Cross-Layer Design of Energy Efficient Coded ARQ Systems

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Abstract—**The energy efficient design of coded automatic-repeat-request (ARQ) systems is studied in this paper. The optimization aims to minimize the energy required for the successfully delivery of one information bit from a transmitter to a receiver. The design is performed by incorporating a wide range of practical system parameters and metrics, such as hardware power consumption, modulation, channel coding, and frame error rate (FER) in the physical layer, and frame length and protocol overhead in the media access control layer. A new log-domain threshold approximation method is proposed to analytically quantify the impacts of the various system parameters on the FER, and the results are used to facilitate the system design. The optimum transmission energy and frame length that minimize the energy per information bit are identified in closed-form expressions as functions of the various practical system parameters. The analytical and simulation results demonstrate that the total energy consumption in a coded ARQ system can be *reduced* by *increasing* the transmission energy during one transmission attempt, and significant energy saving as high as 9.5 dB is achieved with the optimum system.**

I. Introduction

Energy efficient communication can extend the battery life of communication terminals, reduce the energy cost, and make the communication process more environmental friendly.

A large number of energy efficient communication techniques have been developed in the physical (PHY) layer [1] and [2] and the media access control (MAC) layer [3]–[7]. Most PHY layer energy efficient communication techniques are developed by exploiting the trade-off between power efficiency and spectral efficiency through various coding, modulation, and signal processing techniques [1] and [2]. In the MAC layer, the energy consumption can be reduced in a number of ways, such as decreasing the transmission duty cycle [3] and [4], carefully scheduling the transmissions to reduce or avoid collisions [5] and [6], or power controls [7], etc.

Most schemes are developed by following the traditional layered-protocol design approach, and they do not directly take advantage of the interactions among the protocol layers that might be critical to energy efficient communications [8]. A cross-layer power-rate-distortion framework is proposed in [9] by considering the trade-off among source distortion, data rate, and hardware complexity, but with an assumption of error-free channel. A PHY/MAC cross-layer design is considered in [10], where the optimum power assignment for the hybrid automatic-repeat-request (H-ARQ) technique in fading channel is studied to reduce the total average power consumption. The optimization in [10] is performed under the constraint of a targeted outage probability, and it does not consider the effects of practical system parameters such as overhead, modulation, data rate, and bit error rate (BER), etc.

In this paper, we propose a new optimum design of practical ARQ systems to minimize the energy required to successfully deliver an information bit from a transmitter to a receiver through a Rayleigh fading channel. The optimization incorporates a large number of practical system parameters that cover the operations in the hardware, the PHY layer, and the MAC layer, such as the efficiency of the power amplifier, the power consumption of digital hardware, data rate, modulation, frame length, frame error rate (FER), and the protocol overhead, etc.

The system design is performed by jointly optimizing the transmission energy in the PHY layer and the frame length in the MAC layer. For a system employing ARQ, a lower transmission energy does not necessarily mean less total energy consumption, because it might increase the number of retransmissions, thus the total energy required to *successfully* deliver a frame. On the other hand, increasing the transmission energy beyond its optimum operation point will result in a waste of the energy resource. Similarly, a longer frame usually means a higher FER, yet a shorter frame has poor overhead efficiency. To quantify the impacts of transmission energy and frame length, a new log-domain threshold approximation is proposed to build an explicit analytical relationship between the FER and the design parameters. The optimum transmission energy and frame length are expressed as closed-form expressions of the various practical system parameters. The analytical and simulation results demonstrate that significant energy savings are achieved through the optimization.

II. System Model

Consider a transmitter and a receiver separately by a distance d. The information bits at the transmitter are divided into frames. Each frame has L uncoded information bits and L_0 overhead bits. The information bits and overhead bits from the

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transmitter are encoded with a channel encoder with code rate r. For a system employing M-QAM, the number of symbols in each frame is $L_s = \frac{L+L_0}{r \log_2 M}$, where L is chosen in a way such that L_s is an integer.

