# Layered Frequency-Domain Turbo Equalization for Single Carrier Broadband MIMO Systems

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***Abstract*— A new layered frequency-domain turbo equalization (LFDTE) scheme is proposed for single carrier (SC) multiple-input multiple-output (MIMO) systems. The proposed scheme combines the respective advantages of layered detection and turbo equalization to further lower the bit error rate (BER) for high-data-rate communications. Different from the traditional frequency-domain turbo equalization (FDTE) method for MIMO systems, our new detection algorithm employs a layered structure in each turbo iteration, and at each layer a group of best data streams is detected in the form of soft symbols and then canceled from the received signals in frequency domain. The extrinsic information for the detected coded bits gleaned by the current layer is fed back to the previous layers as *a-priori* information utilized for equalization in the next iteration. Since the frequency-domain equalization (FDE) is performed in each frequency bin, the proposed receiver scheme has similar computational complexity compared to the traditional FDTE, but achieves better performance of BER. Simulation results demonstrate that the new proposed scheme outperforms the traditional FDTE without layered detection at the same number of iterations, and the performance becomes better if multiple layers are used in the detection.**

## I. INTRODUCTION

Turbo equalization has been demonstrated an effective method [1] to combat inter-symbol interference (ISI) caused by frequency-selective channels. The turbo principle is proceeding by iteratively exchanging soft information between the equalizer and the decoder, and the performance of the turbo equalizer is significantly improved with the number of iterations. However, for systems with high modulation levels and/or long channel delay spread, the maximum a *posteriori* (MAP)-based equalizer and decoder adopted in [1] become prohibitively complex to implement. The time-domain turbo equalizer based on minimum mean square error (MMSE) criterion with low complexity has been proposed in [2], [3] for single-input single-output (SISO) system, and extended to MIMO systems in [4]. To further reduce computational complexity of the equalizer for severe time-dispersive channels, frequency-domain turbo equalizers (FDTE) have been provided in [5], [6] for SISO systems, and recently FDTE for MIMO systems has been developed in [7] with good performance and acceptable complexity.

Layered equalization structure based on succussive interference cancelation (SIC) is an effective detection method to mitigate the effect of co-channel interference (CCI) for MIMO systems. The concept of layered detection, originally introduced by Vertical Bell Laboratories Layered Space-Time (V-BLAST) system for flat fading channels [8], has been extended to frequency-selective channels in [9], [10] by employing time-domain decision feedback equalizer (DFE) or delayed decision feedback sequence estimator (DDFSE) at each layer of detection. The frequency-domain equalization has also been adopted in a layered detection structure [11] to achieve good performance by incorporating a traditional FDE with DFE within the layered structure. However, when the block length and the memory length of feedback filter are large, the layered structure with FDE-DFE becomes computationally expensive due to inversion of large matrix.

To further improve performance of MIMO systems while keeping low complexity, we apply the powerful frequency-domain turbo equalization into the layered detection structure, and propose a new layered frequency-domain turbo equalization (LFDTE) receiver scheme for single carrier MIMO systems. The proposed receiver scheme combines the respective advantages of frequency-domain turbo equalization (FDTE) and layered detection. The benefit of this joint structure is that lower BER can be achieved with similar computational complexity comparing with the traditional FDTE without layered detection, and less number of iterations is required for LFDTE to approach similar performance in contrast to FDTE.

Our work is different from the FDTE proposed in [7] and LSFE in [11] in that a structure with multiple layers is employed in each iteration of turbo equalization, and a group of best data streams in the MMSE sense is selected for tentative detection at each layer. The detected soft symbols, not hard symbols, at an early layer are used to construct the CCI and the canceled from the received signals. The interference-canceled received signals are fed forward to the subsequent layers. The extrinsic information on the coded bits gleaned in the current layer are fed back to previous layers as the *a-priori* information for the next iterative equalization. The reliability of the detected symbols is steadily increased with the number of iterations. The BER performance of the proposed algorithm is investigated by numerical simulation, compared with the existing FDTE without layered structure. Simulation results show that the proposed LFDTE algorithm outperforms the FDTE algorithm at the same number of iterations and the performance is improved gradually when multiple layers are used in the detection.

