ULTRA-LOW POWER COMPRESSIVE WIRELESS SENSING FOR DISTRIBUTED WIRELESS NETWORKS

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***Abstract*—Wireless sensor networks (WSNs) developed for the monitoring of critical military or civilian infrastructures are expected to have long life cycle with ultra-low power consumption. An ultra-low power wireless sensing scheme is developed by exploiting the unique features of infrastructure monitoring systems, which usually have long latency tolerance, low data rate, and strong correlation among data collected by spatially distributed sensors. The wireless sensor nodes asynchronously transmit measured data through a new exponential-interval media access control (EI-MAC) scheme, which can asymptotically almost surely (a.a.s.) achieve collision-free communication by leveraging on the long latency tolerance and low data rate of the system. Two low power sensing schemes, namely, compressive detection (CD) and compressive transmission (CT), are proposed in recognition of the strong correlation among data samples collected by n spatially distributed sensing nodes. Both the two schemes are fully scalable; have ultra-low power consumption; have less distortion compared to conventional schemes; and allow the sensing nodes to operate asynchronously without central control. Theoretical analysis shows that the normalized mean square distortion of the recovered information scales as $\mathcal{O}(\frac{\log n}{n})$.*

I. Introduction

Wireless sensor networks (WSNs) developed for the monitoring of critical military or civilian infrastructures are endowed with many unique features that are not available in conventional wireless networks. Many of the infrastructures, such as bridges, tunnels, and buildings, have extremely long life cycle in the order of years or decades, with very slow changing rates. As a result, infrastructure monitoring systems have long latency tolerance with ultra-low data rate. In addition, data collected in real world often contain redundancies due to the spatial correlation inherent in the monitored object(s). The redundancy/correlation can be used to facilitate the design of infrastructure monitoring systems.

In this paper, we propose to design ultra-low power, high fidelity wireless sensing schemes by exploiting the unique features of infrastructure monitoring systems. Two wireless sensing schemes, namely, compressive transmission (CT), and compressive detection (CD), are proposed in this paper. In CT, data collected by n sensing nodes are first transmitted asynchronously to one or more compressing nodes, where the redundancy in the data is partly removed by projecting the n-dimension data onto a subspace (transform domain) with dimension $k \ll n$. The k transform domain coefficients are then delivered to a fusion center (FC) to recover the original information. In CD, data from n sensing nodes are directly delivered to a FC, where the information is reconstructed by projecting the received signal onto a k-dimension subspace. It's demonstrated through asymptotic analysis that, for both of the two schemes, if the total transmission power is fixed, and if the inter-sensor distance is much smaller compared to the sensor-FC distance, the optimum value of the transform domain dimension, k, scales with $\log n$, and the normalized mean square distortion (NMSD) of the recovered information scales with $\frac{\log n}{n}$.

The proper operation of the CT and CD schemes requires an effective media access control (MAC) protocol to coordinate the transmission of the spatially distributed sensing nodes. In recognition of the long latency tolerance and low data rate of infrastructure monitoring systems, a new exponential-interval MAC (EI-MAC) scheme is proposed. The EI-MAC scheme results in an extremely low duty cycle for the sensing nodes, and can asymptotically almost surely (a.a.s.) achieve a collision-free communication. The combination of EI-MAC and CT/CD schemes forms an efficient cross-layer infrastructure monitoring system, which is fully scalable, has ultra-low power consumption with high fidelity, and allows spatially distributed sensing nodes to operate asynchronously without central control.

A. Related Works

There are limited works in the literature on the development of ultra-low power wireless communications. In [1], an ultra-low power MAC protocol is proposed by employing a periodic preamble sampling scheme to

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reduce the idle listening time of the sensor nodes. The performance of the MAC protocol of IEEE 802.15.4 [2], a standard for low power/low rate personal area networks (PAN), is investigated in [3]. The above MAC protocols are designed for real time communications with data rates in the order of hundreds of kilo-bits per second, which are much higher than the targeted data rate for infrastructure monitoring system. In [4] – [5], various hardware structures and low power integrated circuits (IC) are proposed for low power communications. The hardware structures are designed from the perspectives of reducing current leakage, more efficient low noise amplifier (LNA), etc. They do not directly take advantage of the unique features of infrastructure monitoring systems.

The newly proposed CT and CD schemes are motivated in part by the recent results on compressive sensing [6], which demonstrate that a small number of random projections of a sparse signal can retain most its salient information. A compressive wireless sensing scheme is proposed in [7], where each sensor transmits a scaled version of its measured data, and the superposition of all the data samples forms a projection onto a transform domain at the FC. The scheme in [7] requires perfect node synchronization, which is extremely difficult to achieve in a spatially distributed network.

