A Modified Fixed Sphere Decoding Algorithm for Under-Determined MIMO Systems

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***Abstract*—A modified FSD algorithm is proposed for under-determined (UD) multiple-input multiple-output (MIMO) systems with N transmit antennas and $M < N$ receive antennas. This paper focuses on the low-complexity detection of coded UD-MIMO systems with iterative turbo detection, where a soft-input soft-output (SISO) MIMO detector exchanges soft information with a SISO decoder. In the first iteration, a modified fixed complexity sphere decoding (FSD) method is developed by utilizing the structure of a UD-MIMO system. The modified FSD employs a new detection ordering scheme that has a lower complexity but a better performance compared to the conventional ordering scheme. From the second iteration and beyond, the MIMO detector is implemented with a generalized serial interference cancelation (GSIC) scheme and a block decision feedback equalizer (BDFE) to further reduce the complexity. Simulation results show that the newly proposed FSD-GSIC-BDFE structure can achieve significant performance gains over existing schemes, especially for systems with high level modulations.**

I. Introduction

An under-determined (UD) linear system has more unknown variables than the number of equations or observations. It can be used to model a wide variety of wireless communication systems, *e.g.*, a spatial multiplexing multiple-input multiple-output (MIMO) system with N transmit anteannas and $M < N$ receive antennas, and the uplink of a infrastructure based wireless network where a N wireless nodes transmit to a base station with $M < N$ antennas, etc. This paper focuses on the low-complexity detection of coded UD-MIMO systems, and the results can be easily extended to other UD communication systems or networks.

The optimum solution of the UD-MIMO system can be obtained through exhaustive search of the set Q^N, where Q is the modulation level. However, the complexity of the optimum detection grows exponentially with Q and N. A large number of low complexity detection methods have been proposed for symmetric ($N = M$) or over-determined ($N < M$) MIMO systems, such as the optimum sphere decoding (SD) [1] with maximum likelihood (ML) detection, the sub-optimum fixed-complexity sphere decoding (FSD) [2] and [3], and the vertical Bell laboratories layered space-time (V-BLAST) [4]. All of the above schemes cannot be directly applied to a UD-MIMO system because they would require the inverse of a rank deficient matrix in the UD-MIMO system.

Several sub-optimum methods have been proposed to solve the UD-MIMO system with affordable complexity. A generalized parallel interference cancelation (GPIC) is proposed in [6], where exhaustive search is performed over the extra $N - M$ signal dimensions. The exhaustive search generates Q^{N-M} parallel symmetric sub-systems, and V-BLAST is used in each sub-system. A generalized sphere decoding (GSD) scheme is proposed in [7] by combining GPIC with SD in the parallel sub-systems. In [8], the metric calculation of sphere decoding is modified to avoid the inversion of a rank deficient matrix, but the method works only for constant-modulo constellation.

Recently, turbo detection is investigated in [9] for coded UD-MIMO system, where the soft-input soft-output (SISO) MIMO detector iteratively exchanges soft information with a SISO decoder. The SISO-MIMO detector is implemented by utilizing GPIC with block decision feedback equalization (BDFE) in the first iteration and a new generalized serial interference cancelation (GSIC) with BDFE in all the other iterations. The system works well for low level modulations, and its performance deteriorates rapidly as the modulation level increases.

In this paper, we propose an efficient MIMO detector for coded UD-MIMO systems with high level modulations. A modified SISO FSD algorithm is developed by tailoring towards the structure of UD-MIMO systems. The modified FSD algorithm divides the transmit antennas into two groups, such that the group with signals of lower signal-to-noise ratio (SNR) will undergo an exhaustive tree search and the group with the stronger signals will go through a low complexity constrained tree search. Therefore, channel ordering is much simplified in comparison to the conventional FSD scheme [2]. The modified FSD algorithm is used in the first iteration of the turbo detection, and the GSIC-BDFE is used in subsequent iterations to futher reduce the complexity. The GSIC-BDFE orders the symbols based on a reliability estimation, which is calculated from the *a priori* input and is dynamically updated as the iterations progress. For low level modulation systems as studied in [9], symbols with higher reliability are detected firstly by treating those with lower reliability as interference. We propose to reverse this order for high level modulation systems, such that symbols with lower reliability will be used in interference cancelation firstly to reduce the residual interference. Simulation results show that the newly proposed FSD-GSIC-BDFE scheme outperforms existing schemes by 2

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dB in flat-fading UD-MIMO channel.