Throughout the paper, we use $[\cdot]^T$, $[\cdot]^H$, $(\cdot)^{-1}$, and $\mathrm{Tr}\{\cdot\}$ to denote the matrix transpose, Hermitian transpose, inverse, and trace respectively.

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## II. System Model and Preliminaries

Consider a broadband MIMO wireless communication system with $n_t$ transmit antennas and $n_r$ receive antennas. As shown in the transmitter structure depicted in Fig. 1, each antenna packages the data independently and radiates the modulated data stream simultaneously at the same carrier frequency. For the $p$-th antenna, the binary information, $b_p \in \{0, 1\}$, is encoded by a channel encoder and permuted randomly by an interleaver to yield the coded bit stream $\mathbf{c}_p$. The coded bits are then mapped into $2^M$−ary symbols based on a symbol alphabet set $\mathcal{S} = \{\alpha_1, \cdots, \alpha_{2^M}\}$, where $\alpha_m$, $m = 1, \cdots, 2^M$, is a modulation symbol with unit power, corresponding to bit pattern $\mathbf{d}_m = [d_{m,1}, \cdots, d_{m,M}]$. Let $x_p(k)$ denote the $k$-th symbol of the $p$-th antenna, then $x_{p,k} = \alpha_m$ if the coded bit vector $[c_{p,Mk-M+1}, \cdots, c_{p,Mk}] = \mathbf{d}_m$. The modulated symbols are grouped into blocks, and each block consists of $N$ symbols. The cyclic prefix (CP), which is the repetition of the last $N_{cp}$ symbols of the block, is appended before each block to avoid inter-block interference (IBI) and make frequency-domain methods applicable at receivers.

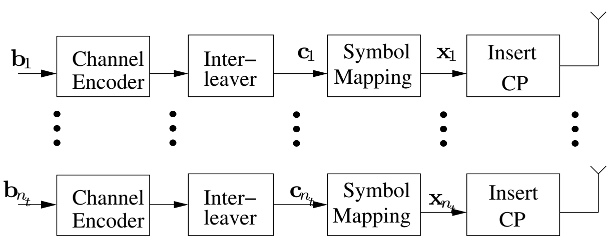


Fig. 1. The MIMO transmitter structure.

The data blocks are transmitted over the frequency-selective channel and distorted by additive white Gaussian noise (AWGN) at the receiver. The baseband equivalent signal received at the $q$-th antenna can be expressed in discrete time domain as

$$y_q(k) = \sum_{p=1}^{n_t} \sum_{l=1}^{L} h_{q,p}(l, k) x_p(k + 1 - l) + w_q(k), \quad (1)$$

where $h_{q,p}(l, k)$ represents the channel impulse response on the $l$-th tap at time $k$, which is the composite of transmit filter, ISI channel and receive filter, and $L$ is the memory length of the channel. Here we assume all the subchannels corresponding to the pairs of transceiver antennas have the same channel length. In general, the channels are time-varying with the instant time, but they can be assumed to be quasi-static within one block if the block duration is less than the channel coherence time. $w_q(k)$ is the white Gaussian noise sampled at the $q$-th receiver antenna with the probability density function (PDF) of $\mathcal{N}(0, \sigma_w^2)$.

Define the received block at the $q$-th antenna after removing CPs as $\mathbf{y}_q = [y_q(1), \cdots, y_q(N)]^T$, the transmitted block by the $p$-th antenna as $\mathbf{x}_p = [x_p(1), \cdots, x_p(N)]^T$, and the noise block as $\mathbf{w}_q = [w_q(1), \cdots, w_q(N)]^T$. The system model in (1) can be represented in matrix format as

$$\begin{bmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_{n_r} \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{1,1} & \cdots & \mathbf{h}_{1,n_t} \\ \vdots & \ddots & \vdots \\ \mathbf{h}_{n_r,1} & \cdots & \mathbf{h}_{n_r,n_t} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_{n_t} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_1 \\ \vdots \\ \mathbf{w}_{n_r} \end{bmatrix}, \quad (2)$$

where $\mathbf{h}_{q,p}$ is the $(q, p)$-th channel matrix given by

$$\mathbf{h}_{q,p} = \begin{bmatrix} h_{q,p}(1) & 0 & \cdots & h_{q,p}(L) & \cdots & h_{q,p}(2) \\ \vdots & \ddots & \ddots & \ddots & \ddots & h_{q,p}(L) \\ h_{q,p}(L) & \ddots & h_{q,p}(1) & 0 & \ddots & 0 \\ 0 & h_{q,p}(L) & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & h_{q,p}(L) & \cdots & \cdots & h_{q,p}(1) \end{bmatrix}. \quad (3)$$

