Random On-Off Accumulative Transmission for Asynchronous Wireless Sensor Networks

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Abstract-In this paper, a random on-off accumulative transmission (R-OOAT) scheme is proposed to achieve collision-tolerant (CT) media access control (MAC) for asynchronous wireless sensor networks. Unlike conventional MAC schemes that discard packages with collisions at receivers, the CT-MAC extracts the salient information from the colliding signals by using the R-OOAT scheme in the physical layer. Nodes employing the R-OOAT deliver information to a base station through asynchronous on-off transmission of multiple identical sub-symbols at random positions. The R-OOAT improves the D-OOAT (D-OOAT) proposed in [1], where sub-symbols are transmitted at *deterministic* positions. Compared to the D-OOAT, the R-OOAT scheme achieves a smaller collision probability and supports more simultaneous users, while it inherits all the advantages of the D-OOAT. Design guidelines of the R-OOAT system are presented for given parameters, such as the probability of collisions and the maximum number of supported users. It is demonstrated by simulation results that the new CT-MAC with R-OOAT scheme can operate at the presence of severe signal collision.

I. INTRODUCTION

A wireless sensor network (WSN) provides autonomous and constant monitoring of certain physical or environmental parameters with a group of spatially distributed sensor nodes equipped with wireless transceivers. Unlike most of the conventional communication systems that strive to maximize the communication throughput or minimize the transmission delay, a WSN can usually operate with long transmission latency and low data rate due to the non-real time nature of the system and the slow variation of the monitored objects. On the other hand, WSNs usually have strict requirements on the power consumption due to the limits imposed by energy sources [2] - [4].

Most of the low-power WSN techniques are developed in the media access control (MAC) layer, where power consumption can be reduced by either decreasing the duty cycle [4], [5], or by carefully coordinating the transmission of the sensor nodes to reduce collisions at the receiver [6] - [8]. All are based on the conventional MAC approach where signals colliding at a receiver will be discarded and retransmitted. This results in a waste of the precious transmission power. In a low-power WSN, we might not afford the luxury of discarding collided messages, which still contain salient information and can help signal detection. Moreover, most of the low-power schemes are

developed by following the traditional layered-protocol design approach. While the layered design approach carries significant advantages for conventional communication networks, it cannot capture the interactions among protocol layers that might be critical to low-power communications.

Since signal processing in the physical layer can help resolve the signal collision in the MAC layer, a cross-layer collisiontolerant MAC (CT-MAC) scheme was proposed in our previous work [1]. The collision tolerance in the MAC layer is achieved by employing an on-off accumulative transmission (OOAT) scheme, where each data symbol is transmitted in the form of multiple identical sub-symbols (accumulative transmission), and two sub-symbols are separated by a fixed silence period (on-off transmission) to reduce the collision probability and duty cycle. In [1], all the nodes follow the same *deterministic* transmission pattern, *i.e.*, all the sub-symbols are equally spaced with a fixed amount of silence period in between. As a result, a group of aligned users will always collide with each other due to their identical transmission patterns, and the system will quickly become saturated as the number of users increases.

To address the above problems, in this paper we propose a new random OOAT (R-OOAT) scheme where the on-off transmission pattern of each node is randomly chosen, such that no two nodes share the same transmission pattern. Compared to the deterministic OOAT (D-OOAT) scheme in [1], the R-OOAT can support more users with a smaller collision probability due to the employment of unique transmission patterns for each node. The design and performance of R-OOAT systems are studied from two perspectives. First, the interactions among the various design parameters of R-OOAT is quantified by investigating the statistical distribution of collision order, which is defined as the maximum number of collisions for a certain system configuration. Second, the maximum number of users that can be supported in the system is identified by studying the structure of the transmission patterns, which are represented as cyclic-shifted binary vectors. The above theoretical results provide a complete set of guidelines on the design of practical R-OOAT systems. In addition, the employment of the unique transmission pattern for each user enables the accurate estimation of the relative delay among the asynchronous users through a time-domain correlation operation. Simulation results demonstrate that the new R-OOAT scheme with CT-MAC outperforms the original D-OOAT scheme, and it can operate robustly at the presence of severe signal collisions.