II. System Model

Fig. 1 shows the block diagram of a UD-MIMO system with N transmit antennas and $M < N$ receive antennas. Independent N bit streams, $\{\mathbf{a}_n\}_{n=1}^N$, are encoded by convolutional encoders to generate the coded bit streams, $\{\mathbf{b}_n\}_{n=1}^N$, which are then interleaved by pseudo-random interleavers to get the interleaved bit streams, $\mathbf{c}_n = \Pi(\mathbf{b}_n)$, for $n = 1, \cdots, N$, where $\Pi(\cdot)$ is the interleaving operator. Every K bits in a coded bit stream are grouped and mapped to a modulation symbol following a modulation constellation set $\mathcal{Q} = \{\chi_q\}_{q=1}^Q$ with cardinality $Q = 2^K$. The modulated symbols, $\mathbf{x} = [x_1, \cdots, x_N]^T \in \mathcal{S}^{N\times 1}$, are transmitted on N transmit antennas.

The signals sampled at the M receive antennas can be represented as

$$\mathbf{y} = \mathbf{Hx} + \mathbf{v} \tag{1}$$

where $\mathbf{y} = [y_1, \cdots, y_M]^T \in \mathcal{C}^{M\times 1}$ and $\mathbf{v} = [v_1, \cdots, v_M]^T \in \mathcal{C}^{M\times 1}$ represent the received signal and the additive white gaussian noise (AWGN), respectively, with $[\cdot]^T$ denoting the matrix transpose operation. The matrix $\mathbf{H} \in \mathcal{C}^{M\times N}$, is the flat-fading MIMO channel matrix, with the (m, n)-th element, $h_{m,n}$, being the channel coefficient between the n-th transmit antenna and the m-th receive antenna.

Turbo detection is employed at the receiver, which consists of a SISO-MIMO detector and N SISO convolutional decoders, separated by deinterleavers and interleavers as shown in Fig. 1. The optimum maximum *a posteriori* (MAP) algorithm is employed by the convolutional decoders. The decoder and the equalizer exchange soft extrinsic information iteratively to improve the performance. The SISO-MIMO detector calculates the *a posteriori* log-likelihood ratio (LLR) of $c_{n,k}$, the k-th bit from the n-th transmit antenna, as,

$$L_{D1}^{n,k} = L_{D1}(c_{n,k}|\mathbf{y}) = \ln \frac{P(c_{n,k} = 0|\mathbf{y})}{P(c_{n,k} = 1|\mathbf{y})} \tag{2}$$

which is used to generate the extrinsic LLR as $L_{E1}^{n,k} = L_{E1}(c_{n,k}) = L_{D1}(c_{n,k}|\mathbf{y}) - L_{A1}^{(n,k)}$. The extrinsic LLR, $L_{E1}^{(n,k)}$, at the output of the SISO-MIMO detector is then deinterleaved as $L_{A2}^{(n,k)} = \Pi^{-1}(L_{E1}^{(n,k)})$, which is used as the *a priori* input to the MAP decoder. The extrinsic information at the output of the MAP decoder, $L_{E2}^{(n,k)}$, is interleaved into $L_{A1}^{(n,k)} = \Pi(L_{E2}^{(n,k)})$, which is used as the *a priori* input to the SISO-MIMO detector at the next iteration. In the first iteration, $L_{A1}(c_{n,k}) = 0$ because there is no *a priori* information. It is assumed that the receiver knows the channel matrix $\mathbf{H}$ exactly.

III. A New FSD-based SISO-MIMO Detector

In this section, a new FSD-based SISO-MIMO detector is proposed for UD-MIMO systems. FSD is a simplified version of the SD algorithm [1]. The conventional FSD is designed for symmetric or over-determined MIMO systems [2]. We will develop a modified FSD algorithm based on the structures of UD-MIMO systems.

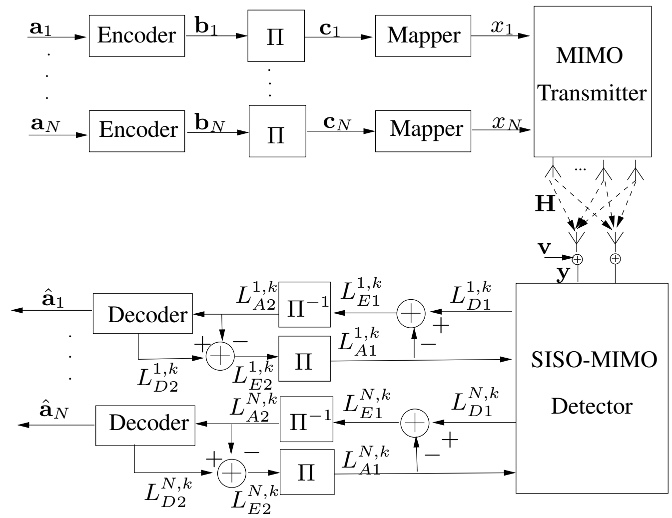

Fig. 1. Turbo-MIMO transceiver block diagram.

