

Low-Complexity Turbo Block Decision Feedback Equalization for MIMO Systems

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Abstract—This paper proposes a low-complexity turbo block decision feedback equalizer (BDFE) with reliability-based successive soft interference cancellation (SSIC) for multiple-input multiple-output (MIMO) systems. Turbo equalization in MIMO systems needs to combat both intersymbol interference (ISI) due to frequency-selective fading and multiplexing interference (MI) among data streams transmitted by different antennas. The proposed MIMO BDFE performs SSIC on both ISI in the time domain and MI in the space domain. For an SSIC receiver, the order in which the symbols are detected and canceled is critical to the system performance, yet it is often overlooked in the literature. We propose a new reliability-based detection ordering scheme, where symbols with higher *a priori* reliability are detected before those with lower *a priori* reliability. The reliability information can be obtained from the *a priori* probability at the input of the soft-decision equalizer. The reliability-based ordering scheme is enabled by the turbo equalization structure, and it yields an extra equalization gain that is unavailable in a conventional equalization system. Furthermore, the adoption of non-linear BDFE in MIMO systems enables a low-complexity *sequence-based* log-likelihood ratio (LLR) evaluation, which is superior to the *symbol-based* LLR evaluation method employed by most other low-complexity turbo equalizers.

I. INTRODUCTION

Turbo equalization is a powerful receiver technique that improves system performance by iteratively exchanging soft information between a soft-decision equalizer and a soft-decision channel decoder. The optimum turbo equalization was proposed in [1], where the equalizer uses the soft-output Viterbi algorithm (SOVA) and the decoder uses Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm [2]. The computational complexity of SOVA and BCJR grows exponentially with the channel length, L , and the modulation level, Q . The complexity is prohibitively high when L and Q are large.

Various sub-optimal, low-complexity soft-decision equalizers have been developed to trade-off complexity with performance for single-input single-output (SISO) systems [3]–[6], and multiple-input multiple-output (MIMO) systems [7]–[9]. In [3]–[5], a linear minimum mean square error (LMMSE) equalizer is used to replace the trellis-based optimum equalizer for SISO systems, and its extensions to MIMO systems are presented in [7], [8]. The linear equalizers achieve low complexity with symbol-based detection, which is inferior to the trellis-based detection employed by SOVA or BCJR algorithms. In [9], pre-filtering is employed to reduce the number of channel states so that the BCJR-based equalization can be performed with reduced complexity. Recently, a turbo

detection structure employing a soft block decision feedback equalizer (BDFE) has been proposed in [6] for SISO systems. By relying on the non-linear structure of BDFE [10], the turbo equalization in [6] enables a low-complexity *sequence-based* log-likelihood ratio (LLR) calculation leading to near-optimal performance, while achieving similar computational complexity as other low-complexity turbo equalizers.

In this paper, we propose a new BDFE-based turbo equalization scheme for MIMO systems. In a spatially multiplexed MIMO system with frequency-selective fading, the interference among transmitted symbols arises from two sources: the inter-symbol interference (ISI) due to time dispersion of the fading, and the spatial multiplexing interference (MI) among data streams transmitted by different antennas. Therefore, the turbo equalizer for MIMO systems needs to deal with both time-domain ISI and space-domain MI. The proposed MIMO BDFE performs successive soft interference cancellation (SSIC) of both ISI and MI. One of the main performance limiting factors of SSIC is error propagation, where an erroneously detected soft symbol will negatively affect the detection of all the subsequent symbols. Therefore, the order in which the symbols are detected during SSIC is critical to the system performance. To minimize error propagation, we propose a reliability-based detection ordering scheme, where symbols with higher reliability will be detected before those with lower reliability. The knowledge of symbol reliability is obtained from the *a priori* probability at the input of the soft-decision equalizer. The reliability information is a unique byproduct of the turbo equalization process and can be obtained at no extra cost. In addition, the *sequence-based* LLR calculation method proposed in [6] is extended to MIMO systems, which collects more reliability information and thus leads to better equalization performance than the *symbol-based* LLR evaluation methods. The SSIC ordering scheme, combined with the MIMO *sequence-based* LLR method, achieves both low computational complexity and near-optimal performance.

The rest of this paper is organized as follows. Section II presents the MIMO system model and the turbo equalizer structure. The turbo MIMO BDFE with reliability-based detection ordering is developed in Section III. Simulation results are presented in Section IV, and Section V concludes the paper.