Let $\mathbf{F}$ denote the normalized Discrete Fourier Transform (DFT) matrix of size $N \times N$ whose $(m, n)$-th element is $\frac{1}{\sqrt{N}} e^{-i2\pi(m-1)(n-1)/N}$, and $\mathbf{F}_{n_r} = \mathbf{I}_{n_r} \otimes \mathbf{F}$ represents the block DFT matrix, where $\mathbf{I}_{n_r}$ is the identity matrix of size $n_r \times n_r$, and $\otimes$ denotes the kronecker product for matrices. By applying $\mathbf{F}_{n_r}$ on both sides of (2), the system model can be described in frequency domain which is expressed as

$$\mathbf{Y}_q = \sum_{p=1}^{n_t} \mathbf{H}_{q,p} \cdot \mathbf{X}_p + \mathbf{W}_q, \quad (4)$$

where $\mathbf{Y}_q$, $\mathbf{X}_p$ and $\mathbf{W}_q$ are the DFT representations of $\mathbf{y}_q$, $\mathbf{x}_p$, and $\mathbf{w}_q$, respectively. $\mathbf{H}_{q,p}$ is a diagonal matrix with the frequency response of the $(q, p)$-th channel on its diagonal entries. At the receiver, the CP-removed data blocks are converted into frequency domain by fast Fourier Transform (FFT), and the traditional frequency-domain turbo equalization [7] can be applied to detect the symbols by iteratively exchanging information on the coded bits between the equalizer and the decoder. The data symbols for all transmit antennas are detected simultaneously in the traditional FDTE structure. In section III, we will employ a layered detection structure in the frequency-domain turbo equalization and propose a new receive scheme for MIMO systems.

## III. Layered Frequency-Domain Turbo Equalization

The proposed layered frequency-domain turbo equalization structure is shown in Fig. 2, where only the first two layers are presented for brevity. In each turbo iteration, a structure with multiple layers of detection is applied, and each layer consists of a frequency-domain equalizer, a channel decoder, and a succussive interference canceler (SIC). At each layer, a group of best data streams in the MMSE sense is detected in the form of soft symbols and then canceled from the received signals in the frequency domain. The interference-canceled signals are fed forward to the next layer as the input, and the extrinsic information on coded bits gleaned in the current layer are fed back to the previous layers as the *a-priori* information in the next turbo iteration. The FDE of each layer takes advantage of the *a-priori* information of coded bits to calculate the coefficients of the equalizer. The soft information on the coded bits are usually evaluated by log-likelihood ratio (LLR). In