B. Notations

The following notations are used in this paper. $\mathbf{a}^T$ denotes the transpose of a column vector $\mathbf{a}$. The l_m-norm of an n-dimension vector $\mathbf{a}$ is defined as $\|\mathbf{a}\|_{l_m} = \left(\sum_{i=1}^n |a_i|^m\right)^{1/m}$. $\mathbb{E}(\cdot)$ denotes mathematical expectation. $a_n = \mathcal{O}(b_n)$ if and only if there exists $0 < M < \infty$ and $n_0 > 0$ such that $|a_n| \leq M|b_n|$ for all $n > n_0$. $a_n = \mathcal{O}(b_n)$ is also denoted as $a_n \preceq b_n$. $a_n \sim b_n$ if and only if both $a_n \preceq b_n$ and $b_n \preceq a_n$.

II. System Model

Consider a wireless sensor network with n sensing nodes uniformly distributed over an area $\mathcal{V}$. The data collected by the i-th node can be modeled as

$$x_i = s_i + w_i, \quad \text{for} \quad i = 1, \cdots, n, \tag{1}$$

where s_i is the desired data related to the location of the i-th node, and the measurement noise, $\{w_i\}_{i=1}^n$, are independently and identically distributed (i.i.d.) random variables (RVs) with zero mean and variance σ_w^2. The average power of one data sample is normalized to unity, *i.e.*, $\frac{1}{n}\mathbb{E}\|\mathbf{s}\|_{l_2}^2 = 1$, where $\mathbf{s} = [s_1, s_2, \cdots, s_n]^T \in \mathcal{R}^{n\times 1}$ is the data vector.

Data in real world often contain redundancies. Therefore, it's reasonable to assume that the data collected by the nodes in a dense wireless network are correlated, and thus are compressible. The data vector is defined as compressible if it can be approximated by a linear combination of $k < n$ orthonormal vectors. Define the k-approximation of $\mathbf{s}$ as

$$\mathbf{s}^{(k)} = \sum_{i=1}^k \theta_i \phi_i. \tag{2}$$

where $\phi_i \in \mathcal{R}^{n\times 1}$, for $i = 1, \cdots, n$, is an orthonormal basis of $\mathcal{R}^n$, and $\theta_i = \mathbf{s}^T\phi_i$ is the projection of $\mathbf{s}$ onto ϕ_i. Without loss of generality, $\{\phi_i\}_{i=1}^n$ is labeled in a way such that $|\theta_1| \geq |\theta_2| \geq \cdots \geq |\theta_n|$.

Definition 1: The data vector $\mathbf{s}$ is defined as β-compressible if its k-approximation satisfies

$$\frac{1}{n}\mathbb{E}\left[\|\mathbf{s} - \mathbf{s}^{(k)}\|_{l_2}^2\right] = \mathcal{O}\left(\xi e^{-\beta k}\right), \tag{3}$$

where the scaling parameters, ξ and β, depend on node density and the nature of the measured data. ■

The measured data will be directly or indirectly transmitted to a FC, where the data will be recovered. Based on a distorted observation of the measured data, the FC obtains an estimate $\hat{\mathbf{s}}$ of $\mathbf{s}$ by minimizing the NMSD, $D = \frac{1}{n}\mathbb{E}\left[\|\mathbf{s} - \hat{\mathbf{s}}\|_{l_2}^2\right]$.

Communication can occur among sensing nodes, or between sensing nodes and a FC. The communication process is assumed to be completely asynchronous, *i.e.*, there is no coordination among nodes regarding the starting time of a transmission. In this case, signals transmitted by two or more nodes might overlap at the receiver, and this leads to collision. Collisions result in data loss and waste of precious transmission power. A simple EI-MAC protocol is proposed to reduce the probability of collision, thus the power consumption.

Definition 2: EI-MAC protocol. Each node transmits at random, with the interval between two consecutive transmission attempts following an independent exponential distribution with mean $\frac{1}{\lambda}$. The message has a fixed length of τ. The average duty cycle of a node, $\lambda\tau$, satisfies $\lambda\tau \sim \frac{1}{n\log n}$, with n being the number of nodes. The transmission schedules of different nodes are independent. In case of a collision, the message will be discarded and no retransmission will be attempted. ■

One important power saving feature of EI-MAC is that no retransmission will be attempted in case of collision. Since the monitored object changes very slowly, there is a strong time domain correlation among packets transmitted by the same sensor. In case a packet from a sensor is lost, the FC can obtain a reasonable estimate of the lost packet through the packets from the same sensor but received in previous rounds. Thus the loss of a very small percentage of packets won't significantly affect the integrity of the reconstructed information. In addition, the following Lemma shows that EI-MAC can actually a.a.s. achieve collision-free communication.