II. SYSTEM DESCRIPTION

Consider a MIMO system with N transmit antennas and M receive antennas as shown in Fig. 1. At the transmitter, N bit streams, with $a_{n,k}$ being the k -th bit of the n -th stream, are multiplexed onto the N transmit branches. The n -th bit stream is encoded, interleaved, and modulated, and the outputs of the encoder, interleaver, and symbol mapper (modulator) are denoted by $b_{n,k}$, $c_{n,k}$, and $x_{n,k}$, respectively. For a Q -ary modulation with the constellation set $\mathcal{S} = \{\chi_q\}_{q=1}^Q$, every $\log_2 Q$ coded bits are mapped onto one modulation symbol, *i.e.*, the group of coded bits, $\{c_{n,(k-1)\log_2 Q + p}\}_{p=1}^{\log_2 Q}$, are mapped to the modulation symbol $x_{n,k}$. The modulated symbols on the n -th branch are transmitted by the n -th transmit antenna in the form of length- N_b blocks as, $\mathbf{x}^{(n)} = [x_{n,1}, \dots, x_{n,N_b}]^t \in \mathcal{S}^{N_b \times 1}$, where $(\cdot)^t$ denotes matrix transpose.

At the receiver, the received block at the m -th receive antenna can be written as

$$y_{m,k} = \sum_{l=0}^{L-1} \sum_{n=1}^N f_{m,n}(l) x_{n,k-l} + v_{m,k} \quad \text{for } k = 1, \dots, N_b \quad (1)$$

where $f_{m,n}(l)$ is the l -th tap of the equivalent discrete-time channel between the n -th transmit antenna and the m -th receive antenna, and $v_{m,k}$ is zero-mean additive white Gaussian noise (AWGN) with variance σ_v^2 . It's assumed that the channel is constant over one block, and there is no inter-block interference (IBI) attributing to the insertion of guard intervals among blocks.

Stacking the received samples from all the receive antennas at time k into a vector, we have $\mathbf{y}_k = \sum_{l=0}^{L-1} \mathbf{F}_l \mathbf{x}_{k-l} + \mathbf{v}_k$, where $\mathbf{y}_k = [y_{1,k}, \dots, y_{M,k}]^t \in \mathcal{C}^{M \times 1}$, $\mathbf{x}_{k-l} = [x_{1,k-l}, \dots, x_{N,k-l}]^t \in \mathcal{S}^{N \times 1}$, and $\mathbf{v}_k = [v_{1,k}, \dots, v_{M,k}]^t \in \mathcal{C}^{M \times 1}$ are the received sample vector, transmitted symbol vector, and noise vector, respectively, and the (m,n) -th element of the channel matrix, $\mathbf{F}_l \in \mathcal{C}^{M \times N}$, is $f_{m,n}(l)$. Stacking $\{\mathbf{y}_k\}_{k=1}^{N_b}$ into a column vector, as $\mathbf{y} = [\mathbf{y}_1^t, \mathbf{y}_2^t, \dots, \mathbf{y}_{N_b}^t]^t \in \mathcal{C}^{MN_b \times 1}$, leads to $\mathbf{y} = \mathbf{F}\mathbf{x} + \mathbf{v}$, where $\mathbf{x} = [\mathbf{x}_1^t, \dots, \mathbf{x}_{N_b}^t]^t \in \mathcal{S}^{NN_b \times 1}$, $\mathbf{v} = [\mathbf{v}_1^t, \dots, \mathbf{v}_{N_b}^t]^t \in \mathcal{C}^{MN_b \times 1}$, and

$$\mathbf{F} = \begin{bmatrix} \mathbf{F}_0 & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{F}_{L-1} & \dots & \mathbf{F}_1 & \mathbf{F}_0 & \dots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{F}_{L-1} & \dots & \mathbf{F}_1 & \mathbf{F}_0 \end{bmatrix} \in \mathcal{C}^{MN_b \times NN_b}. \quad (2)$$