A. The Modified FSD Algorithm for UD-MIMO Systems

Similar to the conventional SD and FSD algorithms, the modified FSD algorithm performs detection by searching a subset of a tree structure. The tree has N layers, and each layer represents a transmit antenna. Denote the root layer as layer N, and the leaf layer as layer 1. Each node on the tree has Q branches leading to Q child nodes, with each branch representing a possible symbol from the constellation set $\mathcal{Q}$. A path from a leaf node on layer 1 leading up to the root node represents a possible transmitted vector $\mathbf{x} \in \mathcal{Q}^{N\times 1}$. The full tree structure has Q^N leaf nodes, therefore there are Q^N paths from the leaf nodes to the root node. The ML detection will exhaustively search all the Q^N paths on the tree. The SD-based algorithms, on the other hand, will search a small subset of the paths in the tree structure around the received signal vector. Details of the modified FSD tailored for UD-MIMO systems are given as follows.

The modified FSD consists of three steps: channel ordering, tree search, and LLR calculation. In channel ordering for the conventional FSD designed for over-determined or symmetric MIMO systems, the columns of the channel matrix $\mathbf{H}$ are permutated based on a certain order as $\mathbf{H}_p = [\mathbf{h}_{p_1}, \cdots, \mathbf{h}_{p_N}]$ such that transmit antenna p_k corresponds to the k-th layer. However, for UD-MIMO systems, the channel matrix $\mathbf{H}$ is rank-deficient thus the channel ordering scheme is no longer applicable to UD-MIMO systems. A new channel ordering scheme is proposed in this paper and detailed information will be presented in the next subsection.

In tree search, the permuted channel matrix $\mathbf{H}_p$ is used to generate the zero-forcing estimate of the symbol vector as

$$\hat{\mathbf{x}} = \mathbf{H}_p^{\dagger}\mathbf{y} = [\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_N]^T. \tag{3}$$

where $\mathbf{H}_p^{\dagger} = (\mathbf{H}_p^H\mathbf{H}_p)^{-1}\mathbf{H}_p^H$ is the pseudo-inverse of $\mathbf{H}_p$, with $[\cdot]^H$ denoting matrix Hermitian.

The zero-forcing estimate $\hat{\mathbf{x}}$ is used as a starting point of the tree search, which attempts to compute the following metric

$$\Delta = \|\mathbf{U}(\mathbf{x} - \hat{\mathbf{x}})\|^2 = \sum_{i=1}^{N} u_{ii}^2 |x_i - z_i|^2, \quad (4)$$

where $\mathbf{x} = [x_1, \cdots, x_N]^T$ is one of the Q^N possible paths from the root to a leaf, with x_i being the symbol on the i-th layer, and

$$z_i = \hat{x}_i - \sum_{j=i+1}^{N} \frac{u_{ij}}{u_{ii}} (x_j - \hat{x}_j). \quad (5)$$

The matrix $\mathbf{U} = \{u_{ij}\} \in \mathcal{C}^{N \times N}$ is an upper-triangular matrix calculated through the Cholesky decomposition of a diagonal-loaded Gram matrix $\bar{\mathbf{G}} = \mathbf{H}_p^H \mathbf{H}_p + \beta \mathbf{I}_N$ as follows

$$\bar{\mathbf{G}} = \mathbf{H}_p^H \mathbf{H}_p + \beta \mathbf{I}_N = \mathbf{U}^H \mathbf{U}, \quad (6)$$

where β is a small positive number, and $\mathbf{I}_N$ is a size-N identity matrix. The original Gram matrix, $\mathbf{G} = \mathbf{H}_p^H \mathbf{H}_p$, is rank-deficient in a UD-MIMO system. Therefore, $\mathbf{G}$ is not positive definite and the Cholesky decomposition does not exist. Adding a small positive number β to the diagonal of $\mathbf{G}$ generates a positive definite approximation of $\mathbf{G}$, and this makes the Cholesky decomposition in (6) possible. The effect of β can be considered as adding some noise to the system, and the performance loss due to the extra noise is negligible if β is small enough, *e.g.*, $\beta = 10^{-6}$.