II. RANDOM ON-OFF ACCUMULATIVE TRANSMISSION

In this section, the system model and operation of the proposed CT-MAC is first presented, and then the optimal receiver is briefly described.

A. System Model

Consider a WSN with N spatially distributed nodes and one base station (BS). Data collected by the sensor nodes are delivered to the BS through a one-hop transmission. The sensor nodes work asynchronously without central control or explicit cooperation.

To achieve collision tolerance in the MAC layer, the wireless nodes employ the R-OOAT scheme in the physical layer as shown in Fig. 1, where each data symbol is transmitted through R identical sub-symbols with a duration of T_0 each, over a symbol period of $T_s = MT_0$, where $M \ge R$. As a result, R out of M sub-symbol positions are occupied during each symbol period, and the duty cycle is $\eta = \frac{R}{M}$. The positions of the occupied sub-symbols, or, the transmission pattern, for the *n*-th user can be expressed by a binary vector of length M, $\mathbf{p}_n = [p_n(1), \cdots, p_n(M)]^T \in \mathcal{B}^{M \times 1}$, where $\mathcal{B} = \{0, 1\}$, with $p_n(m) = 1$ if a sub-symbol is transmitted at the *m*-th sub-symbol location, and $p_n(m) = 0$ otherwise. Denote $\mathcal{P}_M^R = \{\mathbf{p} | \mathbf{p} \in \mathcal{B}^{M \times 1}, w(\mathbf{p}) = R\}$, with $w(\mathbf{p})$ be-

Denote $\mathcal{P}_M^R = \{\mathbf{p} | \mathbf{p} \in \mathcal{B}^{M \times 1}, w(\mathbf{p}) = R\}$, with $w(\mathbf{p})$ being the weight of the binary vector \mathbf{p} . The R-OOAT scheme is defined by the ternary (N, M, R) and the position vector set $\mathcal{Q}_M^R \subseteq \mathcal{P}_M^R$. It is assumed that different nodes use different position vectors. Then the maximum number of nodes that can be supported by the R-OOAT scheme is the same as the cardinality of \mathcal{Q}_M^R . Fig. 1 shows an example system with (N = 5, M = 12, R = 3). Due to the asynchronism among the nodes and the on-off transmission scheme, only a subset of the nodes will mutually interfere with each other at a given sub-symbol position.

Define the collision order at the m-th sub-symbol position as

$$N_{c}(m) = \sum_{n=1}^{N} p_{n}(i_{nm}), \qquad (1)$$

where $i_{nm} = \text{mod}_M(m) - p_{n0}$, with $\text{mod}_M(m)$ being the modular M operator, and p_{n0} is the relative starting position of the *n*-th node due to the node asynchronism. For example, in Fig. 1, $p_{10} = 2, p_{20} = 1, p_{30} = 9, p_{40} = 6$, and $p_{50} = 5$. The collision order is defined as $N_c = \max_m N_c(m)$. We have $N_c = 2$ for the system shown in Fig. 1.

Based on the above discussion, a CT-MAC system with R-OOAT can be represented by

$$y(m) = \sum_{n=1}^{N} \sqrt{E_n} p_n(i_{nm}) h_n s_{nk_{nm}} + z(m)$$
(2)

where E_n is the transmission energy for each sub-symbol, y(m)and z(m) are the received sample and additive white Gaussian noise (AWGN) at the *m*-th sub-symbol, respectively, h_n is the fading coefficient between the node *n* and the BS, s_{nk} is the



Fig. 1. An R-OOAT system with N = 5 nodes, R = 3 repetitions per symbol period, and each symbol contains M = 12 possible sub-symbol positions.

k-th symbol transmitted by the node n, and $k_{nm} = \lceil \frac{m - p_{n0}}{M} \rceil$, with $\lceil a \rceil$ being the smallest integer greater than or equal to a.

