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Low Complexity Turbo Detection of Coded Under-Determined MIMO Systems

Michael L. Walker*, Jun Tao†, Jingxian Wu*, and Yahong Rosa Zheng†
*Dept. of Electrical Engineering, University of Arkansas, Fayetteville, AR 72701, USA
†Dept. of Electrical & Computer Eng., Missouri University of Science & Technology, Rolla, MO 65409, USA

***Abstract*— The turbo detection of a coded *under-determined* multiple-input multiple-output (UD-MIMO) system is studied in this paper. A UD-MIMO system with N transmit antennas and $M < N$ receive antennas has more unknowns than the observations. However, most existing low complexity detectors, such as the vertical Bell Laboratories Layered Space-Time (V-BLAST) or the block decision feedback equalizer (BDFE), work only when $M \geq N$. We propose a new turbo detector structure that combines the existing generalized parallel interference cancellation (GPIC) technique and a new generalized serial interference cancellation (GSIC) technique, with a soft-input soft-output (SISO) BDFE. In the first iteration, the GPIC-BDFE equalization is adopted. From the second iteration and beyond, due to the availability of the *a priori* soft information, the UD-MIMO system can be partitioned into multiple sub-systems by using a reliability-based partition scheme. Then, the GSIC-BDFE equalization, which has a much lower complexity than the GPIC-BDFE equalization, can be used. Simulation results show that the new detection scheme achieves significant performance gain as the iteration progresses with a reasonable complexity.**

I. Introduction

Most existing works on multiple-input multiple-output (MIMO) system with spatially multiplexing focus on the symmetric ($M = N$) or the over-determined ($M > N$) systems [1] - [6], where N and M denotes the number of transmit antennas and the number of receive antennas, respectively. However, in many practical scenarios, under-determined MIMO (UD-MIMO) systems with $N > M$ are preferred over symmetric or over-determined MIMO systems. For instance, the downlink transmission from a base station to a mobile station can usually be modeled as a UD-MIMO system, because a mobile device usually has less antennas than a base station due to the size limitation. A wireless sensor network (WSN) with a large number of low-complexity sensor nodes simultaneously transmitting to a data fusion center (DFC) with less antennas than nodes, can also be modeled as a UD-MIMO system.

Despite the practical importance of UD-MIMO systems, there are very limited works in the literature devoted to such systems. The well known vertical Bell Laboratories Layered Space-Time (V-BLAST) [1] along with its many variations [2] - [4], can not be directly applied to UD-MIMO systems, due to the numerical instability caused by the inversion of a rank deficient matrix. The optimum maximum likelihood sequence estimation (MLSE) detector can be directly applied to a UD-MIMO system. However, its complexity grows exponentially as S^N, where S is the modulation constellation size. Sphere decoding (SD) [7] is proposed to partly reduce the complexity of MLSE, but its complexity is still prohibitively high when N is large. In [8], receiver oversampling is proposed to generate the so-called "virtual receive antennas". This method only works when the coherence time of the channel is comparable to or smaller than the symbol period. Generalized parallel interference cancellation (GPIC) is proposed in [9] to perform exhaustive search over the extra $N-M$ signal dimensions. The exhaustive search generates S^{N-M} parallel symmetric $M \times M$ sub-systems, which are detected with V-BLAST. We denote it as GPIC-VBLAST in this paper. The GPIC is designed primarily for uncoded systems, yet almost all practical wireless systems are protected by channel coding.