Instead of exhaustively calculating the metrics for all the Q^N paths, the FSD only calculates the metrics of a subset of pathes by searching over the tree layer-by-layer. The search starts from the N-th layer at the root of the tree, and it follows a breadth-first approach, *i.e.*, all the metrics at the same layer are calculated and compared before moving on to the next layer. A large number of branches are pruned during the search and only a subset of branches or pathes survive before moving on to the next layer.

Consider a parent node at the i-th layer and on the k-th survival path, the distance metric of the branch from this parent node to one if its Q child nodes is

$$d_{ik}(\chi_q) = u_{ii}^2 |\chi_q - z_{ik}|^2, \chi_q \in \mathcal{S} \quad (7)$$

where $z_{ik} = \hat{x}_i - \sum_{j=i+1}^{N} \frac{u_{ij}}{u_{ii}} (x_{jk} - \hat{x}_j)$, with x_{jk} being the symbol at the j-th layer and on the k-th survival path. The ML detection will keep all the Q paths originating from the same parent node, and use them as the parent nodes for the next layer. The FSD, on the other hand, orders $\{d_{ik}(\chi_q)\}_{q=1}^{Q}$ in an ascending order and only keep the first $n_i \leq Q$ paths as the survival paths. Therefore, the number of survival paths of FSD at the i-th layer is $(n_i \times n_{i+1} \cdots \times n_N)$.

The vector $\mathbf{n}_S = [n_1, \cdots, n_N]^T$ is called node distribution. After searching the entire tree, the total number of survival paths is $K = \prod_{i=1}^{N} n_i$, and the accumulated metric for the k-th path is computed as $\Delta_k = \sum_{i=1}^{N} d_{ik}(x_{ik})$, where x_{ik} denotes the survival symbol of the k-th path at the i-th layer. Among the K survival paths, only ν paths with the smallest metrics Δ_k are selected as the final survival paths for LLR calculation. The choice of the node distribution and the number of final survival paths ν affect the performance and complexity tradeoff of the FSD algorithm. If $n_1 = \cdots n_N = Q$ and $\nu = Q^N$, then the FSD degrades to the regular ML detection. The node distribution and the number of final survival paths can be chosen such that $K << Q^N$ and $\nu < K$ to significantly reduce the complexity of metric Δ_k and LLR calculation, yet still maintain a performance that is very close to that of the optimum ML detection.

In a UD-MIMO system, the values of u_{ii}, for $i = M + 1, \cdots, N$, are usually very small due to the rank deficient structure of $\mathbf{G}$. As a result, the SNR on layers $M+1$ to N are very low. Due to the low SNR, it is very likely that the global optimum path might have unfavorable local metrics at these layers, and is discarded if one or more of these layers use a small n_i. This will degrade the overall system performance even if all the subsequent layers use larger n_i. To address this problem, we propose to preserve the full structure of the tree from layers $M+1$ to N, *i.e.*, $n_i = Q$ for $i = M+1, \cdots, N$. This corresponds to exhaustive tree search at layers $M+1$ to N to ensure that the optimum path is preserved in these layers. The complexity reduction is achieved at layers 1 to M, where $\{n_i\}_{i=1}^{M}$ can choose values less than Q to reduce the complexity.

Therefore, we divide the tree structure of a UD-MIMO system into two parts. The top part, from layers $M+1$ to N (root), utilizes an exhaustive tree search. The bottom part, from layers 1 (leaf) to M, employs a constrained tree search by choosing $n_i < Q$ at these layers. It will be shown by simulations that such a two-part tree search algorithm can achieve a performance that is very close to its optimum counterpart, but with a much lower complexity.

At the end of the tree search, the accumulated metrics of the K survival paths, $\{\Delta_k\}_{k=1}^{K}$, are ordered from low to high as $\Delta_{o_1} \leq \cdots \Delta_{o_k}$, where $[o_1, \cdots, o_k]$ is a permutation of $[1, \cdots, K]$. Define a set that contains the survival paths with the smallest metrics as $\mathcal{P} = \{\mathbf{p}_{o_k}\}_{k=1}^{\nu}$, where $\mathbf{p}_{o_k} = [x_{1o_k}, \cdots, x_{No_k}]^T$ is the symbol vector of the o_k-th path. The set $\mathcal{P}$ is the output of the tree search algorithm.