Lemma 1: For the EI-MAC protocol with parameters λ and τ as defined above, the probability of collision tends to 0 as $n \to \infty$.

Proof: When n is large, the duration of a message, $\tau \sim \frac{1}{n \log n} \frac{1}{\lambda}$, is negligible compared to the average transmission interval, $\frac{1}{\lambda}$. With an exponential interval between two consecutive transmissions, the transmission schedule for any given node can be modeled as a Poisson process with arrival rate λ [8]. Since the transmission schedules of different nodes are independent, the combined transmission schedule of all the n nodes is a Poisson process with arrival rate $n\lambda$. As a result, the interval between any consecutive transmissions from all nodes follows an independent exponential distribution with mean $\frac{1}{n\lambda}$. The probability of no collision is equivalent to the probability that the interval between any two consecutive transmissions is larger than τ, and it can be calculated as

$$P_n = \int_{\tau}^{\infty} f_n(x) dx = e^{-n\lambda\tau}, \quad (4)$$

where $f_n(x) = n\lambda e^{-n\lambda x}$ is the probability density function (pdf) of the transmission interval for a system with n nodes. With $\lambda\tau \sim \frac{1}{n \log n}$, it can be easily shown that the asymptotic probability of no collision is, $P_c = 1 - \lim_{n\to\infty} P_n = 0$. ■

The result in Lemma 1 indicates that the communications in the wireless sensor network can be carried out asynchronously with a collision probability arbitrarily small, thanks to the small duty cycle made possible by the long delay tolerance and low data rate of the infrastructure monitoring system.

III. Asynchronous Compressive Wireless Sensing

Two compressive wireless sensing schemes, compressive detection and compressive transmission, are proposed in this section to achieve ultra-low power high fidelity wireless sensing. To facilitate the analysis, define the radius of the monitored area, $\mathcal{V}$, as $\rho = \inf_{\mathbf{v}_0 \in \mathcal{V}} \sup_{\mathbf{v} \in \mathcal{V}} \|\mathbf{v} - \mathbf{v}_0\|_{l_2}$. The value of $\mathbf{v}_0 \in \mathcal{V}$ that leads to the infimum of $\sup_{\mathbf{v} \in \mathcal{V}} \|\mathbf{v} - \mathbf{v}_0\|_{l_2}$ is defined as the center of $\mathcal{V}$. Denote the distance between the center of $\mathcal{V}$ and the FC as l. It's assumed that $l \gg \rho$, such that all the sensing nodes have approximately the same distance, l, to the FC.

A. Direct Sensing

Before proceeding to the discussion of the new compressive sensing schemes, we first present the results of a direct sensing (DS) scheme, where all the sensing nodes directly transmit the measured data to the FC, and no compression is performed during the transmission or detection process. The results will be used as a benchmark for the evaluation of the new schemes.

If the average power budget of a node is $\bar{P}_i$, with duty cycle $\lambda\tau$, the instantaneous power assumed by a node during actual transmission is thus $P_i = \frac{\bar{P}_i}{\lambda\tau} \sim n \log n \bar{P}_i$. To ensure the system is scalable, it's assumed that the total transmission power, P_t, of all the nodes is fixed. Thus $\bar{P}_i \sim \frac{P_t}{n}$. Correspondingly, the instantaneous transmission power of the i-th sensing node satisfies $P_i \sim \log n \cdot P_t$.

The data collected by the FC of a system employing EI-MAC can be represented as

$$\mathbf{y}_c = \sqrt{\frac{P_t \log n}{l^\alpha}} (\mathbf{s} + \mathbf{w}) + \mathbf{z} \quad (5)$$

where $\mathbf{y}_c = [y_{c1}, \cdots, y_{cn}]^T$, $\mathbf{s} = [s_1, \cdots, s_n]^T$, $\mathbf{w} = [w_1, \cdots, w_n]^T$, and $\mathbf{z} = [z_1, \cdots, z_n]^T$ are size $n \times 1$ vectors containing received samples, data samples, sensing noise, and additive white Gaussian noise (AWGN), respectively, and α is the pathloss exponent. The covariance matrix of $\mathbf{z}$ is $\sigma_z^2 \mathbf{I}_n$, with $\mathbf{I}_n$ being a size $n \times n$ identity matrix.

In DS, the FC performs estimation of the data vector with the least squares (LS) method as, $\hat{\mathbf{s}} = \sqrt{\frac{l^\alpha}{P_t \log n}} \mathbf{y}_c$, and the corresponding NMSD is

$$D_{\text{DS}} = \frac{1}{n} \mathbb{E}\left[\|\mathbf{s} - \hat{\mathbf{s}}\|_{l_2}^2\right] = \sigma_w^2 + \frac{1}{\log n} \frac{l^\alpha \sigma_z^2}{P_t} \sim \sigma_w^2. \quad (6)$$

The results in (6) indicate that the NMSD for DS is lower bounded by the sensing noise variance, σ_w^2.