In this paper, we propose a low-complexity turbo detection scheme for coded UD-MIMO systems. The turbo detection improves the system performance via iterative operation between a soft-input, soft-output (SISO) equalizer and a SISO channel decoder [10] - [12]. The SISO equalizer is developed by combining the existing GPIC structure and a new generalized serial interference cancellation (GSIC) structure, with a SISO block decision feedback equalization (BDFE) [5]. In the first iteration, the GPIC-BDFE equalization is proposed, and it does not require the *a priori* information. From the second iteration and beyond, the $N \times M$ UD-MIMO system is partitioned into $\lceil \frac{N}{M} \rceil$ symmetric or over-determined sub-systems, where $\lceil a \rceil$ represents the smallest integer larger than a. The partition is performed with a new reliability-based partitioning scheme by utilizing the *a priori* information that is unique to the turbo detection. The sub-systems are detected serially with the SISO BDFE. The serial interference cancellation among the sub-systems is performed by using both the *a priori* soft information at the equalizer input, and the *a posteriori* soft information generated by the SISO BDFE. The resulting GSIC-BDFE has a much lower complexity than the GPIC-BDFE. It is shown through simulations that the hybrid GPIC-GSIC structure with the BDFE can achieve significant performance gains as the iterations progress in coded systems, and in uncoded systems the GPIC-BDFE outperforms the GPIC-VBLAST in terms of both the bit error rate (BER) and the complexity.

The rest of this paper is organized as follows. In Section II, the UD-MIMO system model is described. In Section III, the proposed low-complexity turbo detection scheme using GPIC-BDFE and GSIC-BDFE is developed, and its complexity is analyzed and compared with existing schemes. Simulation results are provided in Section IV, and Section V concludes the paper.

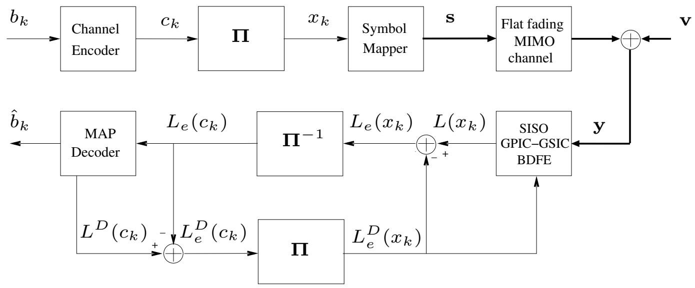

Fig. 1. Block Diagram of A Coded UD-MIMO System.

II. System Model

The block diagram of a coded UD-MIMO system with N transmit antennas and $M < N$ receive antennas is shown in Fig. 1. The information bit stream, $\{b_k\}_k$, is encoded by a convolutional encoder to generate the coded bit stream $\{c_k\}_k$. The coded bits are then interleaved by a pseudo-random interleaver $\mathbf{\Pi}(\cdot)$ as $x_k = \mathbf{\Pi}(c_k)$. For a constellation set $\mathcal{S} = \{\chi_k\}$ with cardinality $S = |\mathcal{S}| = 2^K$, every K interleaved coded bits are mapped to one modulation symbol. The modulated symbols are de-multiplexed onto the N transmitting antennas for transmission, with s_n transmitted by the n-th antenna. The discrete-time samples at the receive antenna are represented by

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{v} \tag{1}$$

where $\mathbf{H} \in \mathcal{C}^{M\times N}$ is the flat-fading MIMO channel matrix with the (m, n)-th element, $h_{m,n}$, being the fading coefficient between the n-th transmit antenna and the m-th receive antenna. The vectors, $\mathbf{s} = [s_1, \cdots, s_N]^T \in \mathcal{S}^{N\times 1}$, $\mathbf{y} = [y_1, \cdots, y_M]^T \in \mathcal{C}^{M\times 1}$, and $\mathbf{v} = [v_1, \cdots, v_M]^T \in \mathcal{C}^{M\times 1}$ are, respectively, the transmitted symbol vector, the received sample vector, and the additive white Gaussian noise (AWGN) vector. The symbol $\mathcal{C}$ is the complex number set and the operation $\mathbf{A}^T$ denotes the matrix transpose. It is assumed that the receiver has ideal knowledge of the channel matrix $\mathbf{H}$.

The turbo receiver consists of a SISO GPIC-GSIC BDFE equalizer and a SISO maximum *a posteriori* (MAP) convolutional decoder, separated by an interleaver and a de-interleaver ($\mathbf{\Pi}^{-1}$) as shown in Fig. 1. The SISO equalizer and the SISO decoder iteratively exchange soft extrinsic log likelihood ratios (LLR) to improve the detection quality. Details on the turbo equalization can be found in [10] - [12].