B. Optimum Receiver

With asynchronous nodes, the symbol from one node usually collides with multiple symbols from multiple nodes, and the colliding symbols change from sample to sample, *e.g.* in Fig. 1, s_{11} collides with s_{41} at sample instant 7, and s_{11} collides with s_{31} at sample instant 11. We denote such interference as co-channel interference between two nodes, and the interference between symbols transmitted at different moments. The presence of CC-ISI determines that the optimum detection should be performed in terms of maximum likelihood sequence estimation (MLSE) with trellis-based algorithm such as the Viterbi algorithm.

Due to the random structure of the position vectors and the asynchronous nature of the system, the CC-ISI changes with respect to time. Therefore, the system will have a time-varying trellis structure. At the BS, the detection in the physical layer is performed by applying the extended Viterbi algorithm with time-varying trellis as described in [1].

In order to establish the time-varying trellis, the receiver needs to know the relative delays among the asynchronous nodes. The receiver can estimate the relative node delays by utilizing the unique structures of the different position vectors. When a new node joins the network, the BS randomly picks a position vector from the set of unused position vectors, and assign the randomly picked vector to the new node. Therefore, the position vector is known to both the transmitter and the receiver. Before transmitting data, node n will first transmit a sequence of pilot symbols, $\mathbf{a}_n = [a_{n1}, \dots, a_{nL}]^T$, with R-OOAT, where L is the number of pilot symbols. The receiver can identify the relative delay, p_{n0} , by performing correlation between the received samples and the position vector, as (c.f. (2))

$$\hat{p}_{n0} = \underset{p \in [0, \cdots, M-1]}{\operatorname{argmax}} \sum_{l=1}^{L} \left| a_{nl} \sum_{m=1}^{M} p_n \left(i_{nm} \right) y((l-1)M + m) \right|, \quad (3)$$

where $i_{nm} = \mod_M(m) - p$. As shown by simulation, the estimation accuracy improves with L.

III. COLLISION TOLERANCE OF R-OOAT

A WSN is called collision-tolerant if the transmitted signals can be recovered at the receiver beyond a certain fidelity measure. For a CT-MAC system with R-OOAT, at any moment, each received sample is the weighted superposition of symbols from up to N_c different nodes. At the mean time, each symbol, s_{nk} , is embedded in R received samples at the receiver. This is similar to a multiple-input multiple-output (MIMO) system with N_c inputs and R outputs. Therefore, the collision tolerance is tightly related to the collision order, N_c , and the repetition order, R.

Due to the asynchronism among the nodes and the randomness of the position vectors used by the nodes, the collision order, N_c , depends on the relative starting position and therefore is a random variable.

If $N_c \leq R$, the system is always collision-tolerant because it is equivalent to a symmetric ($N_c = R$) or consistent overdetermined ($N_c < R$) linear system. If $N_c > R$, then the R-OOAT system is equivalent to an under-determined linear system. In this case, the system is not collision-tolerant if the transmitted signal, s_{nk} , is analog, because an under-determined linear system has infinite analog solutions. On the other hand, if the transmitted signal is digital, then the system can still be collision-tolerant as long as there is a unique intersection between the solution set of the under-determined system and the finite signal set. However, the detection of under-determined system requires complex receiver structures and the performance of under-determined system degrades significantly in the presence of noise and fading.

In practice, to ensure the collision tolerance and the system performance, it is desirable to have a symmetric or overdetermined system $(N_c \leq R)$. Intuitively, the number of repetition has two opposite effects on the collision tolerance. Given the number of nodes and the number of sub-symbol position per symbol, a smaller repetition number means less potential collisions among the users, and this contributes positively to the collision tolerance of the system. On the other hand, a smaller R means a smaller dimension of the received signal, and this contributes negatively to the collision tolerance. The choice of R depends on which of the two effects is dominating, which in turn is determined by the parameters M and N. We have the following proposition regarding the relationship between N_c , R, N, and M.