B. Compressive Detection

The data vector is compressible in the sense that it can be well approximated by $k < n$ transform domain coefficients as described in (2) and (3). Motivated by this fact, we propose to perform estimation of the transform domain coefficients, $\{\theta_i\}_{i=1}^k$, instead of directly estimating the original data vector, s, at the FC. This method is denoted as CD since the compressible feature of the data is utilized during the detection, and the transmission process is the same as that of the DS scheme. It's assumed that the FC has knowledge of the transform domain basis, which can be obtained by analyzing the statistical properties of the received data samples.

Upon receiving the data vector $\mathbf{y}_c$ as given in (5), the FC performs LS estimation of the transform domain coefficients, $\{\theta_i\}_{i=1}^k$, by projecting the received sample vector onto the first k transform domain bases. The estimated data vector can then be written as

$$\begin{aligned}\hat{\mathbf{s}}^{(k)} &= \sqrt{\frac{l^\alpha}{P_t \log n}} \sum_{i=1}^{k} \left(\mathbf{y}_c^T \phi_i\right) \phi_i, \\ &= \mathbf{s}^{(k)} + \sum_{i=1}^{k} \tilde{w}_i \phi_i + \sqrt{\frac{l^\alpha}{P_t \log n}} \sum_{i=1}^{k} \tilde{z}_i \phi_i, \quad (7)\end{aligned}$$

where $\tilde{w}_i = \mathbf{w}^T \phi_i$, $\tilde{z}_i = \mathbf{z}^T \phi_i$. Since $\phi_i \phi_i^T = \mathbf{I}_n$, the variances of $\tilde{w}_i$ and $\tilde{z}_i$ remain σ_w^2 and σ_z^2, respectively.

From (3) and (7), the NMSD of the CD scheme, $D_{\text{CD}} = \frac{1}{n}\mathbb{E}\|\mathbf{s} - \hat{\mathbf{s}}^{(k)}\|_{l_2}^2$, can be written as

$$D_{\text{CD}} = \mathcal{O}\left(\xi e^{-\beta k}\right) + \frac{k}{n}\sigma_w^2 + \frac{k}{n \log n}\delta_z, \quad (8)$$

where $\delta_z = \frac{l^\alpha \sigma_z^2}{P_t}$. In (8), the first term, which is due to the β-compressibility of the data vector, decreases with k, yet the second and third terms, which are due to the sensing and transmission noises, increase with k.

Next we will find the optimum value of k that minimizes D_{CD}, and establish the asymptotic behavior of D_{CD} as $n \to \infty$. Before moving on to the main results, we first present the following Lemma, which will be used during the asymptotic analysis.

Lemma 2: Consider two real valued sequences, $\{\lambda_n\}$ and $\{\mu_n\}$. If $\lambda_n \sim \mu_n$ and $0 < \mu_n < 1$ for all $n > n_0 > 0$, then (i) $\lambda_n^{-1} \sim \mu_n^{-1}$; (ii) $\log \lambda_n^{-1} \sim \log \mu_n^{-1}$; and (iii) $\lambda_n + \lambda_n \log \lambda_n^{-1} \sim \mu_n + \mu_n \log \mu_n^{-1}$.

Proof: The proof is omitted here for brevity. ■

Theorem 1: For a wireless sensor network with EI-MAC, if the data vector is β-compressible and the FC performs compressive detection, then the NMSD satisfies

$$D_{\text{CD}} \preceq \mu_n(1 + \log \xi) + \mu_n \log \mu_n^{-1} \sim \frac{\log n}{n}, \quad (9)$$

where $\mu_n = \frac{1}{\beta}\left(\frac{\sigma_w^2}{n} + \frac{\delta_z}{n \log n}\right)$. The value of k leading to the above asymptotic behavior scales as $k \sim \log n$.

Proof: Define $\bar{D}_{\text{CD}} = \xi e^{-\beta k} + \frac{k}{n}\sigma_w^2 + \frac{k}{n \log n}\delta_z$, then $D_{\text{CD}} \preceq \bar{D}_{\text{CD}}$. It can be easily shown that $\bar{D}_{\text{CD}}$ is convex in k, thus the value of k that minimizes $\bar{D}_{\text{CD}}$ can be obtained by solving $\frac{\partial}{\partial k}\bar{D}_{\text{CD}} = 0$. The result is

$$k = \frac{1}{\beta} \log \left[\beta \xi \left(\frac{\sigma_w^2}{n} + \frac{\delta_z}{n \log n}\right)^{-1}\right] = \frac{1}{\beta} \log(\xi \mu_n^{-1}). \quad (10)$$