III. Low Complexity SISO BDFE with GPIC-GSIC

The new low complexity SISO equalizer with GPIC-GSIC is developed for the UD-MIMO system in this section. In the first iteration, there is no *a priori* information available and the existing GPIC technique is adopted. The GPIC exhaustively searches the extra $(N-M)$-dimension signal space in an UD-MIMO system, resulting in S^{N-M} parallel sub-systems. Each sub-system is then detected by the SISO BDFE in parallel. In the subsequent iterations, the *a priori* information is available and a new GSIC technique is proposed. In GSIC, the UD-MIMO system is partitioned into multiple sub-systems, each is again detected with the SISO BDFE. The interference among sub-systems are serially canceled out by using the tentative soft decisions calculated from both the *a priori* input and the *a posteriori* output. The sub-system partition relies on a newly defined reliability measure, α_n, assigned to each symbol s_n. The calculation of α_n will be discussed later in this section. Details on the GPIC-BDFE and the GSIC-BDFE are presented in the following two subsections, respectively.

A. First Iteration: GPIC-BDFE

The new GPIC-BDFE equalization is developed in this subsection for the first iteration. The SISO BDFE was originally developed in [5] for symmetric or over-determined systems, and it can not be directly applied to UD-MIMO systems. We propose to solve this problem with a new GPIC-BDFE structure, which partitions the UD-MIMO system into multiple parallel non-UD sub-systems.

With the GPIC technique, the channel matrix $\mathbf{H}$ is partitioned into two matrices as, $\mathbf{H}_1 = [\mathbf{h}_{i_1}, \cdots, \mathbf{h}_{i_{N-M}}] \in \mathcal{C}^{M\times(N-M)}$ and $\mathbf{H}_2 = [\mathbf{h}_{i_{N-M+1}}, \cdots, \mathbf{h}_{i_N}] \in \mathcal{C}^{M\times M}$, where $i_1, \cdots, i_N$ is a permutation of $1, \cdots, N$ and $\mathbf{h}_{i_k}$ is the i_k-th column of $\mathbf{H}$. It can be seen that the ordering set, $\{i_n\}_{n=1}^N$, determines how the system is partitioned. After partition, the system model in (1) can be re-written as:

$$\mathbf{y} = \mathbf{H}_1\mathbf{s}_1 + \mathbf{H}_2\mathbf{s}_2 + \mathbf{v} \tag{2}$$

where $\mathbf{s}_1 = [s_{i_1}, \cdots, s_{i_{N-M}}]^T \in \mathcal{S}^{(N-M)\times 1}$ and $\mathbf{s}_2 = [s_{i_{N-M+1}}, \cdots, s_{i_N}]^T \in \mathcal{S}^{M\times 1}$. The $N \times M$ system in (1) is equivalently represented as the superposition of an $(N-M) \times M$ system and an $M \times M$ system.

The GPIC performs exhaustive search of all the $J = S^{N-M}$ possible $\mathbf{s}_1$ as

$$\mathbf{y}_j = \mathbf{y} - \mathbf{H}_1\mathbf{s}_1^{(j)} \text{ for } j = 1, \cdots, S^{N-M}. \tag{3}$$

Thus, by fixing a particular sequence $\mathbf{s}_1^{(j)}$, we can apply the SISO BDFE to equalize the corresponding equivalent symmetric MIMO system $\mathbf{y}_j = \mathbf{H}_2\mathbf{s}_2^{(j)} + \mathbf{v}$.