Proposition 1: Consider a R-OOAT system with (N, M, R). If all the N nodes are transmitting, then the cumulative distribution function (CDF) of the collision order is

$$P(N_c \le u) = \left[\sum_{v=1}^{u} \binom{N-1}{v-1} \frac{R^{v-1}(M-R)^{N-v}}{M^{N-1}}\right]^R, u = 1, \cdots, N \quad (4)$$

Proof: The proof is in Appendix A.

In Fig. 2, $P(N_c \leq R)$ is shown as a function of M with various values of N. The results in Fig. 2 indicate that when M is small, *e.g.* when M < 7 and N = 5, it is desirable to use a smaller R (R = 2) to reduce the number of collisions;



Fig. 2. The probability that the system is not under-determined. (N: number of nodes; M: number of sub-symbol positions per symbol; R: number of repetitions per symbol.)

when M is large, a larger R (R = 3) is preferred because it improves the dimension of the received signal.

Fig. 2 provides a guideline on the choice of the parameters (N, M, R). For instance, if N = 5, picking (M = 12, R = 2) will result in a system that is under-determined for about 25% of the time; while increasing R from 2 to 3 for the same system will reduce the probability of being under-determined to 15%. Increasing M will further reduce the probability of getting an under-determined system, but at the cost of a longer delay if we fix T_0 , or a larger bandwidth if we fix T_s . During the design of the system, the parameters (M, R) can be chosen based on the design objectives of the system, *e.g.*, the number of users N, the targeted $P(N_c \leq R)$, the delay and bandwidth requirement, etc. Once these parameters are given, the values of M and R can be chosen by using the graphic results as shown in Fig. 2.

In summary, the R-OOAT scheme contributes to the collision tolerance of a system from three aspects. First, the random onoff transmission will reduce the collision order, N_c , at the BS. Second, the transmission of R identical sub-symbols results in a R-dimension received signal in the time-domain, which can be used for the detection of the N_c -dimension signal in the spatial-domain. Third, the spreading of the signals in the time-domain enables time diversity. It should be noted that the collision tolerance is achieved by exploiting the low data rate and long latency tolerance of a WSN. Larger value of M results in longer transmission latency or larger bandwidth, but better quality.

IV. CONSTRUCTION OF POSITION VECTOR SET

The position vectors, $\{\mathbf{p}_n\}_{n=1}^N$, plays a critical role on the collision tolerance of the R-OOAT system. Once (N, M, R) is chosen, we can reduce or minimize the collision order, N_c , by carefully construct the set of available position vectors. As indicated before, the position vector set is a subset of \mathcal{P}_M^R that contains all the length-M weight-R binary vectors. The cardinality of the set is thus $|\mathcal{P}_M^R| = {M \choose R}$.

Due to the asynchronism among the nodes, the receiver observes a cyclic-shifted version of \mathbf{p}_n . Define the set that contains all the cyclic shifted versions of \mathbf{p}_n as $\mathcal{O}(\mathbf{p}_n) = \{\mathbf{q} | \mathbf{q} \in \mathcal{P}_M^R; \exists k, (\mathbf{q})_k = \mathbf{p}_n\}$, where $(\mathbf{q})_k = [q(M - k + 1), \cdots, q(M), q(1), \cdots, q(M - k)]^T$ is obtained by cyclic shifting \mathbf{q} to the right by k positions. To ensure that the receiver can distinguish between information from any two nodes with arbitrary relative delay, we should have $\mathcal{O}(\mathbf{p}_n) \cap \mathcal{O}(\mathbf{p}_m) = \emptyset$ if $m \neq n$. Define a subset $\mathcal{Q}_M^R \subseteq \mathcal{P}_M^R$ as

$$\mathcal{Q}_{M}^{R} = \left\{ \mathbf{q}_{n} | \mathbf{q}_{n} \in \mathcal{P}_{M}^{R}; \mathcal{O}(\mathbf{q}_{m}) \bigcap \mathcal{O}(\mathbf{q}_{n}) = \emptyset, \forall m \neq n \right\}.$$
(5)