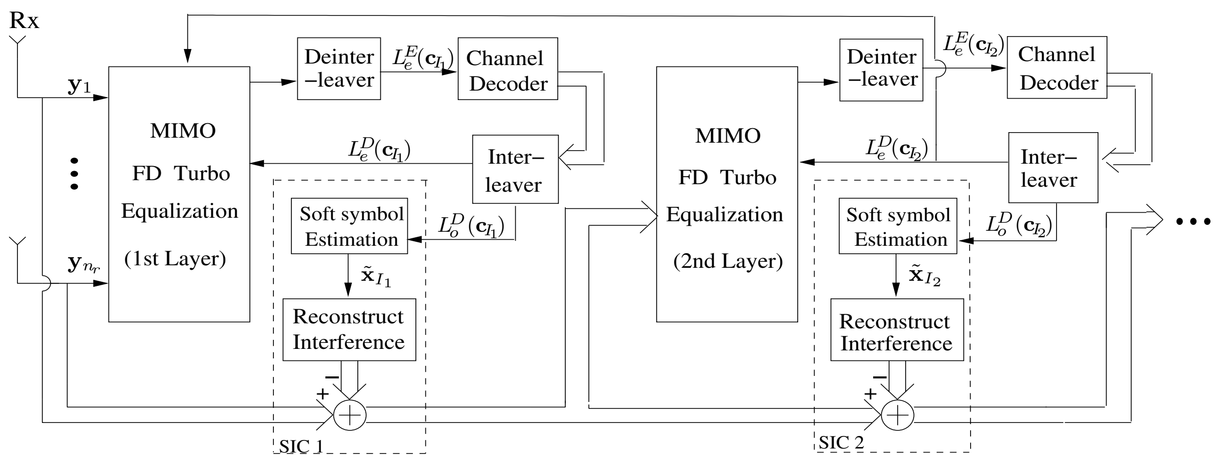


Fig. 2. Layered turbo FDE structure for MIMO systems.

Fig. 2, the extrinsic LLRs generated by the equalizer and the decoder are denoted by $L_e^E(\cdot)$ and $L_e^D(\cdot)$, respectively. The calculation of these extrinsic LLRs is given in [13], which is omitted here for brevity.

### A. Frequency-Domain Turbo Equalization

In the detection of each layer, a frequency-domain turbo equalizer is employed to iteratively equalize the received data and estimate the extrinsic LLR of coded bits based on the equalized symbols. Different from a traditional FDE operating separately from the channel decoder, the FDTE makes use of the *a-priori* information transferred by the decoders. The coefficients of the equalizer are dynamically formulated with the help of the *a-priori* mean and variance of the data symbols. Denote the mean and variance of the symbol $x_p(k)$ as $\mu_{p,k}$ and $\nu_{p,k}$, then $\mu_{p,k}$ can be obtained by

$$\begin{aligned}\mu_{p,k} &= \sum_{\alpha_m \in \mathcal{S}} \alpha_m \cdot P(x_p(k) = \alpha_m) \\ &= \sum_{\alpha_m \in \mathcal{S}} \alpha_m \prod_{j=M(k-1)+1}^{Mk} P(c_{p,j} = d_{m,j'}),\end{aligned} \quad (5)$$

where the probability $P(c_{p,j}{=}d_{m,j'})$ is given by

$$P(c_{p,j}{=}d_{m,j'}){=}\frac{(1-d_{m,j'})e^{L_e^D(c_{p,j})/2}+d_{m,j'}e^{-L_e^D(c_{p,j})/2}}{e^{-L_e^D(c_{p,j})/2}+e^{L_e^D(c_{p,j})/2}}, \quad j'{=}j \bmod M, \quad d_{m,j'} \in \{0,1\}. \quad (6)$$

The variance $\nu_{p,k} = 1 - |\mu_{p,k}|^2$ if the average power of transmit symbols is normalized to 1. For example, a QPSK symbol $\alpha_m$ is selected from an alphabet set $\mathcal{S} = [1, i, -1, -i]$, and its corresponding bit pattern $[d_{m,1}d_{m,2}] \in \{00, 01, 11, 10\}$. The $\mu_{p,k}$ is derived as

$$\begin{aligned}\mu_{p,k} = &\frac{1}{2}\Big(\tanh\big(\frac{1}{2}L_e^D(c_{p,2(k-1)+1})\big)+\tanh\big(\frac{1}{2}L_e^D(c_{p,2k})\big)\Big) \\ &+ i \cdot \frac{1}{2}\Big(\tanh\big(\frac{1}{2}L_e^D(c_{p,2(k-1)+1})\big)-\tanh\big(\frac{1}{2}L_e^D(c_{p,2k})\big)\Big).\end{aligned} \quad (7)$$

By incorporating the *a-priori* information about the symbols to design FDE in the MMSE sense, the estimation of equalized symbols in the frequency and time domain can be represented by the following equations

$$\hat{\mathbf{X}}_{p,k} = K_p^{-1} \cdot \mathbf{U}_p^H \cdot (\mathbf{Y} - \hat{\mathbf{H}} \cdot \bar{\mathbf{X}} + \mu_{p,k}\hat{\mathbf{H}}\mathbf{F}_{n_t}\mathbf{u}_{p,k}), \quad (8)$$

