the k-th symbol from one user might interfere adjacent symbols from the other users due to the signal mis-alignment in the time-domain. The k-th symbols from all the N users, $\{s_{nk}\}_{n=1}^{N}$, can be jointly detected by using a block of uM received signal samples $\mathbf{r}_k = [\mathbf{y}_0^T, \cdots, \mathbf{y}_{u-1}^T]^T \in \mathcal{C}^{uM \times 1}$ with $\mathbf{y}_v = [y_F[vL + k], y_F[vL + K + k], \cdots, y_F[vL + (M - 1)K + k]]^T \in \mathcal{C}^{M \times 1}$. The vector \mathbf{r}_k defined above is obtained by extracting uM elements from the frequency-domain vector \mathbf{y}_F , and it can be alternatively represented as $\mathbf{r}_k = \mathbf{B}_k \mathbf{y}_F$, where $\mathbf{B}_k \in \mathcal{B}^{uM \times uL}$ is obtained by extracting uM rows from a size-uL identity matrix \mathbf{I}_{uL} , with the indices of the extracted rows being vL + mK + k, for $v = 0, \cdots, u - 1$ and $m = 0, \cdots, M - 1$.

From (6), we have

$$\mathbf{r}_k = \sqrt{\frac{E_s}{R}} \mathbf{H}_k \cdot \mathbf{s}_k + \mathbf{w}_k, \tag{9}$$

where $\mathbf{s}_k = [s_{1k}, s_{2k}, \cdots, s_{Nk}]^T \in \mathcal{S}^{N \times 1}$ and $\mathbf{w}_k = \mathbf{B}_k \mathbf{z}_F \in \mathcal{C}^{uM \times 1}$ are the modulation symbol vector and noise vector, respectively, $\mathbf{H}_k = [\mathbf{G}_0^T, \cdots, \mathbf{G}_{u-1}^T]^T$, and $\mathbf{G}_v \in \mathcal{C}^{M \times N}$ is the frequency-domain channel matrix with the (m + 1, n)-th element being $p_n[m]G_{nv}[mK + k]$. Since the elements of \mathbf{w}_k are extracted from \mathbf{z}_F , they are mutually correlated with the covariance matrix $\mathbf{R}_{\mathbf{w}_k} = N_0 \mathbf{B}_k \mathbf{F}_{uL} \mathbf{R}_p \mathbf{F}_{uL}^H \mathbf{B}_k^H$.

The optimum maximum likelihood (ML) detection of (9) is

$$\hat{\mathbf{s}}_{k} = \operatorname*{argmin}_{\mathbf{s}_{k} \in \mathcal{S}^{N}} \left(\mathbf{r}_{k} - \sqrt{\frac{E_{s}}{R}} \mathbf{H}_{k} \mathbf{s}_{k} \right)^{H} \mathbf{R}_{\mathbf{w}_{k}}^{\dagger} \left(\mathbf{r}_{k} - \sqrt{\frac{E_{s}}{R}} \mathbf{H}_{k} \mathbf{s}_{k} \right),$$
(10)

where $\mathbf{R}_{\mathbf{w}_k}^{\dagger}$ is the pseudo-inverse of $\mathbf{R}_{\mathbf{w}_k}$. The ML detection requires the exhaustive search of a set of S^N possible signal vectors, and the complexity grows exponentially with the modulation level S and the number of users N.

A low-complexity detection algorithm is presented here to balance the performance-complexity tradeoff. The suboptimum algorithm is developed by employing an iterative *soft input soft output* (SISO) *block decision feedback equalizer* (BDFE) [20], which performs *soft successive interference cancellation* (SSIC) among the N symbols in s_k .