During the first iteration, there is no *a priori* information available. Therefore, the sub-system partition is determined with the channel matrix $\mathbf{H}$. We adopt the ordering scheme in [9], which is based on the Frobenius norm of the rows of the pseudo-inverse of $\mathbf{H}$, as $\| \left(\mathbf{H}^\dagger\right)_k \|$, where $\mathbf{A}^\dagger$ denotes matrix pseudo-inverse, $(\mathbf{A})_k$ is the k-th row of the matrix $\mathbf{A}$, and $\|\mathbf{a}\|$ is the Frobenius norm of the vector $\mathbf{a}$. We refer to this ordering procedure as *method 1*. It was demonstrated in [9] that such an ordering scheme outperforms another scheme, referred herein as *method 2*, whose ordering is based on the Frobenius norm of the columns of $\mathbf{H}$.

Once the J parallel systems are established, the SISO BDFE can be applied to the J symmetric $M\times M$ systems. The BDFE performs detection through two block filters: a feedforward filter, $\mathbf{W} \in \mathcal{C}^{M\times M}$, and a strict upper triangular feedback filter, $\mathbf{B} \in \mathcal{C}^{M\times M}$, as [5]

$$\tilde{\mathbf{s}}_2^{(j)} = \mathbf{W}\mathbf{y}_j - \mathbf{B}\hat{\mathbf{s}}_2^{(j)} \tag{4}$$

where $\hat{\mathbf{s}}_2^{(j)}$ contains tentative soft decision at the output of the j-th BDFE. The soft decision is defined as the *a posteriori* mean of the symbol, and it can be calculated as

$$\hat{s}_{i_{N-M+m}}^{(j)} = \sum_{k=1}^{S} \chi_k P\left(s_{i_{N-M+m}} = \chi_k | \mathbf{y}_j\right) \tag{5}$$

where $P\left(s_{i_{N-M+m}} = \chi_k | \mathbf{y}_j\right)$ is the *a posteriori* probability (APP) at the output of the j-th BDFE. The adoption of the *a posteriori* soft decision will reduce the effects of error propagation, thus leading to a better system performance.

By using the minimum mean square error (MMSE) criterion, the filters $\mathbf{W}$ and $\mathbf{B}$ can be calculated by [5]

$$\begin{aligned} \mathbf{B} &= \mathbf{U} - \mathbf{I}_M, \\ \mathbf{W} &= \mathbf{U}\mathbf{H}_2^H \left(\mathbf{H}_2\mathbf{H}_2^H + \sigma_0^2 \mathbf{I}_M\right)^{-1} \end{aligned} \tag{6}$$

where $\mathbf{A}^H$ represents matrix Hermitian, $\mathbf{I}_M$ is an identity matrix of size M, and $\mathbf{U} \in \mathcal{C}^{M\times M}$ is an upper triangular matrix with unit diagonal. The matrix $\mathbf{U}$ is obtained from the Cholesky decomposition as $\frac{1}{E_s}\mathbf{I}_M + \frac{1}{\sigma_0^2}\mathbf{H}_2^H\mathbf{H}_2 = \mathbf{U}^H \mathbf{\Delta} \mathbf{U}$, where E_s is the symbol power, σ_0^2 is the noise variance, and $\mathbf{\Delta} \in \mathcal{R}^{M\times M}$ is a diagonal matrix with the m-th diagonal element being δ_m. The symbol $\mathcal{R}$ denotes the real number space.

It is important to note that the two filters defined in (6) depend only on the channel matrix $\mathbf{H}_2$. Therefore they only need to be computed once, and the same filters will be used by all the J parallel sub-systems. On the other hand, in GPIC-VBLAST [9], a matrix pseudo-inverse operation is required for each symbol in each of the J parallel sub-systems, resulting in a total number of JM matrix pseudo-inverse operations. Hence, in the context of an uncoded UD-MIMO system, the complexity of the proposed GPIC-BDFE is much smaller than that of the GPIC-VBLAST.