The m-th symbol observed at the receiver is

$$y_m = \sqrt{E_r} h_m x_m + z_m, \text{ for } m = 1, 2, \cdots, L_s, \quad (1)$$

where E_r is the average energy of a symbol at the receiver, $x_m \in \mathcal{S}$ is the m-th modulated symbol transmitted, $\mathcal{S}$ is the modulation constellation set with the cardinality $M = |\mathcal{S}|$, y_m, h_m, and z_m are the received sample, the fading coefficient between the transmitter and the receiver, and additive white Gaussian noise (AWGN) with single-sided power spectral density N_0, respectively. It is assumed that the system undergoes quasi-static Rayleigh fading, such that the fading coefficient is constant within one frame, and changes from frame to frame.

Define the average E_b/N_0 of an uncoded information bit at the receiver as

$$\gamma_b \triangleq \frac{E_b}{N_0} = \frac{E_r}{r N_0 \log_2 M}. \quad (2)$$

For a transmitter and receiver pair separated by a distance d, the average transmission energy for each symbol at the transmitter can be modelled as [1]

$$E_s = E_r G_1 d^\kappa M_l, \quad (3)$$

where κ is the path-loss exponent, G_1 is the gain factor (including path-loss and antenna gain) at a unit distance, and M_l is the link margin compensating the hardware process variations and other additive background noise or interference.

In addition to the actual transmission energy, we also need to consider the circuit energy per symbol that can be modelled as [1],

$$E_c = \left(\frac{\xi_{_M}}{\eta} - 1\right) E_s + \frac{\beta}{R_s}, \quad (4)$$

where $R_s = \frac{1}{T_s}$ is the gross symbol rate, η is the drain efficiency of the power amplifier, $\xi_{_M}$ is the peak-to-average power ratio (PAPR) of an M-ary modulation signal, β incorporates the effects of baseband processing, such as signal processing, encoding and modulation. For M-ary quadrature amplitude modulated (MQAM) systems with square constellations, $\xi_{_M} \simeq 3(\sqrt{M} - \frac{1}{\sqrt{M}} + 1)$ for $M \geq 4$ [11].

From (2), (3), and (4), the energy required to transmit one information bit during one transmission attempt is

$$E_0 = \frac{L_s}{L}(E_s + E_c) = \frac{L + L_0}{L} \frac{\gamma_b \xi_{_M} N_0 G_d}{\eta} + \frac{\beta}{R_b}, \quad (5)$$

where $G_d = G_1 d^\kappa M_l$, and $R_b = \frac{L}{L_s} R_s$ is the net bit rate of the uncoded information bit.

Due to the effects of channel fading and noise, the receiver might not be able to successfully recover the transmitted signal. The probability that a transmitted frame cannot be recovered equals to FER, which is a function of the γ_b at the receiver, the frame length L_s, the modulation level M, and the channel code. The packet will be retransmitted if the transmitter receive a negative acknowledgement (NACK). Since the retransmissions are independent, the number of retransmissions is a geometric random variable with the parameter FER. The average number of retransmissions is thus

$$\Lambda = \frac{1}{1 - \text{FER}}. \quad (6)$$

The total energy required to successfully deliver an information bit from the transmitter to the receiver can then be calculated by $E_t = \Lambda E_0$, which can be expanded by combining (5) and (6) as

$$E_t = \frac{1}{1 - \text{FER}} \left[\frac{L + L_0}{L} \frac{\gamma_b \xi_{_M} N_0 G_d}{\eta} + \frac{\beta}{R_b}\right]. \quad (7)$$

The total energy per information bit E_t relies on a number of system parameters, including E_b/N_0 at the receiver γ_b, the number of information bits L and the number of overhead bits L_0 per frame, the modulation level M, the net data rate R_b, and the FER that inherently depends on all the above parameters and the code rate r, etc.