Once the set $\mathcal{P}$ is obtained, the LLR at the output of the modified FSD is calculated as follows

$$L_{E1}(c_{n,k}|\mathbf{y}) = \ln \frac{\sum_{\mathbf{x} \in \mathcal{P}_{k,0}} \exp(\frac{-||\mathbf{y} - \mathbf{H}\mathbf{x}||^2}{\sigma^2/2})}{\sum_{\mathbf{x} \in \mathcal{P}_{k,1}} \exp(\frac{-||\mathbf{y} - \mathbf{H}\mathbf{x}||^2}{\sigma^2/2})} \quad (8)$$

where $L_{E1}(c_{n,k}|\mathbf{y})$ is the soft extrinsic information of the k-th bit transmitted by the n-th transmit antenna, and the sets $\mathcal{P}_{k,b}$ is a subset of $\mathcal{P}$, such that it contains all the vectors with $c_{n,k} = b$, for $b = 0, 1$. The complexity of (8) can be reduced significantly with the max-log-map approximation [11].

B. A New Channel Ordering Scheme for the Modified FSD

In this subsection, we proposed a new channel ordering scheme designed specifically for the modified FSD in a UD-MIMO system. Compared to the conventional ordering scheme proposed in [3], the new scheme has a lower complexity but a better performance.

In the modified FSD algorithm for a UD-MIMO system, the layers are divided into two groups, where exhaustive search is performed in the first group that contains layers $M+1$ to N, and low complexity constrained search is performed in the second group that contains layers 1 to M. The motivation behind such a partition is that layers $M+1$ to N have relatively low SNRs. Therefore, one important objective of the channel ordering is the partition of the layers based on their respective reliability, such that layers $M+1$ to N have the lowest reliability. In the first iteration, *a priori* information is not available, so the ordering is based on the channel condition. Since the Cholesky decomposition of the diagonal-loaded Gram matrix is used during the tree search, we propose to measure the channel reliability by using the norms of the columns of the Gram matrix $\mathbf{G} = \mathbf{H}^H\mathbf{H}$.

The new channel ordering is as follows:

1) Calculate the gram matrix as $\mathbf{G} = \mathbf{H}^H\mathbf{H}$.
2) Calculate the norms of the columns of $\mathbf{G}$: $\|\mathbf{g}_i\|, i = 1, \cdots, N$, where $\mathbf{g}_i$ is the i-th column of $\mathbf{G}$.
3) Sort the norms $\{\|\mathbf{g}_i\|\}_{i=1}^N$ in a descending order, such that $\mathbf{g}_{p_1} \geq \cdots \geq \mathbf{g}_{p_N}$.
4) Obtain the ordered channel matrix $\mathbf{H}_p = [\mathbf{h}_{p_1}, \cdots, \mathbf{h}_{p_N}]$.

It is noted that the relative orders among layers $M+1$ to N have no effect on the performance because exhaustive tree search is performed on each of these $N-M$ layers.

C. Second Iteration and Beyond: GSIC-BDFE

The proposed modified FSD is used in the first iteration. From the second iteration and beyond, the GSIC-BDFE scheme [9] is employed to further reduce the complexity.

The columns of the channel matrix $\mathbf{H}$ is first permutated based on the symbol reliability measured from the *a priori* input, then the permuted channel matrix $\mathbf{H}_p$ is partitioned into $D = \lceil \frac{N}{M} \rceil$ symmetric or over-determined sub-systems as follows:

$$\mathbf{y} = \sum_{d=1}^{D} \mathbf{H}_d \mathbf{x}_d + \mathbf{v} \tag{9}$$

where $\mathbf{H}_d = [\mathbf{h}_{d_1}, \cdots, \mathbf{h}_{d_M}] \in \mathcal{C}^{M\times M}$, if $1 \leq d \leq D-1$, and $\mathbf{H}_D = [\mathbf{h}_{D_1}, \cdots, \mathbf{h}_{D'_M}] \in \mathcal{C}^{M\times M'}$, with $d_k = p_{(d-1)M+k}$ and $M' = N-(D-1)M$.