The receiver performs detection by using the received vector \mathbf{y} . For a MIMO receiver employing the trellis-based maximum *a posteriori* (MAP) turbo equalizer, the code bit LLR, $\Lambda[c_{n,k}] \triangleq \ln \frac{P[c_{n,k}=0|\mathbf{y}]}{P[c_{n,k}=1|\mathbf{y}]}$, is calculated as

$$\Lambda[c_{n,k}] = \ln \frac{\sum_{\forall c: c_{n,k}=0} p(\mathbf{y}|\mathbf{c}) \prod_{\forall n', k': k' \neq k \text{ if } n' = n} P(c_{n', k'})}{\sum_{\forall c: c_{n,k}=1} p(\mathbf{y}|\mathbf{c}) \prod_{\forall n', k': k' \neq k \text{ if } n' = n} P(c_{n', k'})} + \Lambda_a[c_{n,k}] \quad (3)$$

$\underbrace{\hspace{10em}}_{\Lambda_e[c_{n,k}]}$

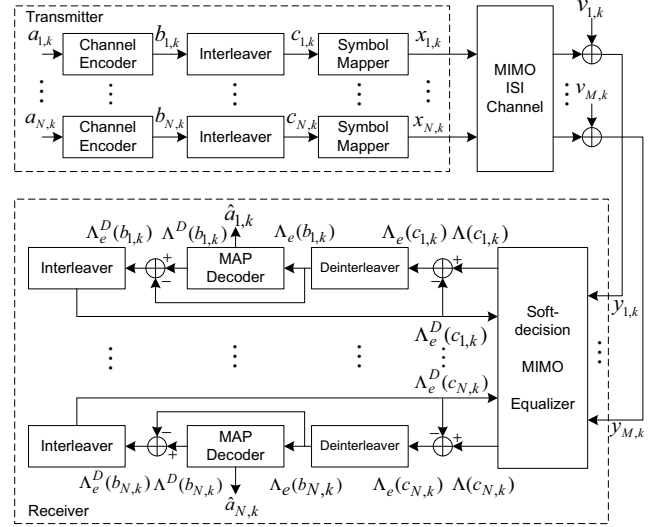


Fig. 1. A MIMO communication system with turbo detection.

where $\Lambda_a[c_{n,k}] = \ln \frac{P[c_{n,k}=0]}{P[c_{n,k}=1]}$ is the *a priori* LLR of $c_{n,k}$, and it can be obtained by interleaving the soft output of the MAP channel decoder from the previous iteration. In the first iteration, there is no *a priori* information, thus $\Lambda_a[c_{n,k}] = 0$.

The extrinsic LLR, $\Lambda_e[c_{n,k}] = \Lambda[c_{n,k}] - \Lambda_a[c_{n,k}]$, at the output of the soft-decision equalizer is deinterleaved, or soft-input, of the MAP channel decoder. The MAP decoder calculates the extrinsic LLR, $\Lambda_e^D[b_{n,k}] = \Lambda^D[b_{n,k}] - \Lambda_e[b_{n,k}]$, for each code bit by exploiting the code trellis structure as well as the *a priori* information $\Lambda_e[b_{n,k}]$. The extrinsic LLR at the output of the channel decoder is interleaved into $\Lambda_e^D[c_{n,k}]$, which is then fed back to the soft-decision equalizer as the *a priori* LLR for the next iteration. Successive iterations are performed to gradually improve the BER performance. In the final iteration, the MAP decoder calculates the LLR $\Lambda^D[a_{n,k}]$ and outputs the corresponding hard decision, $\hat{a}_{n,k}$. It should be noted that the exact calculation of $\Lambda[c_{n,k}]$ in (3) is prohibitively complex given that the complexity is on the order of $\mathcal{O}(Q^{NN_b})$.

III. TURBO MIMO BDFE WITH RELIABILITY-BASED SSIC

A. BDFE with Reliability-based SSIC

In this subsection, we develop the algorithm of turbo MIMO BDFE with reliability-based SSIC, where the symbol reliability information is extracted from the symbol *a priori* probability at the input of the equalizer.