The soft input to the iterative BDFE equalizer is the *a priori* probability of the symbols, $P(s_{nk} = S_i)$, for $n = 1, \dots, N$ and $i = 1, \dots, S$, where $S_i \in S$. The *a priori* information is obtained from the previous detection round with an iterative detection method. The soft output of the equalizer is the *a posteriori* probability of the symbols, $P(s_{nk} = S_i | \mathbf{r}_k)$, for $n = 1, \dots, N$ and $i = 1, \dots, S$. With the soft output at the equalizer, define the *a posteriori* mean, \hat{s}_{nk} , and the extrinsic information, $\beta_{nk}[i]$, of the symbol $s_n(k)$ as

$$\hat{s}_{nk} = \sum_{i=1}^{S} P(s_{nk} = S_i | \mathbf{r}_k) S_i$$
 (11a)

$$\beta_{nk}[i] = \log P(s_{nk} = S_i | \mathbf{r}_k) - \log P(s_{nk} = S_i) (11b)$$

The *a posteriori* mean, \hat{s}_{nk} , is used as soft decisions for the SSIC during the SISO-BDFE process. Details of the SISO-BDFE detection can be found in [20].

In the proposed sub-optimum detection, the SISO-BDFE with SSIC will be performed iteratively. At the first iteration,

the *a priori* probability is initialized to $P(s_{nk} = S_i) = \frac{1}{S}$. The extrinsic information at the output of the *v*-th iteration will be used as the soft input of the (v + 1)-th iteration as $P(s_{nk} = S_i) = c_{nk} \exp[\beta_{nk}[i]]$, where c_{nk} is a normalization constant to make $\sum_{i=1}^{S} P(s_{nk} = S_i) = 1$. At the final iteration, hard decision will be made based on the *a posteriori* probability generated by the SISO-BDFE as

$$\hat{s}_{nk} = \underset{S_i \in \mathcal{S}}{\operatorname{argmax}} P(s_{nk} = S_i | \mathbf{r}_k).$$
(12)

Simulation results show that the performance of the iterative detection algorithm usually converges after 4 iterations.

IV. PERFORMANCE ANALYSIS

Theoretical analysis is performed in this section to quantify the impacts of timing phase offsets on the performance of the proposed FD-OOAT scheme.

A. An Analytic Performance Lower Bound

An analytic performance lower bound on the *bit-error rate* (BER) of the proposed frequency-domain CT-MAC scheme with *binary phase shift keying* (BPSK) is developed by employing the genie-aided detector [21], where a genie provides side information of the symbols from all other users such that interference-free detection can be performed. The genie-aided bound is the same as the exact BER of a single user system, because it assumes that interferences from all other users can be removed. It will be shown through simulations that the bound is very tight even when the number of users is large due to the collision-tolerance properties of the proposed scheme.

With the interference-free assumption, the received signal corresponding to the k-th symbol of the n-th user is

$$\mathbf{r}_{nk} = \sqrt{\frac{E_s}{R}} \mathbf{g}_{nk} \cdot s_{nk} + \mathbf{w}_{nk}, \tag{13}$$

where $\mathbf{r}_{nk} = [\mathbf{y}_{n0}^T, \cdots, \mathbf{y}_{n(u-1)}^T]^T \in \mathcal{C}^{uR \times 1}$ with $\mathbf{y}_{nv} = [y_F[vL+n_1K+k], \cdots, y_F[vL+n_RK+k]]^T \in \mathcal{C}^{R \times 1}$, n_r is the r-th non-zero position in \mathbf{p}_n , $\mathbf{w}_{nk} = [\mathbf{z}_{n0}^T, \cdots, \mathbf{z}_{n(u-1)}^T]^T \in \mathcal{C}^{uR \times 1}$ with $\mathbf{z}_{nv} = [z_F[vL+n_1K+k], \cdots, z_F[vL+n_RK+k]]^T \in \mathcal{C}^{R \times 1}$, and $\mathbf{g}_{nk} = [\tilde{\mathbf{G}}_{n0}^T, \cdots, \tilde{\mathbf{G}}_{n(u-1)}^T]^T \in \mathcal{C}^{uR \times 1}$ with $\tilde{\mathbf{G}}_{nv} = [G_{nv}[n_1K+k], \cdots, G_{nv}[n_RK+k]]^T \in \mathcal{C}^{R \times 1}$ being the channel coefficient vector.