With the filters defined in (6), we have an equivalent system representation from (4)

$$\mathbf{r}_j = \mathbf{W}\mathbf{y}_j = \mathbf{G}\mathbf{s}_2 + \mathbf{e}_j, \tag{7}$$

where $\mathbf{r}_j = [r_1^{(j)}, \cdots, r_M^{(j)}]^T$ and $\mathbf{e}_j = [e_1^{(j)}, \cdots, e_M^{(j)}]^T$ are the receive sample vector and noise sample vector of the equivalent system, respectively, and $\mathbf{G} = \mathbf{B} + \mathbf{I}_M$ is the equivalent channel matrix.

With the equivalent system representation given in (7), the APP for the m-th symbol, $s_{i_{N-M+m}}$, in the j-th parallel sub-system, can be calculated as

$$P\big(s_{i_{N-M+m}}|\mathbf{r}_j\big) = \frac{P(\chi_k)}{A_m} \exp\left[-\frac{1}{\sigma_m^2}\left|\rho_m^{(j)}(s_{i_{N-M+m}})\right|^2\right] \tag{8}$$

where $\sigma_m^2 = \delta_m^{-1}$, $P(\chi_k) = \frac{1}{S}$ for the first iteration, A_m is a normalization constant and $\rho_m^{(j)}(s_{i_{N-M+m}}) = r_m^{(j)} - g_{m,m}s_{i_{N-M+m}} - \sum_{l=m+1}^{M} g_{m,l}\hat{s}_{N-M+l}^{(j)}$, where $g_{m,n}$ is the (m,n)-th element of $\mathbf{G}$.

The APP in (8) is used to replace the APP in (5) for computing the tentative soft decision. Once the tentative soft decision vectors, $\left\{\hat{\mathbf{s}}_2^{(j)}\right\}$, are obtained for all J parallel sub-systems, we will select one survival sub-system with its index determined as follows

$$j_0 = \underset{j}{\operatorname{argmin}} \, \|\mathbf{y} - \mathbf{H}_1\mathbf{s}_1^{(j)} - \mathbf{H}_2\hat{\mathbf{s}}_2^{(j)}\|^2. \tag{9}$$

The corresponding sequence is denoted as $\hat{\mathbf{s}}^{(j_0)} = \left[\mathbf{s}_1^{(j_0)}; \hat{\mathbf{s}}_2^{(j_0)}\right] \in \mathcal{C}^{N\times 1}$, where $[\mathbf{a};\mathbf{b}]$ denotes stacking the two column vectors $\mathbf{a}$ and $\mathbf{b}$ into a single column vector. In the vector $\hat{\mathbf{s}}^{(j_0)}$, the first $N-M$ elements are hard decisions, and the last M symbols are soft decisions. The APP of $\hat{\mathbf{s}}_2^{(j_0)}$ has been calculated in (8). We still need to find the APP of the elements in $\mathbf{s}_1^{(j_0)}$. The APP for the n-th symbol in $\mathbf{s}_1^{(j_0)}$ can be calculated as

$$P(s_{i_n}|\mathbf{y}) = \frac{P(s_{i_n})}{A_{i_n}} \exp\left[-\frac{1}{\sigma_0^2}\|\mathbf{y} - \mathbf{H}\hat{\mathbf{s}}_{i_n}^{(j_0)} - \mathbf{h}_{i_n}s_{i_n}\|^2\right], \tag{10}$$

where $\hat{\mathbf{s}}_{i_n}^{(j_0)}$ is obtained by replacing the n-th element of $\hat{\mathbf{s}}^{(j_0)}$ with 0. The extrinsic bit LLR can then be calculated from the symbol APP as in [5], and is delivered as the input to the MAP decoder.

B. Second Iteration and Beyond: GSIC-BDFE

Soft *a priori* information is available at the input of the equalizer starting from the second iteration. Define the *a priori* mean and variance of the symbols as

$$\begin{aligned} \bar{s}_n &= \sum_{k=1}^{S} \chi_k P(s_n = \chi_k), \\ \sigma_{s_n}^2 &= \sum_{k=1}^{S} |\chi_k - \bar{s}_n|^2 P(s_n = \chi_k) \end{aligned} \tag{11}$$

where the symbol *a priori* probability, $P(s_n = \chi_k)$, can be calculated from the *a priori* LLR, $L_e^D(x_k)$.