The binary position sequences need to be selected from Q_M^R to make sure that no two sequences belong to the same cyclic shift set, $\mathcal{O}(\mathbf{p})$. The maximum number of users that can be supported by an R-OOAT system with parameters M and R is thus equal to the cardinality of Q_M^R as stated in the following theorem.

Theorem 1: For a R-OOAT system with parameters M and R, the maximum number of users that can be supported by the system is $\max(N) = |\mathcal{Q}_M^R|$, where the cardinality $P(M, R) \triangleq |\mathcal{Q}_M^R|$ can be recursively calculated as

$$P(M,R) = \frac{\binom{M}{R}}{M} + \sum_{c \in \mathcal{D}_M^R} V_M^R(c) \left(1 - \frac{1}{c}\right),\tag{6}$$

where \mathcal{D}_M^R is the set of all the common denominators of M and R, $g = \max_{c \in \mathcal{D}(M,R)} c$ is the greatest common denominator (gcd) of M and R. The function $V_M^R(c)$ is recursively defined as

$$V_M^R(c) = P\left(\frac{M}{c}, \frac{R}{c}\right) - \sum_{a \in \mathcal{M}(c)} V_M^R(a), \tag{7}$$

where $\mathcal{M}(c) = \{a | a \in \mathcal{D}_M^R, a > c, a/c \text{ is integer}\}.$

Proof: The complete proof of the theorem is lengthy and is skipped here for brevity. A brief outline of the proof can be found in Appendix B.

Based on the result in Theorem 1, the maximum number of users, $\max(N) = P(M, R)$, that can be supported by a R-OOAT system is shown in Fig. 3. When R = 2, P(M, 2) grows linearly with M. When R > 2, P(M, R) grows exponentially with M, with the slope increases as R increases.

The combination of Theorem 1 and Proposition 1 provides a complete set of guidelines for the choice of N, M, and R. For instance, given N and delay (or bandwidth) requirement, we can first find the range of M and R that supports N from Fig. 3, then pick the specific value of M and R from Fig. 2 to minimize the probability of getting an under-determined system.

V. SIMULATION RESULTS

We first investigate the BER performance of a system with M = 12, R = 3, and various numbers of users N. It is assumed in this example that the receiver has ideal knowledge about the relative delays among the users. As can be seen from the figure, there is only a small performance degradation by increasing N



Fig. 3. Maximum number of nodes supported by a CT-MAC system. (N: number of nodes; M: number of sub-symbol positions per symbol; R: number of repetitions per symbol.)

from 1 to 5. The performance of a system with N = 3 is almost identical to that with N = 1, which is interference-free. For comparison, the performance of the system with D-OOAT [1] is also shown in the figure. At BER = 4×10^{-4} , systems with R-OOAT outperform those with D-OOAT by 1.9 dB when N = 3, and by 1 dB when N = 5. In addition, the R-OOAT can support much more simultaneous users compared to a D-OOAT system.

The impact of non-ideal knowledge of the relative user delay, p_{n0} , is considered in Fig. 5 for a system with N = 3, M = 12, and R = 3. Before transmitting the actual information, each node will first transmit a number of pilot slots for the receiver to estimate its relative delay with respect to other users. Each pilot slot contains 100 symbols. The duration of the pilot slots is considered as the training period. It can be seen from the results in Fig. 5 that the performance improves consistently as the training period increases. When the number of pilot slots increases to 10, the performance of the system with estimated delay is almost the same as that of the ideal system.