$$\hat{x}_p(k) = \mathbf{F}^H(:,k) \cdot \hat{\mathbf{X}}_{p,k}, \quad (9)$$

where $\mathbf{F}(:,k)$ represents the vector composed by the $k$-th column of the DFT matrix $\mathbf{F}$, $\hat{\mathbf{H}}$ is a block matrix containing the estimated frequency-domain responses of all subchannels whose $(q,p)$-th submatrix is denoted as $\hat{\mathbf{H}}_{q,p}$;$\mathbf{u}_{p,k}$ is an unit vector with length of $Nn_t$, whose $\big((p-1)N+k\big)$-th element is 1 and others are 0; $\bar{\mathbf{X}} = \mathbf{F}_{n_t}[\bar{\mathbf{x}}_1^T, \cdots, \bar{\mathbf{x}}_{n_t}^T]^T$ with $\bar{\mathbf{x}}_p = [\mu_{p,1}, \cdots, \mu_{p,N}]^T$; $K_p = 1 + \frac{1-\bar{\nu}_p}{N}\mathrm{Tr}\{\hat{\mathbf{H}}^H(:,(pN-N+1:pN))\mathbf{U}_p\}$, where $\hat{\mathbf{H}}(:,(pN-N+1:pN))$ denotes the matrix composed of the $(pN-N+1)$-th to $pN$-th columns of $\hat{\mathbf{H}}$, $\bar{\nu}_p = \frac{1}{N}\sum_{k=1}^{k=N}\nu_{p,k}$; the equalizer coefficients $\mathbf{U}_p$ for the $p$-th data stream are given by

$$\mathbf{U}_p = (\sigma^2\mathbf{I}_{Nn_r}+\mathbf{H}\cdot\bar{\mathbf{V}}\cdot\mathbf{H}^H)^{-1}\cdot\mathbf{H}\cdot\begin{bmatrix}\mathbf{O}_{(p-1)N\times N} \\ \mathbf{I}_N \\ \mathbf{O}_{(n_t-p)N\times N}\end{bmatrix}, \quad (10)$$

where $\mathbf{O}_{m\times n}$ represents a zero matrix with size of $m \times n$, and $\bar{\mathbf{V}}$ is a diagonal matrix with size of $Nn_t \times Nn_t$, defined by

$$\bar{\mathbf{V}} = \mathrm{diag}\Big\{\bar{\nu}_1\mathbf{I}_N, \quad \cdots, \quad \bar{\nu}_{n_t}\mathbf{I}_N\Big\}. \quad (11)$$

Once the equalized symbols are obtained, an assumption is made that the equalized symbol is approximately subject to Gaussian distribution given the transmitted symbol [12], which is represented by

$$\hat{x}_p(k) = \rho_p x_p(k) + \eta_p, \quad (12)$$

where $\rho_p$ is a scalar and $\eta_p$ is a complex white Gaussian noise with zero mean and variance $\sigma_p^2$. The conditional probability density function $P(\hat{x}_p(k)|x_p(k) = \alpha_m)$ is represented by

$$P(\hat{x}_p(k)|x_p(k) = \alpha_m) = \frac{1}{\pi\sigma_p^2}\exp\Big(-\frac{|\hat{x}_p(k) - \rho_p\alpha_m|^2}{\sigma_p^2}\Big). \tag{13}$$

By virtue of Gaussian approximation of the equalized symbols, the extrinsic LLR for the coded bits can be calculated [13].

### *B. Layered Structure for Turbo Detection*

We assume that the MIMO receiver consists of $R$ layers (stages), and at the $r$-th layer, there are $\mathbf{J}_r$ candidate data streams remained as the input after the previous $r-1$ layers' detection, and $\mathbf{I}_r$ data streams will be detected as the output. Hence, we have $\sum_{r=1}^{R}\mathbf{I}_r = n_t$, $\mathbf{J}_r = n_t - \sum_{j=1}^{r-1}\mathbf{I}_j$, and $\mathbf{I}_r \leq \mathbf{J}_r$. For the purpose of constructing the interference signals at the $r$-th layer, the soft values of the $\mathbf{I}_r$ branches of symbols and the corresponding channel impulse responses should be estimated, and the generated soft interference signal is canceled out from the received signal. The channel responses can be estimated by traditional pilot-based methods. The soft symbol estimation can be obtained by (5), but with $P(c_{p,j}=0) = \frac{e^{L_o^D(c_{p,j})}}{1+e^{L_o(c_{p,j})}}$ and $P(c_{p,j}=1) = \frac{1}{1+e^{L_o^D(c_{p,j})}}$, in which $L_o^D(c_{p,j})$ is the *a-posteriori* LLR for the coded bit $c_{p,j}$ provided by the decoder. Denote $\tilde{\mathbf{x}}_p = [\tilde{x}_p(1), \cdots, \tilde{x}_p(N)]^T$ as the block of soft symbols of the $p$-th transmit antenna and $\tilde{\mathbf{X}}_p$ as its frequency-domain representation, then the interference-canceled signals at the $r$-th layer can be written in frequency domain as