The D sub-systems are equalized serially with sub-system D being equalized first and sub-system 1 equalized last. The SISO BDFE of sub-system d is performed over the following equivalent system

$$\mathbf{y}^{(d)} = \mathbf{y} - \sum_{j=1}^{d-1} \mathbf{H}_j \bar{\mathbf{x}}_j - \sum_{j=d+1}^{D} \mathbf{H}_j \hat{\mathbf{x}}_j \tag{10}$$

where $\{\bar{\mathbf{x}}_j\}_{j=1}^{d-1}$ is the *a priori* mean calculated from the *a priori* information from the previous iteration, and $\{\hat{\mathbf{x}}_j\}_{j=d+1}^{D}$ is the *a posteriori* soft decisions of the sub-systems that have been processed in the current iteration.

Since the GSIC-BDFE is employed after the first iteration, the symbol reliability is measured by using an *a priori* reliability metric $\alpha_n = \frac{1}{\sigma_{x_n}^2}$, where $\sigma_{x_n}^2$ is the *a priori* variance of the symbol x_n at the GSIC-BDFE input. A larger α_n means a higher reliability. In [9], the columns of the channel matrix are ordered in an ascending order based on the reliability factor α_n. This ordering scheme is proposed for low level modulations such as binary phase shift keying (BPSK), but it does not work well for high level modulations.

We propose to reverse the detection order in GSIC-BDFE with high level modulations, *i.e.*, order the reliability measure from high to low, such that the sub-system containing the columns with the lowest reliability measures are detected first. The *a priori* information generated from the previous iteration contributes to the detection in the d-th sub-system from two aspects: 1) the *a priori* information of symbols in the d-th sub-system is used to formulate the BDFE filters; 2) the *a priori* information of symbols in sub-systems 1 to $(d-1)$ is used for soft interference cancelation as shown in (10). For high level modulations, intuitively the reliability of SIC is more critical because a less reliable serial interference cancelation (SIC) will lead to large residual interferences due to a larger constellation size. Ordering the reliability measure from high to low means the soft decisions in sub-systems 1 to $d-1$ have higher reliability than those in sub-system d. Such an ordering scheme will maximize the reliability of SIC, thus improve the reliability of the d-th sub-system's *a posteriori* soft decisions, which are also used for the SIC for sub-systems 1 to $d-1$. The above discussions are verified through the simulation results.

IV. Complexity Analysis

In this section, the complexity of the proposed FSD-GSIC-BDFE scheme is compared to that of the conventional GPIC-GSIC-BDFE in [9]. The analysis is based on the number of complex multiplications. Since both schemes use similar operations after the first iteration, the complexity comparison in the first iteration is studied.

The computational complexity of the proposed scheme is mainly contributed by two sources, the channel ordering and the tree search. The LLR calcuation has a much lower complexity because the number of survival paths at the end of the tree search is usually small, and it can employ the max-log-map simplification to further reduce the complexity.

The conventional channel ordering scheme proposed in [3] requires the inversion of N size $N\times N$ matrices. This results in a complexity in the order of $\mathcal{O}(N^4)$. The new ordering scheme proposed in this paper requires the multiplication between a $N \times M$ matrix and a $M \times N$ matrix, and N complex multiplications for the norm calculation. The total number of complex multiplications due to ordering is thus $MN^2 + N^2$.

Before the tree search, we calculate the zero-forcing estimate as in (4), and perform the Cholesky decomposition to get $\mathbf{U}$ as in (6). These two operations require $(\frac{1}{3}N^3 + 2MN^2 + 2N^2 + MN)$ complex multiplications.

The tree search calculates the metrics of survival paths as in (7) along a constrained tree structure. The number of complex multiplications incurred by this operation is

$$\sum_{l=1}^{N}(\prod_{m=l}^{N} n_m + (N-l)\prod_{m=l+1}^{N} n_m) \tag{11}$$

where the term $\prod_{m=l}^{N} n_m$ corresponds to the calculation in (7), and the term $(N-l)\prod_{m=l+1}^{N} n_m$ corresponds to the calculation of z_{ik}. The number of multiplications in (11) is a little different from the calculation in [2] because we only consider the number of complex multiplications instead of real multiplications.

The complexity of the conventional GPIC-BDFE scheme are contributed by three parts: the permutation of channel matrix $\mathbf{H}$, the formulation of the BDFE filters [9, eqn. (6)], and the BDFE filtering operation [9, eqn. (7)]). The ordering and permutations of the column of $\mathbf{H}$ rely on the calculation of the norms of the rows of the pseudo-inverse of $\mathbf{H}$, and the total number of complex multiplications incurred by these operations is $\frac{1}{3}N^3 + MN^2 + 2N^2 + MN$. The formulation of the BDFE filters require a Cholesky decomposition of a $M \times M$ matrix, and several matrix multiplications. It incurs $\frac{8}{3}M^3 + 5M^2$ complex multiplications. The BDFE filtering in one sub-system requires M^2 complex multiplications, and the filtering process needs to be performed for $J = Q^{N-M}$ sub-systems.