Substituting (10) into the definition of $\bar{D}_{\text{CD}}$ leads to

$$\bar{D}_{\text{CD}} = \mu_n(1 + \log \xi) + \mu_n \log \mu_n^{-1}. \quad (11)$$

It's easy to show that $\mu_n \sim \frac{1}{n}$. From Lemma 2, we have $k \sim \log n$, and $\bar{D}_{\text{CD}} \sim \frac{1}{n} + \frac{\log n}{n}$. Since $\lim_{n\to\infty}\left(\frac{1}{n} + \frac{\log n}{n}\right) / \frac{\log n}{n} = 1$, we have $\frac{1}{n} + \frac{\log n}{n} \sim \frac{\log n}{n}$, and this completes the proof. ■

It can be seen from the results in Theorem 1 that the distortion tends to 0 as $n \to \infty$. Therefore, the limit on NMSD imposed by sensing noise variance, σ_w^2, as shown in (6) for the DS scheme, is removed in the CD scheme by exploiting the special compressible structure of the data vector during the detection process at the FC. Therefore, as $n \to \infty$, the newly proposed CD scheme can achieve a better NMSD with less transmission power compared to the DS scheme.

C. Compressive Transmission

We propose an alternative sensing method that utilizes the compressibility of the data vector during the process of transmission. The CT scheme is implemented with the help of a hierarchical two-hop sensing scheme, where the nodes are classified into two categories, sensing nodes and compressing nodes. The sensing nodes collect the measured data at their respective locations, and then transmit them asynchronously to the compressing nodes. Upon collecting data from all the sensing nodes, the compressing nodes perform LS estimation of the transform domain coefficients, and deliver the estimated transform domain coefficients to the FC.

The compressing operation can be performed independently by one node, or collaboratively by up to k nodes, where $k < n$ is the dimension of the compressed signal. To simplify notation, it is assumed in the following discussion that one node is dedicated as the compressing node. Results obtained under this assumption are directly applicable to a system with multiple compressing nodes. In the following analysis, we consider the worst case scenario that all the sensing nodes have equal distance to the compressing node at $d = 2\rho$, with ρ being the radius of the monitored area $\mathcal{V}$.

Based on the EI-MAC protocol, the information retrieved at the compressing node from the i-th sensing node can be modeled as

$$y_{si} = \sqrt{\frac{P_s}{d^\alpha}}(s_i + w_i) + u_i, \qquad \text{for } i = 1, \cdots, n, \quad (12)$$

where P_s is the instantaneous transmission power of a sensing node, and u_i is AWGN with variance σ_u^2. Upon collecting information from all the sensing nodes, the compressing node projects the received vector, $\mathbf{y}_s = [y_{s1}, \cdots, y_{sn}]^T$, onto the first k bases of the transform domain as

$$\vartheta_i = \mathbf{y}_s^T \boldsymbol{\phi}_i = \sqrt{\frac{P_s}{d^\alpha}}(\theta_i + \tilde{w}_i) + \tilde{u}_i, \text{ for } i = 1, \cdots, k, \quad (13)$$

where $\tilde{u}_i = \mathbf{u}^T \boldsymbol{\phi}_i$ with $\mathbf{u} = [u_1, \cdots, u_n]^T$, and the variance of $\tilde{u}_i$ remains σ_u^2. It's assumed during the analysis that the compressing node has ideal knowledge of the transform domain basis. This information either can be extracted by the compressing node through data analysis, or can be obtained from the FC in the communication downlink [7].

The average power, $P_\vartheta = \frac{1}{k}\sum_{i=1}^k \mathbb{E}|\vartheta_i|^2$, is

$$P_\vartheta = \frac{P_s}{d^\alpha}\left(\frac{1}{k}\sum_{i=1}^{k}\sigma_{\theta_i}^2 + \sigma_w^2\right) + \sigma_z^2 \le \frac{P_s}{d^\alpha}\left[\frac{n}{k} + \sigma_w^2\right] + \sigma_u^2, \quad (14)$$

where $\sigma_{\theta_i}^2 = \mathbb{E}|\theta_i|^2$, and the last equality is based on the fact that $\frac{1}{n}\mathbb{E}\|\mathbf{s}\|_{l_2}^2 = \frac{1}{n}\sum_{i=1}^n \sigma_{\theta_i}^2 = 1$.