From the system model in (13), R repetitions of each symbol is equivalently transmitted over uR sub-channels, which is equivalent to a *single-input multiple-output* (SIMO) system. The SIMO system has correlated channel taps and is corrupted by colored noise.

The channel coefficient vector, \mathbf{g}_{nk} , can be represented as

$$\mathbf{g}_{nk} = \sqrt{L} \mathbf{B}_{nk} \cdot \mathbf{F}_{uL} \cdot \mathbf{h}_{n,1}, \tag{14}$$

where $\mathbf{h}_{n,1}$ is the first column of the matrix \mathbf{H}_n , and $\mathbf{B}_{nk} \in \mathcal{B}^{uR \times uL}$ is a binary matrix, with the $(vR+r, vL+n_rK+k+1)$ -th element being 1, for $r = 1, \dots, R$, $v = 0, \dots, u-1$, and all other elements being zero.

The auto-correlation matrix, $\mathbf{R}_{nk} = \mathbb{E}[\mathbf{g}_{nk}\mathbf{g}_{nk}^{H}]$, is

$$\mathbf{R}_{nk} = L \mathbf{B}_{nk} \mathbf{F}_{uL} \mathbf{R}_{\tilde{h}n} \mathbf{F}_{uL}^H \mathbf{B}_{nk}^T, \qquad (15)$$

where $\mathbf{R}_{\tilde{h}n} = \mathbb{E}(\mathbf{h}_{n,1}\mathbf{h}_{n,1}^H)$. $\mathbf{R}_{\tilde{h}n}$ can be written as a block between the samples in \mathbf{w}_{nk} is usually very small. To measure matrix as

$$\mathbf{R}_{\tilde{h}n} = \begin{bmatrix} \mathbf{0}_{l_n \times l_n} & \mathbf{0}_{l_n \times ul_c} & \mathbf{0}_{l_n \times l_r} \\ \mathbf{0}_{ul_c \times l_n} & \mathbf{R}_{hn} & \mathbf{0}_{ul_c \times l_r} \\ \mathbf{0}_{l_r \times l_n} & \mathbf{0}_{l_r \times ul_c} & \mathbf{0}_{l_r \times l_r} \end{bmatrix},$$
(16)

where $l_r = L - l_n - ul_c$, and the (l_1, l_2) -th element of $\mathbf{R}_{hn} \in \mathcal{C}^{ul_c imes ul_c}$ is $c_n[l_1, l_2] = \mathbb{E}\left[h_{nT}[l_1]h_{nT}^*[l_2]\right]$, and it can be calculated from [18, eqn. (17)].

The covariance matrix $\mathbf{R}_{\mathbf{w}_{nk}}$ of the colored noise \mathbf{w}_{nk} is

$$\mathbf{R}_{\mathbf{w}_{nk}} = N_0 \mathbf{B}_{nk} \mathbf{F}_{uL} \mathbf{R}_{\varphi} \mathbf{F}_{uL}^H \mathbf{B}_{nk}^T.$$
(17)

The covariance matrix might be rank deficient. Define the pseudo-inverse of the noise covariance matrix $\mathbf{R}_{\mathbf{w}_{nk}}$

$$\mathbf{R}_{\mathbf{w}_{nk}}^{+} = \mathbf{U}_{nk} \mathbf{\Lambda}_{nk}^{-1} \mathbf{U}_{nk}^{H} \in \mathcal{C}^{uR \times uR}, \qquad (18)$$

with $\mathbf{U}_{nk} = [\mathbf{u}_{nk,1}, \mathbf{u}_{nk,2}, \cdots, \mathbf{u}_{nk,v_k}] \in \mathcal{C}^{uR \times v_k}, \mathbf{\Lambda}_{nk} = \text{diag}[\lambda_{nk,1}, \lambda_{nk,2}, \cdots, \lambda_{nk,v_k}] \in \mathcal{C}^{v_k \times v_k}$, where v_k is the number of non-zero eigenvalues of $\mathbf{R}_{\mathbf{w}_{nk}}$, $\mathbf{\Lambda}_{nk}$ is a diagonal matrix with $\{\lambda_{nk,i}\}_{i=1}^{v_k}$ being the non-zero eigenvalues of $\mathbf{R}_{\mathbf{w}_{nk}}$, and $\{\mathbf{u}_{nk,i}\}_{i=1}^{v_k}$ are the corresponding orthonormal eigenvectors.