The complexity of the proposed algorithm is higher than that of the conventional GPIC-BDFE. For example, when $M = 2, N = 6$ for QPSK modulation, and $n_{\mathbf{s}} = [1\ 2\ 4\ 4\ 4\ 4]$ is used as node distribution, the number of complex multiplications for the proposed scheme is 6954 while the number of complex mulitplications for the GPIC-BDFE scheme is 1848. It will be demonstrated by simulation that the proposed scheme can achieve significant performance gains over the original GPIC-BDFE at the cost of complexity.

V. Simulation Results

Simulations are performed to verify the performance of the newly proposed FSD-GSIC-BDFE scheme in a UD-MIMO system with $N = 6$ transmit antennas and $M = 2$ receive antennas. The information bits are encoded with a rate $1/2$ systematic convolutional code with the generator polynomial $G = [7, 5]_8$. The block length is 1024. The flat-Rayleigh-fading MIMO channel model is employed.

The bit-error rate (BER) results for quadrature-phase shift keying (QPSK) and eight-phase shift keying (8PSK) are shown in Figs. 2 and 3, respectively. The node distributions for the QPSK and 8PSK modulated systems are $[1\ 3\ 4\ 4\ 4\ 4]$ and $[1\ 4\ 8\ 8\ 8\ 8]$, respectively. The number of survival paths after the tree search is $\nu = 16$ for both configurations.

The proposed FSD-GPIC-BDFE scheme achieves significant performance gains over the conventional GPIC-GSIC-BDFE scheme, for both QPSK and 8PSK systems. If we consider the results from the 5-th iteration, the proposed scheme outperforms the GPIC-GSIC-BDFE by 2.2 dB at the BER = 10^{-3} for the QPSK system, and by 4.0 dB at the BER = 10^{-3} for the 8PSK system. In addition, the results

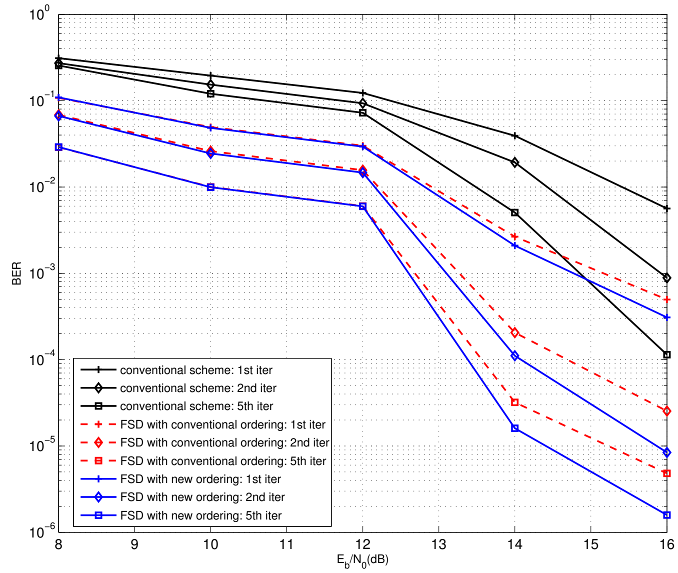

Fig. 2. Simulation results of QPSK encoded system with $N = 6$ and $M = 2$.

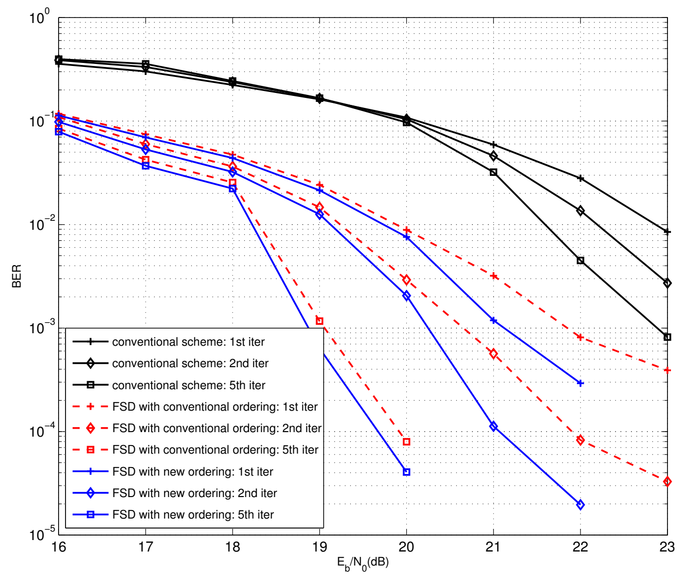

Fig. 3. Simulation results of 8PSK encoded system with $N = 6$ and $M = 2$.