VI. CONCLUSION

A cross-layer CT-MAC scheme with R-OOAT was proposed for an asynchronous WSN. The collision tolerance in the MAC layer is enabled through physical layer operations, which include a simple R-OOAT scheme at the wireless nodes, and a MLSE detector with time-varying trellis structure at the BS. The design of the R-OOAT scheme was studied by investigating the interactions among the parameters from two perspectives: minimizing the probability that the system is under-determined, and maximizing the number of users that can be supported by a given system. Design guidelines for R-OOAT system were developed based on the analytical results. It was demonstrated through simulations that the proposed CT-MAC with R-OOAT can reliably operate at the presence of severe signal collision.

APPENDIX

A. Proof of Proposition 1

At each sub-symbol index, the probability that a node will transmit a sub-symbol is $\frac{R}{M}$. Without loss of generality, it is assumed that user n transmits at sub-symbol locations m_1, \dots, m_R . At each of the above R sub-symbol locations, the probability that there are exactly (u-1) other nodes colliding with node n follows a binomial distribution,

$$P(N_c(m_k) = u|n) = {\binom{N-1}{u-1}} \frac{R^{u-1}(M-R)^{N-u}}{M^{N-1}}.$$
 (8)

Since $N_c = \max \{N_c(m_k)\}\)$, we have $P(N_c \leq u|n) = P(N_c(m_k) \leq u, k = 1, \cdots, R|n)$, or,

$$P(N_c \le u|n) = \left[\sum_{v=1}^{u} P(N_c(m_k) = v|n)\right]^R,$$
(9)

where it is assumed that $\{N_c(m_k)\}_{k=1}^R$ are independent and identically distributed (i.i.d.), and the assumption can be justified by the node asynchronism and the randomness of the position vectors. Since the above analysis is true for all the N nodes, we can get (4) by combining (8) and (9).

B. An Outline of the Proof of Theorem 1

The outline of the proof relies on the following definition. *Definition 1: Cyclic Order*. The cyclic order of a sequence $\mathbf{q} \in \mathcal{Q}_M^R$ is defined as the smallest integer k that satisfies $\mathbf{q} = (\mathbf{q})_k$.

By shifting a sequence, $\mathbf{p} \in \mathcal{P}_M^R$, to the right by 1 position for M consecutive times, we can get M sequences, and some of the M sequences might be duplicates if the cyclic order of the sequence is less than M. Therefore we can get a total number of P(M, R)M sequences by right shifting all the sequences in \mathcal{Q}_M^R by 1 position for M consecutive times. $\binom{M}{R}$ sequences out of the P(M, R)M sequences are unique, and the remaining $P(M, R)M - \binom{M}{R}$ are duplicates of one of the unique sequences.

If the cyclic order of \mathbf{p} is k, then k out of the M sequences obtained by cyclic shifting \mathbf{p} are unique (part of the $\binom{M}{R}$ sequences), and the remaining M - k sequences are duplicates of one of the k unique sequences.

It can be proved that: 1) the cyclic order of the sequences in \mathcal{Q}_M^R must be in the form of $\frac{M}{c}$, $c \in \mathcal{D}_M^R$; 2) there are totally $V_M^R(c)$ unique sequences in \mathcal{Q}_M^R with cyclic order $\frac{M}{c}$. Then the total number of duplicate sequences from sequences with cyclic order $\frac{M}{c}$ is $V_M^R(c) \left(M - \frac{M}{c}\right)$. The total number of duplicate sequences is thus $\sum_{c \in \mathcal{D}_M^R} V_M^R(c) \left(M - \frac{M}{c}\right)$, which equals to $P(M, R)M - {M \choose R}$, and (6) follows immediately.

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Fig. 4. BER performance of a system with M = 12, R = 3, and ideal knowledge of user delay.



Fig. 5. Impact of training duration on the BER performance of a system with N = 3, M = 12, and R = 3.

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