The value of γ_b has two opposite effects on E_t. On one hand, FER is a decreasing function in γ_b. Therefore, increasing γ_b will decrease the average number of retransmissions Λ, thus reduce E_t. On the other hand, E_0 is a strictly increasing function in γ_b, thus it translates a positive relationship between γ_b and E_t.

A similar observation can also be obtained for the relationship between E_t and L. Λ translates a positive relationship between E_t and L because FER is an increasing function in L for a given channel code and modulation scheme, whereas E_0 is a decreasing function in L.

Therefore, it is critical to identify the optimum values of γ_b and L that can achieve minimal energy per information bit.

III. Optimum System Design

The optimum system design that can minimize E_t under the constraints of fixed M, R_b and L_0 are studied in this section.

A. FER with a Log-Domain Linear Threshold Approximation

In this subsection, an accurate approximation of the FER of coded systems in quasi-static Rayleigh fading is obtained with the threshold-based method originally presented in [12]. Furthermore, we propose a new log-domain linear approximation method for the calculation of the threshold value required for the FER approximation. The threshold-based method with the newly proposed log-domain linear approximation explicitly build a connection between the FER and the various system parameters.

With the threshold-based method [12], the FER of a coded system in a quasi-static Rayleigh fading channel can be accurately approximated by

$$\text{FER} \simeq 1 - \exp\left(-\frac{\gamma_\omega}{\gamma_b}\right), \quad (8)$$

where γ_ω is a threshold value that can be calculated as

$$\gamma_\omega = \left[\int_0^\infty \frac{1 - \text{FER}_G(\gamma)}{\gamma^2} d\gamma\right]^{-1}, \quad (9)$$

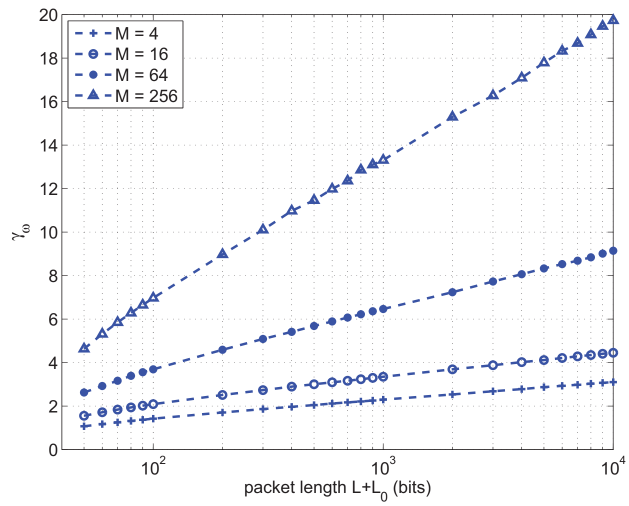

Fig. 1. γ_ω as a function of $L + L_0$

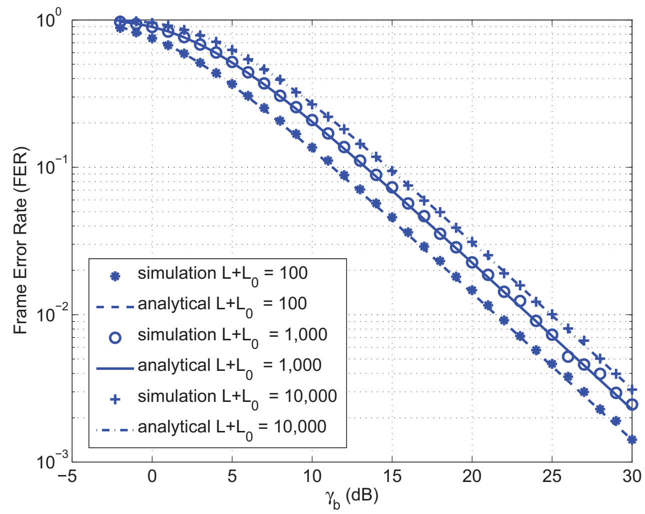

Fig. 2. Comparison of the simulation FER with the analytical approximation in (11).

where $\text{FER}_G(\gamma)$ is the FER in an AWGN channel.