With the soft *a priori* information, the reliability information can be calculated. Define the reliability measure of s_n as $\alpha_n = 1/\sigma_{s_n}^2$, which is based on the fact that a lower *a priori* variance means a smaller deviation from the *a priori* mean, which has been used as the tentative soft decision. Then we can order the reliability information in ascending order as: $\alpha_{i_1} \leq \alpha_{i_2} \leq \cdots \leq \alpha_{i_N}$, and partition the system into $D = \lceil \frac{N}{M} \rceil$ parallel subsystems as follows

$$\mathbf{y} = \sum_{d=1}^{D} \mathbf{H}_d\mathbf{s}_d + \mathbf{v}, \tag{12}$$

where $\mathbf{H}_d = [\mathbf{h}_{d_1}, \cdots, \mathbf{h}_{d_M}] \in \mathcal{C}^{M\times M}$, if $1 \leq d \leq D-1$, and the matrix $\mathbf{H}_D = [\mathbf{h}_{D_0}, \cdots, \mathbf{h}_{D_{M'}}] \in \mathcal{C}^{M\times M'}$, with $d_k = i_{(d-1)M+k}$, $D_k = i_{(\lceil \frac{N}{M} \rceil - 1)M+k}$ and $M' = N - (\lceil \frac{N}{M} \rceil - 1)M$.

In (12), the UD-MIMO system is represented as the superposition of D sub-systems, with $D-1$ symmetric sub-systems with channel matrix $\{\mathbf{H}_d\}_{d=1}^{D-1}$, and 1 over-determined or symmetric sub-system with channel matrix $\mathbf{H}_D$. Due to the availability of the tentative soft decision, we can directly perform soft interference cancellation (IC) among the sub-systems without resorting to the exhaustive search used in GPIC.

Therefore, we propose a new GSIC structure, where the D sub-systems are equalized serially, with sub-system D being equalized first, and sub-system 1 equalized last. During the equalization of the sub-system d, the interference from sub-systems 1 to $d-1$ is canceled by using the *a priori* information obtained from the previous iteration, and the interference from sub-systems $d+1$ to D is canceled by using the soft tentative decision obtained through the SISO BDFE equalization performed in the current iteration.

The SISO BDFE of sub-system d is performed over the following equivalent system

$$\mathbf{y}^{(d)} = \mathbf{y} - \sum_{j=1}^{d-1} \mathbf{H}_j \bar{\mathbf{s}}_j - \sum_{j=d+1}^{D} \mathbf{H}_j \hat{\mathbf{s}}_j, \tag{13}$$

where $\{\bar{\mathbf{s}}_j\}_{j=1}^{d-1}$ contains the *a priori* soft decision from the previous iteration, and $\{\hat{\mathbf{s}}_j\}_{j=d+1}^{D}$ contains the *a posteriori* soft decision from the current iteration.

The BDFE used in the GSIC-BDFE structure takes advantage of the *a priori* mean and the *a priori* variance defined in (11), thus it is different from the BDFE used in the first iteration. Based on the system model in (13), the BDFE is similar to that based on the model (3). Details are referred to (4) to (10).

C. Complexity Analysis

The complexity of the proposed GPIC-GSIC BDFE is analyzed and compared with the existing GPIC-VBLAST scheme. The computation complexity arises from two sources. The first one is the ordering of the symbols and the calculation of the equalizer matrices; and the second one is the process of the symbol detection. For simplicity, the calculation of the soft LLR is not included since it is similar for both detection schemes. The comparison is based on the total number of complex multiplication (CM). As shown in the previous subsections, the detection of a UD-MIMO system is equivalent to the detections of multiple decomposed symmetric or over-determined sub-systems. Therefore, it is sufficient to investigate one sub-system.