Based on the *a priori* probabilities of the transmitted symbols $\{P(x_{n,k} = \chi_q)\}_{q=1}^Q$, the *a priori* mean and variance of $x_{n,k}$ are calculated, respectively, as

$$\bar{x}_{n,k} = \sum_{q=1}^Q \chi_q P(x_{n,k} = \chi_q) \quad (4a)$$

$$\sigma_{n,k}^2 = \sum_{q=1}^Q (\chi_q - \bar{x}_{n,k})^2 P(x_{n,k} = \chi_q) \quad (4b)$$

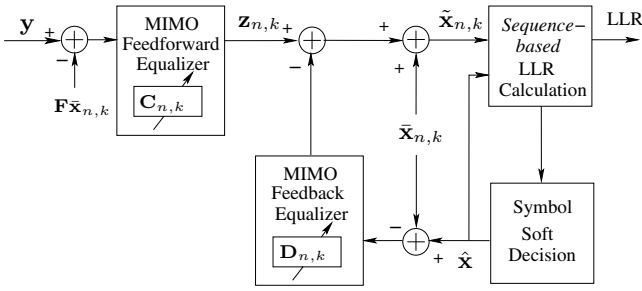


Fig. 2. Soft-decision MIMO BDFE (when the k -th symbol of the n -th stream is equalized.)

where the symbol *a priori* probability, $P(x_{n,k} = \chi_q)$, can be obtained from the *a priori* probabilities of its mapping bits as $P(s_{n,k} = \chi_q) = \prod_{p=1}^{\log_2 Q} P[c_{n,(k-1)\log_2 Q + p} = c_{q,p}]$, where $\{c_{q,p}\}_{p=1}^{\log_2 Q}$ denotes the mapping bits of χ_q . Define the *a priori* mean vector as $\bar{\mathbf{x}} = [\bar{x}_{1,1}, \dots, \bar{x}_{N,1}, \dots, \bar{x}_{1,N_b}, \dots, \bar{x}_{N,N_b}]^t$.

The structure of the MIMO BDFE with SSIC is shown in Fig. 2, where a feedforward filter, $\mathbf{C}_{n,k} \in \mathcal{C}^{NN_b \times MN_b}$, and a zero-diagonal strict upper triangular feedback filter, $\mathbf{D}_{n,k} \in \mathcal{C}^{NN_b \times NN_b}$, are employed, leading to

$$\tilde{\mathbf{x}}_{n,k} = \mathbf{C}_{n,k} (\mathbf{y} - \mathbf{F}\bar{\mathbf{x}}_{n,k}) - \mathbf{D}_{n,k} (\hat{\mathbf{x}} - \bar{\mathbf{x}}_{n,k}) + \bar{\mathbf{x}}_{n,k} \quad (5)$$

where $\bar{\mathbf{x}}_{n,k}$ is obtained by setting $\bar{x}_{n,k} = 0$ in $\bar{\mathbf{x}}$ to avoid positive feedback, and leaving all the other elements unchanged. The interference cancellation is performed with the *a posteriori* tentative soft decision vector $\hat{\mathbf{x}} = [\hat{x}_{1,1}, \dots, \hat{x}_{N,1}, \dots, \hat{x}_{1,N_b}, \dots, \hat{x}_{N,N_b}]^t \in \mathcal{C}^{NN_b \times 1}$ with $\hat{x}_{n,k}$ defined as

$$\hat{x}_{n,k} = \sum_{q=1}^Q \chi_q P(x_{n,k} = \chi_q | \mathbf{y}). \quad (6)$$

Using soft decision during interference cancellation will partly reduce the negative effects of error propagation.

The upper triangular structure of $\mathbf{D}_{n,k}$ determines that the interference cancellation is performed successively by following the reverse order of the elements in \mathbf{x} , *i.e.*, the last element in \mathbf{x} is detected first, and the first element in \mathbf{x} is detected last. With SSIC, if an erroneous or low-quality decision is made for one symbol, the result will negatively affect the performance of the detection of all the subsequent symbols. Therefore, the order in which the symbols are detected during BDFE plays a crucial role on the performance of the entire system. It's desirable to perform detection over symbols that will render a more reliable soft detection earlier during the SSIC process. In traditional hard-decision BDFE, there is no symbol reliability information available before the equalization starts, thus the detection order is usually fixed. On the other hand, with the iterative detection enabled by turbo equalization, the soft-decision BDFE obtains the *a priori* probability of each symbol from the output of the previous iteration, and the reliability information of the symbols can be extracted from the *a priori* probability before the equalization takes place.

Motivated by the *a priori* information that is unique to turbo detection, we propose a new BDFE with reliability-based SSIC, where the symbols will be reordered based on their reliability information, such that the high-reliability symbols will be detected before the low-reliability symbols during SSIC. We propose to measure the symbol reliability by using the symbol *a priori* variance as defined in (4b), and a lower *a priori* variance means a higher reliability. The variance of a random variable (RV) measures the expected squared deviation of the RV from its mean. Since the soft detection is performed with the symbol mean, it is proper to measure the symbol reliability by its symbol variance.