The estimated coefficient vector, $\boldsymbol{\vartheta}^{(k)} = [\vartheta_1, \cdots, \vartheta_k]^T$, is transmitted to the FC. The signal received at the FC can be written as

$$\mathbf{y}_c = \sqrt{\frac{P_c}{l^\alpha P_\vartheta}}\boldsymbol{\vartheta}^{(k)} + \mathbf{z}, \quad (15)$$

where P_c is the instantaneous transmission power of the compressing node. The data vector can then be estimated at the FC by performing the LS detection on $\mathbf{y}_c$ as

$$\begin{aligned}\hat{\mathbf{s}}^{(k)} &= \sqrt{\frac{(dl)^\alpha P_\vartheta}{P_c P_s}}\sum_{i=1}^{k} y_{ci}\boldsymbol{\phi}_i, \\ &= \mathbf{s}^{(k)} + \sum_{i=1}^{k}\tilde{w}_i\boldsymbol{\phi}_i + \sqrt{\frac{d^\alpha}{P_s}}\sum_{i=1}^{k}\tilde{u}_i\boldsymbol{\phi}_i + \sqrt{\frac{(dl)^\alpha P_\vartheta}{P_c P_s}}\sum_{i=1}^{k} z_i\boldsymbol{\phi}_i.\end{aligned}$$

The NMSD for $\hat{\mathbf{s}}^{(k)}$ can be represented by

$$D_{\text{CT}} = \mathcal{O}\left(\xi e^{-\beta k}\right) + \frac{k}{n}\sigma_w^2 + \frac{k}{n}\frac{d^\alpha \sigma_u^2}{P_s} + \frac{k}{n}\frac{(dl)^\alpha P_\vartheta \sigma_z^2}{P_c P_s}. \quad (16)$$

The scaling behavior of D_{CT} is presented in the following Theorem.

Theorem 2: For a wireless sensor network with EI-MAC and a fixed total transmission power, if the data vector is β-compressible, then the normalized mean square distortion, D_{CT}, satisfies

$$D_{\text{CT}} \preceq \eta_n(1 + \log\xi) + \eta_n \log \eta_n^{-1} + \delta_c \sim \frac{\log n}{n}, \quad (17)$$

where $\eta_n = \frac{1}{n}\frac{1}{\beta}(\sigma_w^2 + \delta_s)(1 + \delta_c)$, with $\delta_s = \frac{d^\alpha \sigma_u^2}{P_s}$ and $\delta_c = \frac{l^\alpha \sigma_z^2}{P_c}$. The value of k leading to the above asymptotic behavior scales as $k \sim \log n$.

Proof: Substituting (14) into (16), we have $D_{\text{CT}} \preceq \bar{D}_{\text{CT}}$, with

$$\bar{D}_{\text{CT}} = \xi e^{-\beta k} + \frac{k}{n}\left(\sigma_w^2 + \delta_s + \delta_c\sigma_w^2 + \delta_c\delta_s\right) + \delta_c. \quad (18)$$

It's easy to verify that $\bar{D}_{\text{CT}}$ is convex in k. Thus the value of k minimizing $\bar{D}_{\text{CT}}$ can be obtained by solving $\frac{\partial}{\partial k}\bar{D}_{\text{CT}} = 0$, and the result is

$$k = \frac{1}{\beta}\log\left(\xi\eta_n^{-1}\right). \quad (19)$$

Substituting (19) expression into (18) leads to

$$\bar{D}_{\text{CT}} = \eta_n(1 + \log\xi) + \eta_n \log\eta_n^{-1} + \delta_c. \quad (20)$$

For a system with duty cycle $\frac{1}{n\log n}$, the total average transmission power for n transmissions by the sensing nodes is $P_{t1} = \frac{nP_s}{n\log n}$. Similarly, the total average transmission power for k transmissions by the compressing node is $P_{t2} = \frac{kP_c}{n\log n}$. Thus,

$$\begin{aligned} P_s &= \log n \cdot P_{t1}, & (21a)\\ P_c &= \frac{n}{k}\log n \cdot P_{t2}. & (21b)\end{aligned}$$

Substituting (21) into the definition of η_n leads to

$$g(n)h_l(n) \le \eta_n \le g(n)h_u(n). \quad (22)$$

where $g(n) = \frac{1}{n}\frac{1}{\beta}\left(\sigma_w^2 + \frac{d^\alpha\sigma_u^2}{\log n \cdot P_{t1}}\right)$, $h_l(n) = \left[1 + \frac{l^\alpha\sigma_z^2}{n\log n \cdot P_{t2}}\right]$, and $h_u(n) = \left[1 + \frac{l^\alpha\sigma_z^2}{\log n \cdot P_{t2}}\right]$. It's easy to show that $g(n)h_l(n) \sim g(n)h_u(n) \sim n^{-1}$, thus $\eta_n \sim n^{-1}$. Combining the above analysis with (19) and Lemma 2, we have $k \sim \log n$.