Fig. 1 shows γ_ω as a function of $L + L_0$ under various modulation schemes. The channel code is a rate $r = \frac{1}{2}$ convolutional code with the generator polynomial $[5, 7]_8$ and constraint length 3. It is observed from the figure that γ_ω can be modelled as a linear function of $\log(L+L_0)$, with the slope and intercept determined by the different modulation schemes. Similar linear relationships are also observed for other channel codes. Therefore, we propose to model γ_ω as

$$\gamma_\omega \simeq k_{_M} \log(L + L_0) + b_{_M}, \tag{10}$$

where $k_{_M}$ and $b_{_M}$ are the slope and intercept determined by the modulation scheme and the actual channel code. The value of $k_{_M}$ and $b_{_M}$ can be estimated by performing the least squares (LS) method on the results in Fig. 1. For the $M = 4$, we have $k_4 = 0.3744$ and $b_4 = -0.31$.

Combining (8) and (10) leads to a new FER approximation

$$\text{FER} \simeq 1 - (L + L_0)^{-\frac{k_M}{\gamma_b}} \exp\left(-\frac{b_{_M}}{\gamma_b}\right). \tag{11}$$

Fig. 2 compares the actual FER obtained through simulation with the corresponding analytical approximation by using (11), under different values of $L + L_0$, for systems with $M = 4$. The convolutional code is the same as the one used in Fig. 1. Excellent agreements are observed between the actual simulation results and their analytical approximations. Therefore, the analytical expressions in (8) and (10) give a very accurate approximation of the actual FER.

B. Optimum γ_b

The optimum value of γ_b at the receiver that minimizes E_t is studied in this subsection.

Before proceeding to the actual optimization, we present the following theorem about convexity, which will be used in identifying the optimum system parameters.

Theorem 1: Consider a decreasing function $f(x)$ and an increasing function $g(x)$. If both $f(x)$ and $g(x)$ are convex, then $f(x)g(x)$ is convex.

Proof: Consider $0 < x_1 < x_2$ and $\alpha \in [0, 1]$. Define $\theta_1 = \alpha f(x_1)g(x_1) + (1-\alpha)f(x_2)g(x_2)$, and $\theta_2 = f(\alpha x_1 + (1-\alpha)x_2)g(\alpha x_1 + (1-\alpha)x_2)$. Since $f(x)$ and $g(x)$ are convex, we have $\theta_2 \geq \theta_3$ with θ_3 defined as

$$\theta_3 = [\alpha f(x_1) + (1-\alpha)f(x_2)][\alpha g(x_1) + (1-\alpha)g(x_2)] \tag{12}$$

Since $\theta_1 = \theta_1(1 - \alpha + \alpha)$, the term θ_1 can be alternatively represented as

$$\begin{aligned} \theta_1 \quad = \quad & \alpha^2 f(x_1)g(x_1) + (1-\alpha)^2 f(x_2)g(x_2) + \\ & \alpha(1-\alpha)\left[f(x_1)g(x_1) + f(x_2)g(x_2)\right] \end{aligned} \tag{13}$$

From (12) and (13), we have

$$\frac{\theta_3 - \theta_1}{\alpha(1-\alpha)} = [f(x_1) - f(x_2)][g(x_2) - g(x_1)] \geq 0. \tag{14}$$

Therefore $\theta_2 \geq \theta_3 \geq \theta_1$, and this completes the proof. ■

We can prove that Λ in (6) is a decreasing function in γ_b, and it is convex in γ_b by showing that $\frac{\partial^2 \Lambda}{\partial \gamma_b^2} \geq 0$, and details are omitted here for brevity. It is straightforward to show that E_0 in (5) is an increasing and convex function in γ_b. Therefore, based on the results in Theorem 1, we have the following corollary about the convexity of $E_t = \Lambda E_0$.