With the analysis above, define the *a priori* reliability as $\rho_{n,k} = 1/\sigma_{n,k}^2$ for the symbol $x_{n,k}$. Sorting $\rho_{n,k}$ in ascending order leads to

$$\boldsymbol{\rho} = [\rho_{\langle 1 \rangle}, \rho_{\langle 2 \rangle}, \dots, \rho_{\langle NN_b \rangle}]^t \quad (7)$$

such that $\rho_{\langle m \rangle} \leq \rho_{\langle n \rangle}$ for $m < n$, and $\langle \cdot \rangle$ is an index mapping operator defined as

$$\langle \cdot \rangle: \{1, \dots, NN_b\} \rightarrow \{(1, 1), \dots, (N, N_b)\} \quad (8)$$

The reliability-based ordering leads to an alternative system representation $\mathbf{y} = \mathbf{F}'\mathbf{x}' + \mathbf{v}$, where $\mathbf{x}' = [x_{\langle 1 \rangle}, x_{\langle 2 \rangle}, \dots, x_{\langle NN_b \rangle}]^t$, $\mathbf{F}' = [\mathbf{f}_{\langle 1 \rangle}, \mathbf{f}_{\langle 2 \rangle}, \dots, \mathbf{f}_{\langle NN_b \rangle}]$, with $\mathbf{f}_{\langle k \rangle}$ being a column of \mathbf{F} .

Define the reordered *a priori* mean vector and *a priori* covariance matrix as

$$\bar{\mathbf{x}}' = [\bar{x}_{\langle 1 \rangle}, \bar{x}_{\langle 2 \rangle}, \dots, \bar{x}_{\langle NN_b \rangle}]^t, \quad (9a)$$

$$\boldsymbol{\Sigma}' = \text{diag} \{ \sigma_{\langle 1 \rangle}^2, \sigma_{\langle 2 \rangle}^2, \dots, \sigma_{\langle NN_b \rangle}^2 \} \quad (9b)$$

where $\text{diag}\{\mathbf{a}\}$ is a diagonal matrix with the elements of \mathbf{a} on its main diagonal. Similar to (5), the MIMO BDFE with reliability-based ordering SSIC for the detection of the g -th symbol in \mathbf{x}' can be written as

$$\tilde{\mathbf{x}}'_g = \mathbf{C}_g (\mathbf{y} - \mathbf{F}'\bar{\mathbf{x}}'_g) - \mathbf{D}_g (\hat{\mathbf{x}}' - \bar{\mathbf{x}}'_g) + \bar{\mathbf{x}}'_g \quad (10)$$

where $\tilde{\mathbf{x}}'_g$, $\bar{\mathbf{x}}'_g$, $\hat{\mathbf{x}}'_g$ follows the same ordering scheme of \mathbf{x}' , and $\bar{\mathbf{x}}'_g$ is obtained by setting the mean of the g -th symbol in $\bar{\mathbf{x}}'$ as zero.

B. Calculation of \mathbf{C}_g and \mathbf{D}_g

With the common assumption of perfect decision feedback, *i.e.*, $\hat{\mathbf{x}}' = \mathbf{x}'$, the error vector of BDFE can be written as

$$\mathbf{e}_g = \mathbf{C}_g \mathbf{y}_g - (\mathbf{D}_g + \mathbf{I}_{NN_b}) (\mathbf{x}' - \bar{\mathbf{x}}'_g) \quad (11)$$

where $\mathbf{y}_g = \mathbf{y} - \mathbf{F}'\bar{\mathbf{x}}'_g$ and \mathbf{I}_{NN_b} is an identity matrix of size NN_b . Minimizing the mean square error $\mathbb{E}[\mathbf{e}_g^h \mathbf{e}_g]$ leads to the solutions of the feedforward matrix and feedback matrix as [6]

$$\mathbf{C}_g = \mathbf{R}_g \boldsymbol{\Sigma}'_g \mathbf{F}'^h [\mathbf{F}'\boldsymbol{\Sigma}'_g \mathbf{F}'^h + \sigma_v^2 \mathbf{I}_{NN_b}]^{-1}, \quad (12a)$$