Thus, from (20) and Lemma 2, we have $\bar{D}_{\text{CT}} \sim \frac{1}{n} + \frac{\log n}{n} + \frac{1}{n} \sim \frac{\log n}{n}$. This completes the proof. ■

The proof of Theorem 2 doesn't specify how the total power, $P_t = P_{t1} + P_{t2}$, should be allocated between the transmissions by the sensing nodes and the compressing nodes, respectively. Define $\zeta = \frac{P_{t1}}{P_t}$, we will next identify the value of ζ that can asymptotically minimize $\bar{D}_{\text{CT}}$.

Substituting (21) into (18), we have

$$\bar{D}_{\text{CT}} = e^{-\beta k} + \frac{k}{n}\left[\sigma_w^2 + \underbrace{\frac{1}{\log n}\frac{d^\alpha\sigma_u^2}{\zeta P_t} + \frac{1}{\log n}\frac{l^\alpha\sigma_z^2}{(1-\zeta)P_t}}_{f_1(\zeta)} + \underbrace{\frac{k}{n\log n}\frac{l^\alpha\sigma_z^2\sigma_w^2}{(1-\zeta)P_t} + \frac{k}{n(\log n)^2 P_t^2}\frac{(ld)^\alpha\sigma_u^2\sigma_z^2}{\zeta(1-\zeta)}}_{f_2(\zeta)}\right].$$

It's clear from the above equation that the optimum value of ζ depends on $f_1(\zeta) + f_2(\zeta)$. Since $f_2(\zeta)$ scales to 0 much faster compared to $f_1(\zeta)$ with the increase of n, when n is large, it's sufficient for us to evaluate the optimum value of ζ by using $f_1(\zeta)$ alone.

It's easy to show that $f_1(\zeta)$ is convex in $\zeta \in (0,1)$, thus the value of ζ that minimizes $f_1(\zeta)$ can be obtained by solving $\frac{d}{d\zeta} f_1(\zeta) = 0$, and the result is

$$\zeta = \begin{cases} \frac{d^\alpha \sigma_u^2 - \sqrt{(dl)^\alpha \sigma_u^2 \sigma_z^2}}{d^\alpha \sigma_u^2 - l^\alpha \sigma_z^2}, & d^\alpha \sigma_u^2 \neq l^\alpha \sigma_z^2, \\ 0.5, & d^\alpha \sigma_u^2 = l^\alpha \sigma_z^2 \end{cases} \tag{23}$$

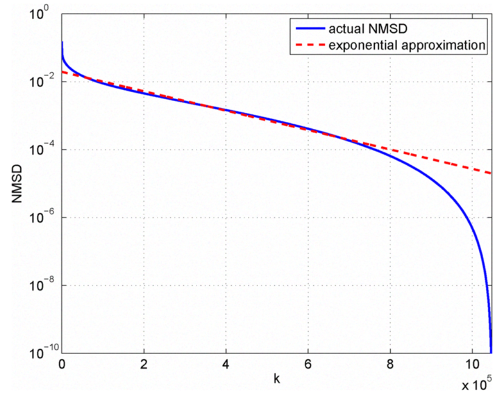
Comparing the results in Theorems 1 and 2 indicates that, if $\rho \ll l$, the newly proposed CD and the CT sensing schemes have similar scaling performances in terms of NMSD. This result is quite surprising, because it is commonly believed that transmitting compressed information can lead to better power efficiency than transmitting uncompressed information, due to the high energy cost of communication relative to computation. The similar performance between the CT and CD schemes can be explained by the fact that the salient information of a sparse signal is preserved in the transform domain coefficients, and the signal-to-noise ratio (SNR) per transform domain is similar for both the CT and the CD schemes.

IV. Numerical Examples

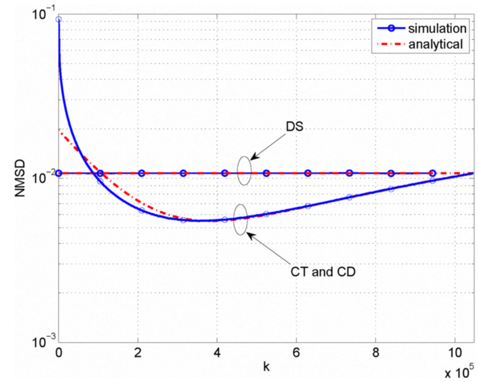
To qualitatively demonstrate the fidelity of the sensing schemes, a (1024×1024)-pixel airport image [9] as show in Fig. 2(a) is used as the data to be sensed. The image is rich in edges, textures, and details, such that it can effectively demonstrate the performance of the proposed schemes. The main parameters used in the examples are defined as follows. The signal-to-sensing noise ratio (SSNR) is defined as $\gamma_w = \frac{P_t}{\sigma_w^2}$. The SNR at the FC is defined as $\gamma_c = \frac{P_t}{\sigma_z^2 l^\alpha}$, which is equivalent to the SNR observed at the FC if all the power is employed by a single transmitter at distance l from the FC. It's assumed that $\sigma_u^2 = \sigma_z^2$, $\alpha = 3.6$, $\rho = 1$, and $l = 10$. The total power is normalized to $P_t = 1$.