Define the noise whitening matrix $\mathbf{D}_{nk} = \mathbf{\Lambda}_{nk}^{-1/2} \mathbf{V}_{nk}^{H}$. Applying \mathbf{D}_{nk} to both sides of (18) yields an equivalent system

$$\bar{\mathbf{r}}_{nk} = \sqrt{\frac{E_s}{R}} \bar{\mathbf{g}}_{nk} \cdot s_{nk} + \bar{\mathbf{w}}_{nk}, \qquad (19)$$

where $\bar{\mathbf{r}}_{nk} = \mathbf{D}_{nk}\mathbf{r}_{nk}$, $\bar{\mathbf{g}}_{nk} = \mathbf{D}_{nk}\mathbf{g}_{nk}$, and $\bar{\mathbf{w}}_{nk} =$ $\mathbf{D}_{nk}\mathbf{w}_{nk}$ with the covariance matrix of $\bar{\mathbf{w}}_{nk}$ being $\mathbf{R}_{\bar{\mathbf{w}}_{nk}}$ $\mathbf{D}_{nk}\mathbf{R}_{\mathbf{w}_{nk}}\mathbf{D}_{nk}^{H}=N_{0}\mathbf{I}_{v_{k}}.$

The SNR of (19) can be written as

$$\gamma = \mathbf{g}_{nk}^{H} \mathbf{R}_{\mathbf{w}_{nk}}^{+} \mathbf{g}_{nk} \frac{\gamma_{0}}{R}, \qquad (20)$$

where $\gamma_0 = \frac{E_s}{N_0}$ is the SNR without fading. For systems with BPSK and Rayleigh fading, the error probability for s_{nk} is [12]

$$P_{nk}(E) = \frac{1}{\pi} \int_0^{\frac{\pi}{2}} \prod_{r=1}^{L_{nk}} \left[1 + \frac{\delta_{nkr}\gamma_0}{R\sin^2\theta} \right]^{-1} d\theta, \quad (21)$$

where L_{nk} is the rank of the product matrix, $\mathbf{D}_{nk}\mathbf{R}_{nk}\mathbf{D}_{nk}^{H}$, and δ_{nkr} , for $r = 1, \dots, L_{nk}$, are the corresponding nonzero eigenvalues. The average BER can then be calculated as $P(E) = \frac{1}{NK} \sum_{n=1}^{N} \sum_{k=1}^{K} P_{nk}(E).$

In the above analysis, the order of multipath diversity is \hat{L}_{nk} , which is the rank of $\mathbf{D}_{nk}\mathbf{R}_{nk}\mathbf{D}_{nk}^{H}$. The matrix \mathbf{R}_{nk} corresponds to correlation of the frequency domain channel coefficients, and $\mathbf{D}_{nk}^H \mathbf{D}_{nk} = \mathbf{R}_{\mathbf{w}_{nk}}^+$ is the pseudo-inverse of the noise covariance matrix, $\mathbf{R}_{\mathbf{w}_{nk}}$.

The off-diagonal elements of the matrix $\mathbf{R}_{\mathbf{w}_{nk}}$ are contributed by the correlation of the colored noise. The uR elements in the noise vector, \mathbf{w}_{nk} , are extracted from the size-uLfrequency-domain noise vector \mathbf{z}_{F} based on the transmission pattern \mathbf{p}_n , and there is at least K sub-channels between any two samples in \mathbf{w}_{nk} . As a result, the mutual correlation the mutual correlation of the samples in \mathbf{w}_{nk} , define a metric