Corollary 1: For the FER given in (11), the total energy per information bit, E_t, in (7) is convex in γ_b. ■

Once we establish the convexity of E_t in γ_b, the optimum γ_b can be solved as stated in the following corollary.

Corollary 2: In a quasi-static Rayleigh fading channel, if the FER is given in (11), then the optimum γ_b that minimizes E_t is

$$\hat{\gamma}_b = \frac{1}{2}\left(\gamma_\omega + \sqrt{\gamma_\omega^2 + 4\gamma_\omega \frac{B}{A}\frac{L}{L+L_0}}\right) \tag{15}$$

where $A = \frac{\xi_M N_0 G_d}{\eta}$, and $B = \frac{\beta}{R_b}$.

Proof: Since E_t is convex in γ_b, the optimum γ_b that minimize E_t can be obtained by solving $\frac{\partial E_n}{\partial \gamma_b} = 0$, which yields

$$\gamma_b^2 - \gamma_\omega \gamma_b - \gamma_\omega \frac{B}{A} \frac{L}{L + L_0} = 0 \quad (16)$$

The result in (15) can be obtained by solving (16). ■

It should be mentioned here that the optimum γ_b is the average E_b/N_0 at the receiver. Correspondingly, the optimum energy per symbol required at the transmitter is

$$\hat{E}_s = \hat{\gamma}_b \times N_0 \times r \times \log_2 M \times G_d, \quad (17)$$

where $\hat{\gamma}_b$ is the optimum value calculated from (15).

C. Optimum L

The optimum number of information bits L that minimizes E_t is studied in this subsection.

Similar to the results in Corollary 2, the optimum solution of L relies on the convexity of E_t. However, the direct proof of the convexity of E_t with respect to L is quite tedious. To simplify analysis, we can show that E_t is convex in $\xi = \log(L + L_0)$.

We can prove that 1) Λ in (6) is an increasing and convex function in ξ; and 2) E_0 in (5) is a decreasing and convex function in ξ, and details are omitted here for brevity. Therefore, based on Theorem 1, we have the following corollary regarding the convexity of E_t with respect to ξ.

Corollary 3: For the FER given in (11), the total energy per information bit E_t in (7) is convex in $\xi = \log(L + L_0)$. ■

Based on the convexity of E_t in L, the optimum L is stated as follows.

Corollary 4: In a quasi-static Rayleigh fading channel, if the FER is given in (11), then the optimum L that minimize E_t satisfies the following equality

$$\hat{L} = \frac{\sqrt{A^2(k_M + \gamma_b)^2 + 4Ak_M B} - A(k_M - \gamma_b)}{2k_M(A\gamma_b + B)} \gamma_b L_0 \quad (18)$$

where $A = \frac{\xi_M N_0 G_d}{\eta}$, and $B = \frac{\beta}{R_b}$.

Proof: The optimum L is obtained by solving $\frac{\partial E_n}{\partial \xi} = 0$, which yields

$$k_M(A\gamma_b + B)L^2 + A\gamma_b L_0 (k_M - \gamma_b) L - A\gamma_b^2 L_0^2 = 0 \quad (19)$$

The result in (18) can be obtained by solving (19). ■

It is worth pointing out that even though the result in Corollary 4 is obtained through $\frac{\partial E_n}{\partial \xi} = 0$, it is exactly the same as solving $\frac{\partial E_n}{\partial L} = 0$ because $\frac{\partial \xi}{\partial L} = \frac{1}{L+L_0} \neq 0$.

D. Joint Optimum γ_b and L

In (15) and (18), the optimum value of γ_b is expressed as a function of L and vice versa. The global optimum operation point can be achieved by jointly optimizing γ_b and L.