$$\mathbf{D}_g = \mathbf{R}_g - \mathbf{I}_{NN_b} \quad (12b)$$

where $\boldsymbol{\Sigma}'_g$ is obtained by setting $\sigma_{\langle g \rangle}^2 = 1$ in $\boldsymbol{\Sigma}'$ while leaving all the other elements unchanged, $\mathbf{R}_g \in \mathcal{C}^{NN_b \times NN_b}$

is a unit-diagonal upper triangular matrix obtained from the Cholesky decomposition $\Sigma_g'^{-1} + \frac{1}{\sigma_v^2} \mathbf{F}'^h \mathbf{F}' = \mathbf{R}_g^h \Delta_g \mathbf{R}_g$, and $\Delta_g \in \mathcal{C}^{NN_b \times NN_b}$ is a diagonal matrix. With the equalizer matrices in (12), an equivalent system model is obtained as (c.f. (11))

$$\mathbf{z}_g \triangleq \mathbf{C}_g \mathbf{y}_g = \mathbf{R}_g (\mathbf{x}' - \bar{\mathbf{x}}_g') + \mathbf{e}_g. \quad (13)$$

C. Calculation of Extrinsic LLR based on Equalized Symbols

The equivalent MIMO system representation in (13) enables a *sequence-based* LLR calculation, where the individual symbol, $x_{\langle g \rangle}$, can be detected by gleaning information from the entire block \mathbf{z}_g . In the i -th iteration of the turbo equalization, the *sequence-based a posteriori* probability (APP) of $x_{\langle g \rangle}^{(i)}$ conditioned on \mathbf{z}_g is

$$P(x_{\langle g \rangle}^{(i)} | \mathbf{z}_g) = \frac{p(\mathbf{z}_g | x_{\langle g \rangle}^{(i)}) P(x_{\langle g \rangle}^{(i)})}{p(\mathbf{z}_g)}. \quad (14)$$

The conditional probability $p(\mathbf{z}_g | x_{\langle g \rangle}^{(i)})$ can be expressed as

$$p(\mathbf{z}_g | x_{\langle g \rangle}^{(i)}) = p(\mathbf{z}_g | \hat{\mathbf{x}}_g^{(i)}) \quad (15)$$

with

$$\hat{\mathbf{x}}_g^{(i)} = [\hat{x}_{\langle 1 \rangle}^{(i-1)}, \dots, \hat{x}_{\langle g-1 \rangle}^{(i-1)}, x_{\langle g \rangle}^{(i)}, \hat{x}_{\langle g+1 \rangle}^{(i)}, \dots, \hat{x}_{\langle NN_b \rangle}^{(i)}] \quad (16)$$

where $\hat{x}_{\langle p \rangle}^{(i-1)}$ and $\hat{x}_{\langle p \rangle}^{(i)}$ denote the tentative soft decisions from the $(i-1)$ -th and i -th iteration, respectively. The upper triangular structure of \mathbf{R}_g determines that the symbol $x_{\langle p \rangle}$ is detected before $x_{\langle g \rangle}$ with $g < p$, and thus the tentative detections of those symbols from the current iteration can be used during the detection of $x_{\langle g \rangle}$ as illustrated in (16). The equality in (15) holds because the tentative soft decisions of symbols other than $x_{\langle g \rangle}$ are deterministic during the detection of $x_{\langle g \rangle}$.

Based on the approximation that \mathbf{e}_g in (13) is a random vector with independent Gaussian elements, we have

$$p(\mathbf{z}_g | x_{\langle g \rangle}^{(i)}) \approx \prod_{j=1}^{NN_b} \frac{1}{\pi \alpha_g^j} \exp \left\{ -\frac{|\beta_g^j|^2}{(\alpha_g^j)^2} \right\} \quad (17)$$

where $(\alpha_g^j)^2 = \delta_{j,j}^{-1}$ with $\delta_{j,j}$ being the j -th diagonal element of Δ_g , and β_g^j is defined in (18) at the top of the next page. In (18), $r_{j,l}$ is the (j,l) -th element of \mathbf{R}_g , and z_g^j is the j -th element of \mathbf{z}_g . During the calculation of (17), we only need to consider $j \in [1, g]$ since the terms β_g^j with $j > g$ are independent of $x_{\langle g \rangle}^{(i)}$.