$$\rho = \frac{1}{NK} \sum_{n=1}^{N} \sum_{k=1}^{K} \frac{\|\mathbf{R}'_{\mathbf{w}_{nk}}\|_2}{\|\mathbf{R}_{\mathbf{w}_{nk}}\|_2},$$
(22)

where $\mathbf{R}'_{\mathbf{w}_{nk}}$ is a diagonal matrix obtained by setting all offdiagonal elements of $\mathbf{R}_{\mathbf{w}_{nk}}$ to 0, and $\|\mathbf{A}\|_2$ is the Frobenius norm of the matrix A. The metric $0 \le \rho \le 1$ measures the percentage of energy on the diagonal of $\mathbf{R}_{\mathbf{w}_{nk}}$, and $\rho = 1$ means that $\mathbf{R}_{\mathbf{w}_{nk}}$ is a diagonal matrix. Table 1 shows the values of $1 - \rho$ with u = 2, M = 12, R = 2, and various values of K. It is clear that ρ is very close to 1, and the difference between ρ and 1 decreases as K increases. The results in Table 1 demonstrate that the off-diagonal elements of $\mathbf{R}_{\mathbf{w}_{nk}}$ are negligible compared to its diagonal elements.

If we ignore the off-diagonal elements of $\mathbf{R}_{\mathbf{w}_{nk}}$ and approximate the noise vector \mathbf{w}_{nk} as white noise with correlation matrix $\mathbf{R}'_{\mathbf{w}_{nk}}$, then we can simplify the error performance analysis. With the white noise assumption, the SNR in (20) can be approximated by

$$\gamma_{nk}' = \frac{\gamma_0}{R} \sum_{v=0}^{u-1} \sum_{r=1}^{R} |G_{nv}[n_r K + k]|^2 \phi_{nk}[vR + r], \quad (23)$$

where $\phi_{nk}[r] = q_{nk}^{-1}[r]$ if $q_{nk}[r] \neq 0$ with $q_{nk}[r]$ being the rth diagonal element of $\mathbf{R}_{\mathbf{w}_{nk}}$, and $\phi_{nk}[r] = 0$ otherwise. The error probability in (21) can then be approximated by using the eigenvalues of the product matrix $\mathbf{D}'_{nk}\mathbf{R}_{nk}\mathbf{D}'^{H}_{nk}$, with $\mathbf{D}'_{nk} =$ diag $\left\{\phi_{nk}^{1/2}[0], \cdots, \phi_{nk}^{1/2}[uR-1]\right\}$ being a diagonal matrix. The BER results calculated with the white approximation in (23) is very close to the exact genie-aided bound in (21), from our simulation since ρ is very close to 1.

B. Impacts of Relative Delays

In this subsection, a theoretical framework is provided to study the impacts of the relative delays among the users on the performance of the proposed FD-OOAT scheme. From the analysis in the previous subsection, the performance of the system is dominated by the statistical properties of the SNR γ'_{nk} defined in (23), which in turn depends on the squared amplitude of the channel coefficients, $|G_{nv}[m]|^2$. It should be noted that the power and the auto-correlation of the noise components are independent of the relative delays τ_n as evident in (17).

The relative delay can be expressed as $\tau_n = l_n T_2 + \tau_{n0}$, where l_n represents the mis-alignment among the users, and $\tau_{n0} \in [0, T_2]$ is the timing phase offset of the sampler. It is clear from (7) that l_n has no impact on the squared amplitude $|G_{nv}[m]|^2$. Next we will study the impact of τ_{n0} on $|G_{nv}[m]|^2$.

TABLE I The metric $1 - \rho$ under different K (M = 12, R = 2 and u = 2).