of the FSD-GSIC-BDFE scheme employing the conventional ordering scheme in [3] are also shown in the figures. The new ordering scheme consistently outperforms the conventional ordering scheme in all system configurations. For the QPSK system, the performance gain of the new ordering system is about 0.1 dB; the performance gain increases to about 0.2 dB for the 8PSK system. It should be noted that the newly proposed ordering scheme has a much lower complexity compared to the original ordering scheme. It can be concluded from these results that the proposed FSD-GSIC-BDFE scheme can achieve higher performance gains for systems with high level modulations.

VI. CONCLUSIONS

A new FSD-based method was proposed for the turbo detection of coded UD-MIMO systems. The modified FSD was used in the first iteration, and a GSIC-BDFE method was adopted in the subsequent iterations. The modified FSD partitioned the N layers of a tree structure into two groups, and exhaustive search was used for the first group of $N - M$ layers, and low complexity constrained search was used for the second group of M layers. The layer partition and ordering are performed with a new ordering scheme that has less complexity but better performance than the ordering scheme in the conventional FSD. Complexity analysis and simulation results demonstrated that the proposed scheme achieves significant performance gains over existing schemes, at the cost of increase in complexity. The performance improvement is more pronounced for systems with higher modulation levels.

VII. ACKNOWLEDGMENT

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REFERENCES

[1] U. Fincke and M. Pohst, "Improved methods for calculating vectors of short length in a lattice, including a complexity analysis," *Math. Comput.*, vol. 44, pp. 463C471, Apr. 1985.

[2] L. G. Barbero and J. S. Thompson, "Extending a Fixed-Complexity Sphere Decoder to Obtain Likelihood Information for Turbo-MIMO Systems," *IEEE Trans. Veh. Technol*, vol. 57, no. 5, pp. 2804-2814, Sept. 2008.

[3] L. G. Barbero and J. S. Thompson,"Fixing the Complexity of the Sphere Decoder for MIMO Detection," *IEEE Trans. Wireless Commun.*, vol. 7, no. 6, pp. 2131 - 2142, Jun. 2008.

[4] G. J. Foschini, "Layered Space-time Architecture for Wireless Communication in a Fading Environment When Using Multiple Antennas," *Bell Lab. Tech. J.*, vol. 1, pp. 41-59, 1996.

[5] D. Wubben, R. Bohnke, V. Kuhn, and K. D. Kammeyer, "MMSE Extension of V-BLAST Based on Sorted QR Decomposition," in *Proc. IEEE Vehicular Technol. Conf.*, 2003, vol. 1, pp. 508-512.

[6] Z. Luo, S. Liu, M. Zhao, and Y. Liu, "Generalized parallel interference cancellation algorithm for V-BLAST systems," *Proc. IEEE Int. Conf. Commun.*, 2006, pp. 3207-3212.

[7] M. Damen, K. Abed-Meraim, and J. C. Belfiore, " Generalised Sphere Decoding for Asymmetrical Space-time Communication Architecture," *Electronics Lett.*, vol. 36, pp. 166-167, Jan. 2000.

[8] Tao Cui and Chintha Tellambura, "An Efficient Generalized Sphere Decoder for Rank-Deficient MIMO Systems," *IEEE Commun. Lett.*, vol. 9, no. 5, pp. 423-425, May. 2005.

[9] Michael L. Walker, Jun Tao, Jingxian Wu, and Yahong Rosa Zheng, "Low complexity turbo detection of coded under-determined MIMO systems," in *Proc. IEEE Int. Conf. Commun. 2011*, Jun. 2011.

[10] J. Wu and Y. R. Zheng, " Low Complexity Soft-input Soft-output Block Decision Feedback Equalization," *IEEE J. Select. Areas. Commun.*, vol. 26, no. 2, pp. 281-289, Feb. 2008.

[11] P. Robertson, E. Villebrun, and P. Hoeher, "A Comparison of Optimal and Sub-optimal MAP Decoding Algorithms Operating in the Log Domain," in *Proc. IEEE Int. Conf. Commun. 1995*, pp. 1009-1013, 1995.