For an $M \times M$ sub-system using BDFE detection, the calculation of the equalizer coefficients in (6) involves one Cholesky decomposition for $\mathbf{B}$ and one back-substitution for $\mathbf{W}$. The incurred complexity is thus in the order of $\mathcal{O}(\frac{3}{2}M^3)$. The estimation of all the M transmitted symbols involves two $(M \times M) \times (M \times 1)$ matrix multiplications (MM), incurring a complexity on the order of $\mathcal{O}(M^2)$. The overall complexity order scales with $\mathcal{O}(\frac{3}{2}M^3)$.

For an $M \times M$ sub-system using V-BLAST detection, the determination of the detection ordering and the calculation of the equalizer coefficients involves M matrix inversion of size M, and $\frac{(M-1)(M+2)}{2}$ times of $M \times 1$ Frobenius norm calculations. The total number of CM required is thus $M^4 + \frac{M(M-1)(M+2)}{2}$, and the complexity order is $\mathcal{O}(M^4)$. The estimation of all the M transmitted symbols involves M times of $(1 \times M) \times (M \times 1)$ MM and $(M-1)$ times of $(1 \times 1) \times (M \times 1)$ MM. The total number of CM is thus $M^2 + M(M-1)$, and the complexity scales with $\mathcal{O}(M^2)$. Therefore, the overall complexity is in the order of $\mathcal{O}(M^4)$.

As shown in the previous subsection, there are $J = S^{N-M}$ parallel sub-systems with the GPIC technique. Therefore, in the first iteration, the complexity orders for the GPIC-BDFE and the GPIC-VBLAST are $\mathcal{O}(\frac{3}{2}JM^3)$ and $\mathcal{O}(JM^4)$, respectively. Obviously, in the first iteration, the GPIC-BDFE has a lower complexity than the GPIC-VBLAST since $M \geq 2$ for a MIMO system. From the second iteration, the GSIC-BDFE performs a reliability-based ordering and only detects $D = \lceil \frac{N}{M} \rceil$ sub-systems. The reliability-based ordering incurs a complexity order of $\mathcal{O}(N^2)$ with classic sorting algorithms. The overall complexity is thus in the order of $\mathcal{O}(\frac{3}{2}DM^3+N^2)$. For a practical UD-MIMO system, we have $D \ll J$, thus the complexity of GSIC-BDFE is drastically reduced and is much lower than that of the GPIC-BDFE. For example, in a 7×3 system, $D = \lceil \frac{7}{3} \rceil = 3$ compared with $J = S^{N-M} = 4^4 = 256$ with a QPSK modulation scheme. The GPIC-VBLAST performs non-iterative one-time equalization, thus the complexity analysis is unavailable starting from the second iteration. The complexity comparison is summarized in Table I.

TABLE I
COMPLEXITY COMPARISON

Iteration \ Scheme	GPIC-GSIC with BDFE	GPIC with V-BLAST
1	$\mathcal{O}(\frac{3}{2}JM^3)$	$\mathcal{O}(JM^4)$
≥ 2	$\mathcal{O}(\frac{3}{2}DM^3 + N^2)$	-

IV. SIMULATION RESULTS

Simulation results are presented in this section to demonstrate the performance of the proposed hybrid GPIC-GSIC technique with SISO BDFE. Before examining the performance of the turbo equalizer in a coded system, we first compare the performance of the newly proposed GPIC-BDFE scheme with the existing GPIC-VBLAST scheme presented in [9] in an uncoded 7×3 UD-MIMO system. The results are presented in Fig. 2. The results obtained from the optimum MLSE receiver is also shown as a reference. It can be seen from the figure that the GPIC-BDFE outperforms the GPIC-VBLAST by 5 dB at a BER of 10^{-4}. In addition, the GPIC-BDFE is only 0.7 dB away from the optimum MLSE receiver in the uncoded system. It should be noted that the GPIC-BDFE has a lower complexity compared to GPIC-VBLAST, yet it achieves a better performance. The results in Fig. 2 also demonstrate the superiority of ordering method 1 over ordering method 2, in both the GPIC-BDFE and the GPIC-VBLAST.