K	1	10	20	50	100
$1-\rho$	4.9×10^{-3}	2.3×10^{-4}	8.9×10^{-5}	4.0×10^{-5}	1.8×10^{-5}

Define the *discrete-time Fourier transform* (DTFT) of the T_2 -spaced discrete-time CIR, $h_{nT}[l]$, as

$$H_{nT}(f) = \sum_{l=0}^{ul_c-1} h_{nT}[l] e^{-j2\pi l f}, 0 \le f \le 1$$
(24)

Since $h_{nT}[l] = h_{nc}(lT_2 - \tau_{n0})$, based on the sampling theorem, the DTFT can be expressed as

$$H_{nT}(f) = \frac{1}{T_2} \sum_{i=-\infty}^{\infty} \mathcal{H}_{nc}\left(\frac{f-i}{T_2}\right) \exp\left(-j2\pi\tau_{n0}\frac{f-i}{T_2}\right),\tag{25}$$

where $\mathcal{H}_{nc}\left(\frac{f}{T_2}\right)$ is the Fourier transform of the CIR $h_{nc}(t)$. From (7), (24), and (25), we can write the frequency-domain channel coefficient, $G_{nv}[m]$, as

 $e^{-j2\pi \frac{l_n \cdot (vL+m)}{m}\infty} \qquad (vL+m)$

$$G_{nv}[m] = \frac{e^{-j2\pi} \overline{T_2 \sqrt{u}}}{T_2 \sqrt{u}} \sum_{i=-\infty} \mathcal{H}_{nc} \left(\frac{vL+m}{uLT_2} - \frac{i}{T_2}\right) e^{-j2\pi\tau_{n0}\frac{vL+m-uLi}{uLT_2}}$$
(26)

The CIR, $h_{nc}(t)$, includes the effects of the physical channel and the transmit and receive filters. From (2), we have $\mathcal{H}_{nc}\left(\frac{f}{T_2}\right) = \mathcal{P}_1\left(\frac{f}{T_2}\right)\mathcal{G}_n\left(\frac{f}{T_2}\right)\mathcal{P}_2\left(\frac{f}{T_2}\right)$, where $\mathcal{P}_i\left(\frac{f}{T_2}\right)$ and $\mathcal{G}_n\left(\frac{f}{T_2}\right)$ are the Fourier transforms of $\varphi_i(t)$ and $g_n(t)$, respectively. If the roll-off factor of the transmit and receive filters is α , then the frequency domain support of $\mathcal{P}_i\left(\frac{f}{T_2}\right)$ is $\left|\frac{f}{T_2}\right| \leq \frac{1+\alpha}{2T_1}$, or $|f| \leq \frac{1+\alpha}{2u}$. All practical systems have at most 100% excessive bandwidth, *i.e.*, $\alpha \leq 1$. Therefore, $\mathcal{H}_{nc}\left(\frac{f}{T_2}\right) = 0$ for $|f| > \frac{1}{u}$.

 $\begin{aligned} \mathcal{H}_{nc}\left(\frac{f}{T_2}\right) &= 0 \text{ for } |f| > \frac{1}{u}. \\ I) \ u &= 1: \text{ For a system without oversampling, we have } \\ T_1 &= T_2, \text{ and the frequency-domain support of } \mathcal{P}_i\left(\frac{f}{T_1}\right) \text{ and } \\ \mathcal{H}_{nc}\left(\frac{f}{T_1}\right) \text{ are } |\frac{f}{T_1}| < \frac{1+\alpha}{2T_1}. \text{ Due to the excessive bandwidth } \\ \text{of the transmitted signal when } \alpha > 0, \text{ the sampling operation } \\ \text{at the receiver causes spectrum aliasing as shown in (25) and } \\ (26). \text{ It is apparent from (26) that the frequency-domain channel coefficient is a function of <math>\tau_{n0}. \text{ Therefore, the performance } \\ \text{of the system with } u = 1 \text{ will be affected by } \tau_{n0}. \end{aligned}$

2) $u \geq 2$: The frequency-domain support of $\mathcal{P}_i\left(\frac{f}{T_2}\right)$ and $\mathcal{H}_{nc}\left(\frac{f}{T_2}\right)$ are $\left|\frac{f}{T_2}\right| < \frac{1+\alpha}{2uT_2} \leq \frac{1}{2T_2}$ for $\alpha \leq 1$. Therefore, the sampling rate $\frac{1}{T_2}$ is at least twice as much as the signal bandwidth, and there is no spectrum aliasing after sampling. The channel coefficient in (26) is simplified to

$$G_{nv}[m] = \frac{e^{-j2\pi \frac{l_n \cdot (vL+m)}{uL}}}{T_2\sqrt{u}} \mathcal{H}_{nc}\left(\frac{vL+m}{uLT_2}\right) e^{-j2\pi\tau_{n0}\frac{vL+m}{uLT_2}}.$$
(27)