$$\begin{bmatrix} \mathbf{Y}_1^{r+1} \\ \vdots \\ \mathbf{Y}_{n_r}^{r+1} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_1^{r} \\ \vdots \\ \mathbf{Y}_{n_r}^{r} \end{bmatrix} - \sum_{p'} \hat{\mathbf{H}}_{p'} \cdot \tilde{\mathbf{X}}_{p'}, \quad p' \in \mathcal{D}_{\mathbf{I}_r}, \tag{14}$$

where $\mathbf{Y}_q^r$ and $\mathbf{Y}_q^{r+1}$ represent the input signal at the $q$-th receiver branch for the $r$-th layer and $(r+1)$-th layer, respectively, $\hat{\mathbf{H}}_{p'} = [\hat{\mathbf{H}}_{1,p'}, \cdots, \hat{\mathbf{H}}_{n_r,p'}]^T$, and $\mathcal{D}_{\mathbf{I}_r}$ represents a subset of the detected transmit antennas at the $r$-th layer whose cardinality is $\mathbf{I}_r$.

The interference-canceled received signals at the $r$-th layer are fed forward to the subsequent layer as the input signal. The extrinsic LLRs gleaned at the current layer $L_e^D(\mathbf{c}_{\mathbf{I}_r})$ are sent back to the previous layers for the next iteration of turbo equalization.

## IV. Numerical Results

In this section, we present a numerical example to illustrate our proposed algorithm provides very good performance for fading channels with long delay spread. We considered a MIMO wireless system with 4 transmit antennas and 4 receive antennas. Binary information bits were encoded by a rate-1/2 convolutional encoder with coding generators $(1 + D + D^2 + D^3, 1 + D^2 + D^3)$. The encoded bits were interleaved in a random fashion, and modulated to QPSK and 8PSK symbols with a symbol period $T_s = 0.5\mu s$. The block length $N = 1024$, and modulated data blocks were transmitted over a 20-tap frequency-selective Rayleigh fading channels with an exponential decaying power delay profile. The average power of subchannels for the first and second transmit antenna is four times of the other antennas, which appears unbalanced channel conditions. At the receiver end, the proposed LFDTE structure is employed for iterative detection. The channels were perfectly known at the receiver. BCJR algorithm is used as the decoding scheme, and the FDTE is performed on each single frequency tone for simplicity. The performance of 2 layers and 4 layers detections were investigated in comparison with the 1 layer detection which is actually the traditional FDTE in the same iterations. The soft QPSK and 8PSK symbols of the 4-th antenna for 1 layer detection with 1 iteration are shown in Fig. 3(a) and Fig. 3(c), respectively, and the soft QPSK and 8PSK symbols for 4 layer detection with 3 iterations are shown in Fig. 3(b) and Fig. 3(d). As depicted in Fig. 3(a) and Fig. 3(c), the soft symbols scatter more widely and there is no distinct separation for different modulation symbols. But when multiple layers and more iterations are applied on the detection of symbols, the soft symbols focus on the modulation symbols as shown in Fig. 3(b) and Fig. 3(d), and it is easier to make a decision on the equalized symbols.