To determine (14), the symbol *a priori* probability, $P(x_{\langle g \rangle}^{(i)})$, can be calculated based on the *a priori* probabilities of its composing bits, and are initialized as $P(x_{\langle g \rangle}^{(i)}) = \frac{1}{Q}$ during the first iteration. The value of $p(\mathbf{z}_g)$ can be obtained by using the normalization $\sum_{q=1}^Q P(x_{\langle g \rangle}^{(i)} = \chi_q | \mathbf{z}_g) = 1$. Once the symbol APP in (14) is determined, the APP of the mapping bits can also be calculated. Denote the mapped double index of $\langle g \rangle$ as (n_g, k_g) , then the APP of the

mapping bits, $\left\{ c_{n_g, (k_g-1) \log_2 Q + p}^{(i)} \right\}_{p=1}^{\log_2 Q}$, of symbol $x_{\langle g \rangle}^{(i)}$ can be evaluated as

$$P \left[c_{n_g, (k_g-1) \log_2 Q + p}^{(i)} = b | \mathbf{z}_g \right] = \sum_{x_{\langle g \rangle}^{(i)} \in \mathcal{S}_p^{(b)}} P(x_{\langle g \rangle}^{(i)} | \mathbf{z}_g) \quad (19)$$

where $\mathcal{S}_p^{(b)} \triangleq \{\chi_q | \chi_q \in \mathcal{S} : c_{q,p} = b\}$ with $b \in \{0, 1\}$, for $p = 1, \dots, \log_2 Q$. The LLR of code bit can then be calculated as

$$\Lambda \left[c_{n_g, (k_g-1) \log_2 Q + p}^{(i)} \right] = \ln \frac{\sum_{x_{\langle g \rangle}^{(i)} \in \mathcal{S}_p^{(0)}} P(x_{\langle g \rangle}^{(i)} | \mathbf{z}_g)}{\sum_{x_{\langle g \rangle}^{(i)} \in \mathcal{S}_p^{(1)}} P(x_{\langle g \rangle}^{(i)} | \mathbf{z}_g)}. \quad (20)$$

The extrinsic LLR of the code bit is obtained by subtracting the *a priori* LLR $\Lambda_a \left[c_{n_g, (k_g-1) \log_2 Q + p}^{(i)} \right]$ out of (20). Finally, since it is computationally expensive to calculate $P(x_{\langle g \rangle}^{(i)} | \mathbf{y})$, we use $P(x_{\langle g \rangle}^{(i)} | \mathbf{z}_g)$ for evaluating the tentative soft decision $\hat{x}_{\langle g \rangle}^{(i)}$ in (6) during the equalization process.

D. Low-complexity Approximate Soft-decision MIMO BDFE

One of the main computational burdens of the proposed algorithm comes from the calculation of the equalizer matrices in (12a) and (12b), which need to be updated for each symbol. The updating of the matrices \mathbf{C}_g and \mathbf{D}_g involves two matrix inversions and one Cholesky decomposition. A closer observation reveals that the symbol-wise filter updating is solely due to the dependence on the second-order *a priori* information Σ_g' . Therefore, computational complexity can be significantly reduced by replacing Σ_g' with the constant autocovariance matrix Σ' , which leads to the same filter matrices, \mathbf{C} and \mathbf{D} , for all the symbols to be equalized. It is demonstrated through simulations that employing constant equalizer matrices for all the symbols doesn't apparently degrade the equalization performance.

IV. SIMULATION RESULTS

Simulation results are presented in this section to demonstrate the performance of the proposed turbo MIMO BDFE detection scheme. We study a 2×2 MIMO system with quadrature phase shift keying (QPSK) modulation. The data is transmitted in packets. Each packet carries $N_{\text{pkt}} = 10,000$ information bits, which are encoded by a rate 1/2 non-systematic convolutional code with a constraint length 4 and a generator polynomial $[G_1, G_2] = [17, 13]_{\text{oct}}$. The encoded packet is further divided into blocks of size $N_b = 100$. The soft-decision MIMO BDFE is operated over blocks, and the MAP decoder is performed over an entire packet. The frequency-selective Rayleigh fading has $L = 10$ symbol-spaced equal-power channel taps, with normalized energy of one. The MIMO channel is constant within one transmitted packet, and changes from packet to packet. For each SNR value, 30 packets are simulated for the BER calculation.