The squared amplitude of the channel coefficient can then be expressed as

$$|G_{nv}[m]|^2 = \frac{1}{T_2\sqrt{u}} \left| \mathcal{H}_{nc}\left(\frac{vL+m}{uLT_2}\right) \right|^2.$$
(28)

It is interesting to note that $|G_{nv}[m]|^2$ is independent of the user mis-alignments l_n or the timing phase offset τ_{n0} . Since the system performance is dominated by the squared amplitude

of the channel coefficient as shown in the SNR defined in (23), the user mis-alignments or timing phase offset has a very small, if any, impact on the performance of the system when $u \ge 2$. Specifically, for systems with at most 100% excessive bandwidth, an oversampling factor of 2 is sufficient to avoid spectrum aliasing at the receiver, thus removes the impacts of τ_{n0} . The above analysis is corroborated by simulation results with both optimum and sub-optimum detectors.

V. SIMULATION RESULTS

In the simulation examples, the sample period at the transmitter is set to $T_1 = 3.69 \ \mu s$, and RRC filters with a rolloff factor $\alpha = 1.0$ is used for both the transmit and receive filters. The relative delays among the users, τ_n , is uniformly distributed between $[0, KT_1]$ with K = 50 unless stated otherwise. The frequency-selective fading channel follows the *Typical Urban* (TU) power delay profile (PDP) [22].

Fig. 2 shows the BER results of the proposed CT-MAC system under various system configurations. There are M =12 sub-channels and R = 2 repetitions for each symbol. The sub-optimum BDFE detection is performed with 4 iterations. The analytical results are obtained with both (21) and the white approximation as in (23), and the two results overlap. We have the following observations about the results. First, when N = 1, the analytical and simulation results match perfectly for both u = 1 and 2. Second, with the BDFE receiver, increasing N has less impacts on the oversampled system with u = 2 than the system with u = 1. At BER = 2×10^{-3} , increasing N from 1 to 10 results in a 1.5 dB and a 0.8 dB performance loss for systems with u = 1and u = 2, respectively. This indicates that the proposed FD-OOAT system can operate properly even when there are a large number of users and collisions. In addition, when u = 2 and N = 10, the sub-optimum BDFE receiver achieves almost the same performance as the optimum ML receiver, but with a much lower complexity. Third, the oversampled system consistently outperforms the system without oversampling. The performance improvement is contributed by the additional multipath diversity and the insensitivity to the timing phase offset due to the oversampling operation. At BER = 2×10^{-3} and N = 10, the oversampled system outperforms its nonoversampled counterpart by 5.6 dB when BDFE is used.

The effects of the receiver timing phase offset on the system performance are studied through simulations in Fig. 3 for single-user systems and Fig. 4 for multi-user systems, respectively. In Fig. 3, there are R = 2 repetitions and M = 12 sub-channels per symbol. To have a better understanding on the effects of timing phase offset, it is assumed that τ_{n0} is fixed at 0 or $0.5T_2$ in Fig. 3. The performance of the system with u = 1 varies as τ_{n0} changes, yet the performance of the oversampled system is independent of τ_{n0} .

A similar observation is obtained in Fig. 4 for systems with multiple users, where the BER is shown as a function of τ_{n0} . The mis-alignment among the asynchronous users, l_n , is uniformly distributed between [0, uK]. The E_b/N_0 is 10 dB. The BER of the oversampled system stays constant



Fig. 2. BER performance comparison of systems with M = 12 sub-channels per symbol, R = 2 repetitions, and different number of users.

regardless of the values of τ_{n0} , for both the optimum and sub-optimum algorithms with different number of users. On the other hand, the BER of the system with u = 1 is a function of τ_{n0} . The simulation results corroborate the theoretical analysis that twice oversampling is sufficient to remove the effects of τ_{n0} for a system with at most 100% excessive bandwidth. Therefore, the proposed oversampled FD-OOAT scheme can operate effectively at the presence of both multiuser interference, user mis-alignment, and timing phase offset.