Since E_t is convex in both γ_b and L, the joint optimum values can be obtained by treating (15) and (18) as a system of two equations with two variables in γ_b and L. The analytical results are very tedious and are omitted here for brevity.

Alternatively, the joint optimum values of γ_b and L can be efficiently calculated by iteratively invoking (15) and (18).

TABLE I
SIMULATION PARAMETERS

L_0	48 bits
Bit Rate	300 kbps
η	0.35
β	310.014 mw
$N_0/2$	-174 dBm/Hz
G_1	30 dB
κ	3.5
M_l	40 dB

Given an initial value L, we can calculate the optimum γ_b by using (15), the output of which is then used to update the value of L with (18). This procedure can be performed iteratively, and it will converge to the joint optimum value of γ_b and L that achieves the global minimum energy consumption.

IV. Numerical Results

Numerical results are presented in this section. The simulation parameters are summarized in Table 1.

Fig. 3 shows E_t as a function of γ_b, with various values of $L + L_0$. The distance is $d = 100$ m. The optimum values of $\hat{\gamma}_b$ for different L calculated from (15) are marked on the figure as the optimum operation points. It can be seen from the figure that E_t is a convex function in γ_b. The optimum operation points obtained from the analytical results match perfectly with the simulation results. If $\gamma_b < \hat{\gamma}_b$, the FER is so high such that the total energy consumption is dominated by the effect of the retransmissions. In this case, we can *reduce* the total energy consumption by *increasing* γ_b. For example, for $L + L_0 = 10,000$, increasing γ_b from -2 to 5 dB will result in an energy saving of 9.5 dB. When $\gamma_b > \hat{\gamma}_b$, E_t increases almost linearly with γ_b because the FER is low enough such that the effect of retransmission is negligible. The result demonstrates that a higher E_b/N_0 does not necessarily mean a better performance. Significant energy saving can be achieved with carefully choosing the operation point.

In Fig. 4, E_t is plotted as a function of $L + L_0$ under various values of γ_b. The distance is $d = 100$ m. The optimum

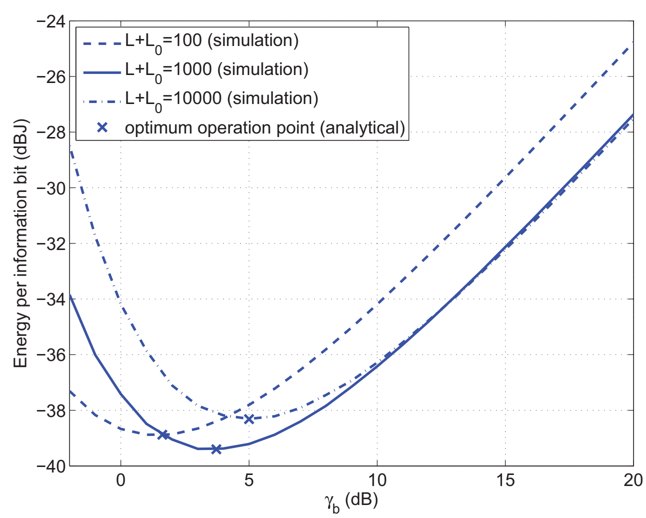

Fig. 3. Energy per information bit E_t v.s. γ_b at the receiver.

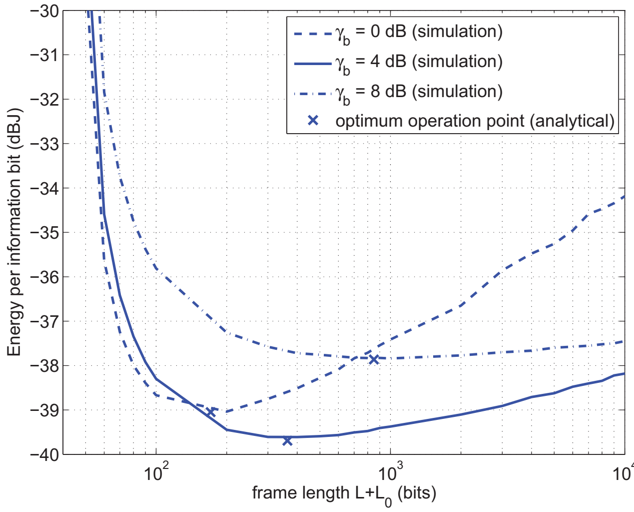

Fig. 4. Energy per information bit E_t v.s. number of bits per frame $L + L_0$.