The BER curves for the two data streams transmitted by the two transmit antennas are plotted in Figs. 3 and 4, respectively. The performances of turbo MIMO BDFE with

$$\beta_g^j = \begin{cases} z_g^j - \sum_{l=j}^{NN_b} r_{j,l} \left[\hat{x}_{\langle l \rangle}^{(i)} - \bar{x}_{\langle l \rangle} \right], & g < j \leq NN_b, \\ z_g^j - r_{g,g} x_{\langle g \rangle}^{(i)} - \sum_{l=g+1}^{NN_b} r_{g,l} \left[\hat{x}_{\langle l \rangle}^{(i)} - \bar{x}_{\langle l \rangle} \right], & j = g, \\ z_g^j - r_{j,g} x_{\langle g \rangle}^{(i)} - \sum_{l=j}^{g-1} r_{j,l} \left[\hat{x}_{\langle l \rangle}^{(i-1)} - \bar{x}_{\langle l \rangle} \right] - \sum_{l=g+1}^{NN_b} r_{j,l} \left[\hat{x}_{\langle l \rangle}^{(i)} - \bar{x}_{\langle l \rangle} \right], & 1 \leq j < g. \end{cases} \quad (18)$$

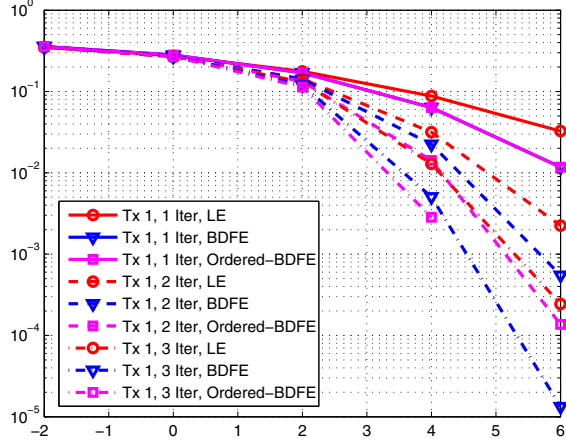


Fig. 3. 2×2 MIMO: BER for Tx 1

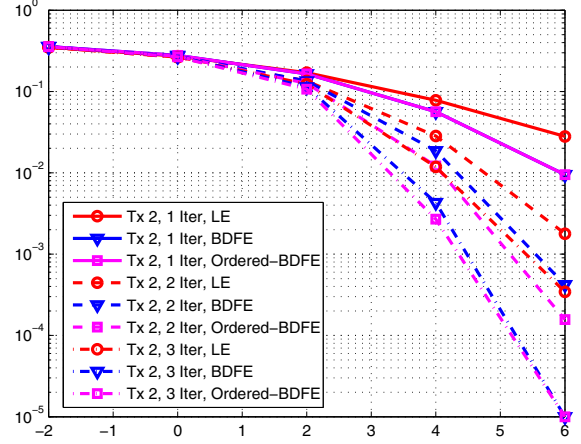


Fig. 4. 2×2 MIMO: BER for Tx 2

and without reliability-based SSIC are compared to turbo MIMO linear equalizer (LE) [7]. We have three observations on the results. First, as expected, the BER improves consistently as the iterations progress in all the three cases. Second, adopting the reliability-based ordering scheme during SSIC leads to faster converging speed and better BER performances compared to systems with a fixed symbol detection order. It is noted that in the first iteration, there is no *a priori* reliability information available (thus no need for ordering), so the two BDFE BER curves overlap. Third, both turbo MIMO BDFE schemes outperform the turbo MIMO LE, attributing to the *sequence-based* LLR evaluation during the equalization.

V. CONCLUSION

A low-complexity soft block decision feedback equalizer with reliability-based successive soft interference cancellation was proposed for turbo detection in MIMO systems. Compared to existing low-complexity turbo detection schemes, the proposed scheme has two advantages. First, the reliability-based SSIC detects symbols with higher reliability before those with lower reliability, leading to better interference cancellation with less error propagation. Second, the adoption of the non-linear BDFE enables a near-optimum *sequence-based* LLR evaluation scheme that yields better performance compared to *symbol-based* LLR evaluations. It was demonstrated through simulations that the reliability-based detection ordering for SSIC yields a turbo equalizer with a better BER performance and a faster converging rate compared to systems with fixed detection order.

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