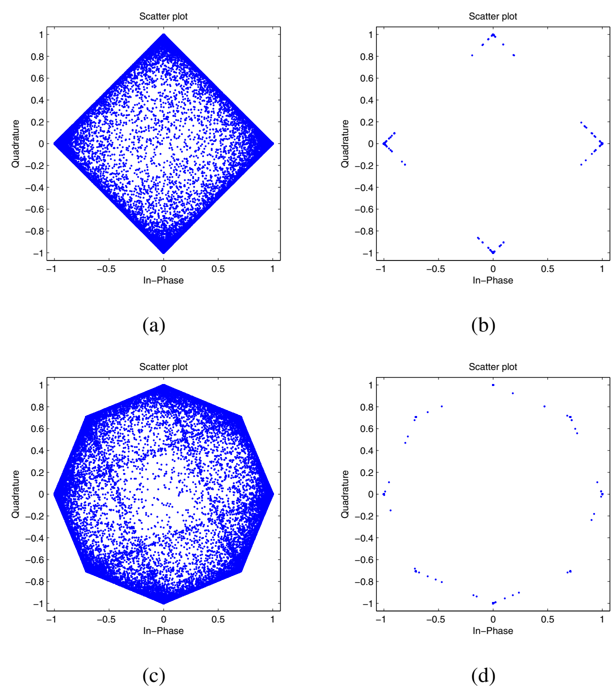


Fig. 3. (a) QPSK soft symbols for 1 layer with 1 iteration at SNR=4 dB. (b) QPSK soft symbols for 4 layer with 3 iterations at SNR=4 dB. (c) 8PSK soft symbols for 1 layer with 1 iteration at SNR=10 dB. (d) 8PSK soft symbols for 4 layer with 3 iterations at SNR=10 dB.

The bit error rates are shown in the Fig. 4 for QPSK and in the Fig. 5 for 8PSK. As shown in these two figures, the performance gain for 2 layers and 4 layers detection is significant if only one iteration is performed, compared with the 1 layer detection. As the iteration proceeds, the gain is decreased, but the performance tends to be better as the number of layers and iterations is increasing. It sufficiently demonstrates that the proposed LFDTE algorithm can provide

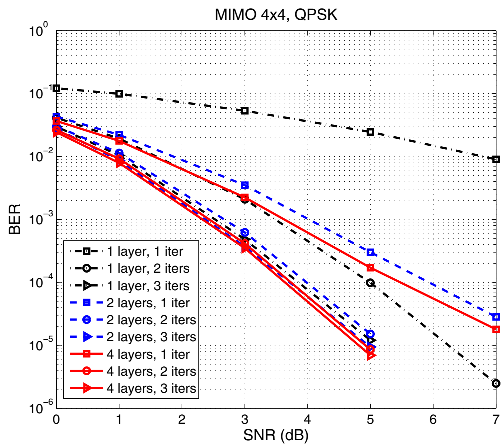


Fig. 4. BER v.s. SNR for FDTE and LFDTE.

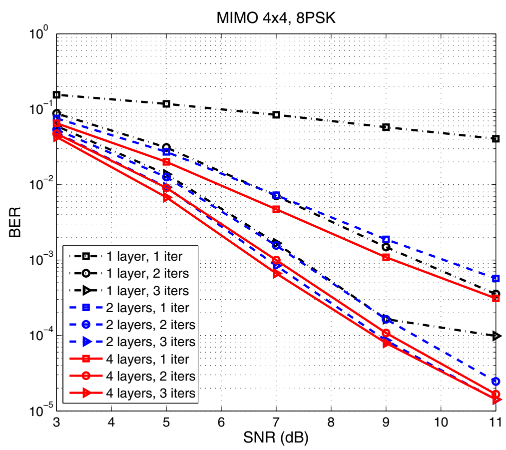


Fig. 5. BER v.s. SNR for FDTE and LFDTE.

better performance than the existing FDTE algorithm for a MIMO communication system over severely time dispersive channels. It is also seen that after 3 iterations, the improvement of performance with multiple layers is not significant. Therefore, given the requirement of system performance and complexity, more layers with less iterations or less layers with more iterations can be employed in the detection to achieve similar performance.

## V. Conclusion

In this paper, we proposed a new low-complexity receiver structure, namely layered frequency-domain turbo equalization (LFDTE) for single carrier MIMO systems. The detection scheme takes advantages of layered detection and turbo equalization, and in each turbo iteration the selected data streams at current layer are detected in terms of soft symbols which are canceled from the received signals. The performance of LFDTE is improved with number of iterations and layers. Through simulations, we conclude that the LFDTE outperforms the traditional FDTE without layer detection for the same iterations, and it achieves better performance over long delay spread channels if multiple layers are used.

## Acknowledgments

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