Next we examine the performance of the coded UD-MIMO system with the new hybrid GPIC-GSIC technique. The transmitted data is divided into blocks of 1024 bits that are encoded with a non-recursive systematic convolutional code

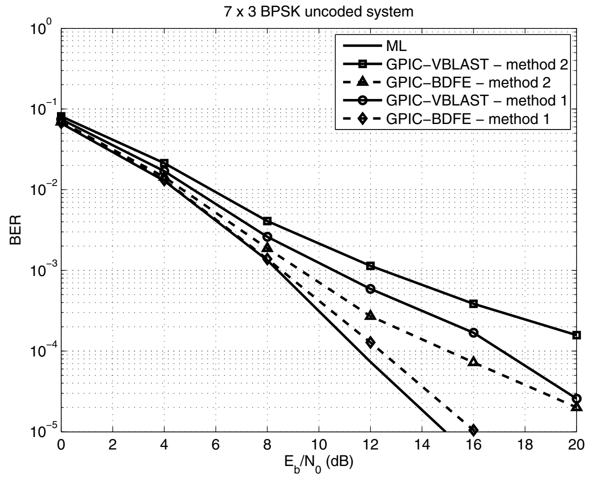

Fig. 2. BER performance of BPSK uncoded system with $N = 7$ and $M = 3$.

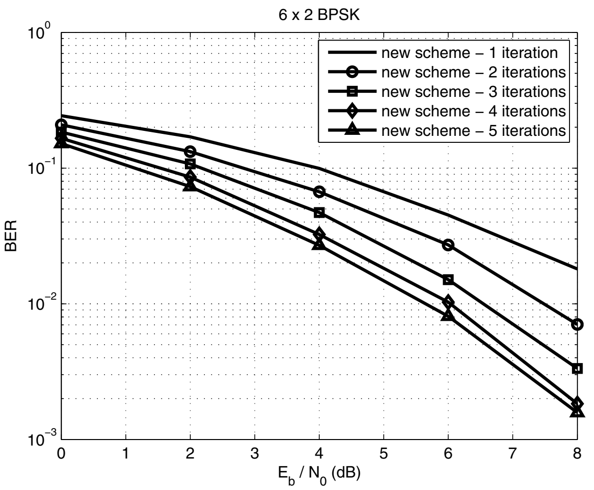

Fig. 3. BER performance of BPSK coded system with $N = 6$ and $M = 2$.

with polynomial generator $G = [7, 5]_8$. The BPSK modulation is employed. In Fig. 3, the results of a system employing 6 transmit antennas and 2 receive antennas are shown. For such a system configuration, the BDFE is performed 16 times in the first iteration corresponding to the 2^{6-2} parallel sub-systems with GPIC, and 3 times in each of the subsequent iterations for the $\frac{6}{2}$ serial sub-systems with GSIC. It can be seen from the figure that the performance improves consistently as the iterations progress. At the BER level of 2×10^{-2}, the fifth iteration outperforms the first iteration by more than 3 dB.

V. Conclusion

A new turbo detection scheme was proposed for coded UD-MIMO systems. The equalization of the UD-MIMO system was performed with a SISO BDFE combined with a new hybrid GPIC-GSIC technique. The GPIC-BDFE was proposed for the first iteration to account for the S^{N-M} parallel sub-systems due to the exhaustive search of the extra $(N - M)$ symbols. The GSIC-BDFE was proposed for all the subsequent iterations, where tentative soft decisions from the previous iteration and the current iteration can be used to perform interference cancellation among the $\lceil \frac{N}{M} \rceil$ serial sub-systems. The partition of the serial sub-systems is based on a reliability metric defined by using the *a priori* symbol variance, which is a unique byproduct of the turbo equalization. Simulation results demonstrated that the GPIC-BDFE system significantly outperforms the existing GPIC-VBLAST system in an uncoded system in terms of both BER and complexity, and the new GPIC-GSIC BDFE scheme achieves good performance in coded UD-MIMO system with reasonable complexity.

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