Fig. 5 demonstrates the impacts of the number of iterations on the *frame error rate* (FER) with the sub-optimum BDFE detector through simulations. There are N = 10 active users, R = 2 repetitions and M = 12 sub-channels per symbol. The largest performance gain is achieved at the second iteration and the performance converges at the fourth iteration for systems with u = 1 or u = 2. At the fourth iteration and FER = 4×10^{-2} , the FER performance of the oversampled system outperforms the one without oversampling by 5.6 dB, which is consistent with the BER improvement observed in Fig. 2.

Fig. 6 shows the normalized throughput as a function of the normalized offered load for various MAC schemes. For the FD-OOAT system, there are M = 10 sub-channels and R = 2 repetitions per symbol. All other systems have M = 10slots per frame. The normalized offered load of all systems is calculated as $G = \frac{N}{M}$. The normalized throughput is defined as the amount of data successfully delivered to the receiver per unit time per unit bandwidth. The normalized throughput for the FD-OOAT scheme is calculated as $\frac{N}{M}(1 - \text{FER})$. Details of the calculation of the normalized offered load and normalized throughput can be found in [10]. For the slotted ALOHA, CRDSA, and IRSA systems, the simulations are performed under the assumption of noise-free communication, i.e., the only source of errors for these systems is the unresolvable signal collisions among the users. Results obtained under the noise-free assumption represent the best possible performance



Fig. 3. The effects of the receiver timing phase offset on the BER performance of the system (There are N = 1 user, M = 12 sub-channels per symbol, and R = 2 repetitions).



Fig. 4. BER v.s. timing phase offset $(E_b/N_0 = 10 \text{ dB})$. There are M = 12 sub-channels per symbol, and R = 2 repetitions).

under any channel configurations. On the other hand, the results of the proposed FD-OOAT systems are obtained in a frequency-selective fading channel with $E_b/N_0 = 15$ dB. As shown in the figure, the slotted ALOHA, CRDSA and IRSA achieve their respective peak throughput when $G \leq 1$, and the throughput drop dramatically when G > 1. The proposed FD-OOAT scheme achieves the maximum throughput 1.03 bps/Hz at G = 1.6 when u = 1. For the oversampled system with u = 2, the maximum throughput 2.06 bps/Hz is achieved at G = 2.6. Therefore, the FD-OOAT system can be overloaded by supporting more users than the number of sub-channels, yet all the other MAC schemes must operate with $G \leq 1$. Employing FD-OOAT increases both the number



Fig. 5. FER performance of systems with the BDFE receiver (There are N = 10 users, M = 12 sub-channels, and R = 2 repetitions).



Fig. 6. Normalized throughput v.s. normalized offered load.

of users supported and peak throughput. In addition, timedomain oversampling allows the FD-OOAT system to support 60% more users than the system with u = 1, and improves the throughput by 100%.

VI. CONCLUSIONS

A cross-layer CT-MAC scheme with frequency-domain OOAT and time-domain oversampling has been proposed for broadband wireless networks operating in frequency-selective fading. With the help of time-domain oversampling, the proposed scheme can operate without precise synchronization, and it is insensitive to timing phase offsets between the sampling clocks at the transmitter and receiver. Simulation results demonstrated that 1) the performance of the oversampled FD-OOAT system was insensitive to user mis-alignment or sampler timing phase offset; 2) significant multipath diversity gain was achieved with the oversampled FD-OOAT scheme; 3) the proposed scheme achieved a high spectral efficiency by supporting a large number of simultaneous broadband users. An oversampled FD-OOAT with M sub-channels per symbol could support up to N = 2.6M simultaneous users and has a normalized throughput peak at 2.06 bps/Hz with BPSK.

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