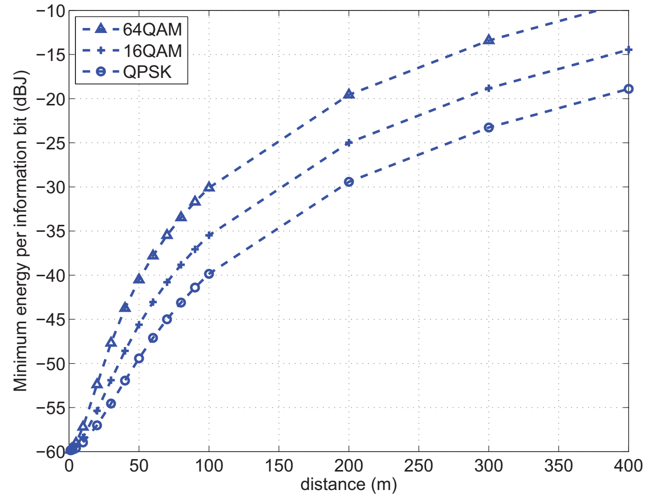

Fig. 5. Minimum energy per information bit as a function of distance.

values of $\hat{L}$ for different γ_b are calculated from (18), and are marked on the figure. As expected, E_t is convex in $\log(L+L_0)$. Again, excellent agreement is observed between the analytical optimum operation points and the simulation results. When $L < \hat{L}$, the energy consumption is dominated by the overhead. Thus significant energy saving can be achieved by slightly increasing L. When $L > \hat{L}$, the slope of E_t with respect to $\log(L + L_0)$ decreases as γ_b increases. This is because the impact of increasing L on FER becomes smaller at higher γ_b. Therefore, system operates at lower γ_b is more sensitive to the frame length.

In the last example, the global optimum E_t is shown as a function of the transmitter-receiver distance d, for systems employing different modulation schemes. The joint optimum $(\hat{\gamma}_b, \hat{L})$ are obtained by iteratively invoking (15) and (18), and the results are then used to calculate the optimum E_t. For example, at $d = 100$ m, the optimum values are (2.66 dB, 230 bits), (4.29 dB, 243 bits), and (6.77 dB, 191 bits) for QPSK, 16-QAM, and 64-QAM, respectively. Lower level modulation has better energy performance at the cost of worse spectral efficiency. Increasing M from 2 to 4, or from 4 to 6, results in approximately 5 dB energy loss, when $d \geq 100$ m.

V. Conclusions

The energy efficient design of coded ARQ systems operating in a quasi-static Rayleigh fading channel has been studied in this paper. A new log-domain threshold approximation method has been proposed to analytically quantify the impacts of receiver E_b/N_0 and frame length on the FER, and the results have been used to facilitate the system optimization. The optimum transmission energy and frame length that minimize the energy per information bit have been obtained in closed-form expressions, and they incorporate the effects of a large number of practical system operation parameters in hardware, the PHY layer, and the MAC layer. From the analytical and simulation results, we have the following observations: 1) The total energy consumption in ARQ can be *reduced* by *increasing* the transmission energy in one transmission attempt; 2) systems operating at higher E_b/N_0 are less sensitive to the frame length; 3) increasing the modulation level by a factor of 4 leads to approximately 5 dB energy loss; 4) significant energy savings (as high as 9.5 dB) can be achieved through the proposed optimum system design.

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