First we verify the β-compressibility of the image used in the examples. Fig. 1(a) illustrates the NMSD between the original image, which is shown in Fig. 2(a), and its k-approximation. It's clear from the figure that when $k \ll n$, the NMSD can be accurately modeled by the function $\xi e^{-\beta k}$, which is depicted as a dashed curve in the figure. The parameters, ξ and β, are solved by applying the LS methods over the experimental NMSD value when $k < \frac{n}{2}$, and the results are $\xi = 0.0196$, $\beta = 6.58 \times 10^{-6}$.

The NMSD between the original data and the data recovered from different sensing methods is shown in Fig. 1(b). The parameters are $\gamma_w = \gamma_c = 20$ dB. The markers are obtained from empirical simulation. The dashed curves for the DS, CD, and CT methods

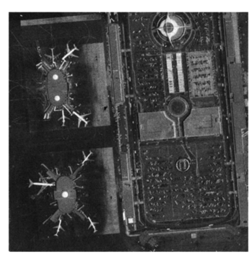


(a) NMSD of the k-approximation.

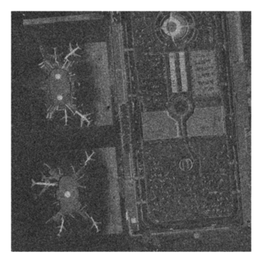


(b) NMSD v.s. k

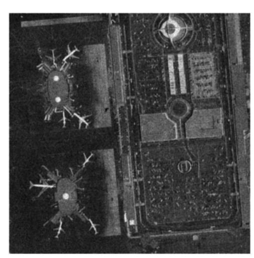
Fig. 1. β-compressibility of the sensing data.



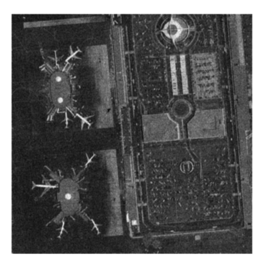
(a) Original Image



(b) DS



(c) CD



(d) CT

Fig. 2. Comparison of images recovered from different sensing methods.

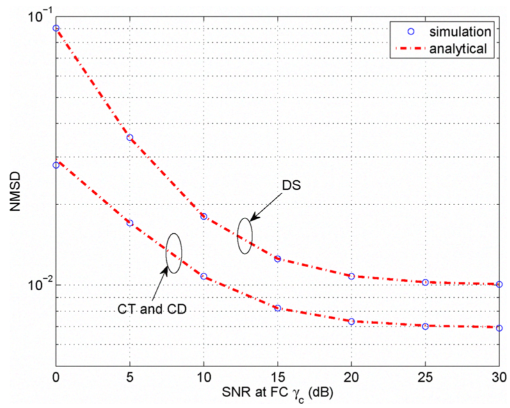


Fig. 3. NMSD v.s. SNR at the FC

are obtained from (6), (8), and (16), respectively. It's interesting to note that the NMSD results for the CD and the CT schemes are almost the same. This can be explained by the fact that the noise at the compressing node is negligible compared to that at the FC given $l \gg \rho$ and $\sigma_c^2 = \sigma_s^2$. For the CD and CT schemes, the value of k that minimizes the NMSD is 385,202, which is far less than the actual data dimension, $n = 1024 \times 1024 = 1,048,576$. The empirical value of k also matches the optimum value of k predicted through analytical approximations given in (10) and (19) for the CD scheme and CT scheme, respectively.

Fig. 2 compares the original image with the recovered images at the FC with different sensing schemes. The parameters are: $\gamma_w = 10$ dB and $\gamma_c = 0$ dB. Under this configuration, the optimum value of k is $31,106$, which is calculated from (10) and (19). It's obvious from Fig. 2 that the image recovered from DS is buried by the sensing noise, yet the images recovered from the CD and CT schemes preserve most of the key features of the original image, including the edges and textures. With the parameters used in this example, the NMSD for the DS, CD, and CT schemes are 0.1790, 0.0249, and 0.0247, respectively.

The last example demonstrates the distortion behaviors of various sensing schemes as functions of γ_c. The SSNR is $\gamma_w = 20$ dB. When $\gamma_c \to \infty$, the NMSD of the DS method is lower bounded by $\sigma_w^2 = 0.01$ as in (6), and the NMSD of the CD and CT schemes is lower bounded by $\xi e^{-\beta k} + \frac{k}{n}\sigma_w^2$ as in (8) and (16). For NMSD at 10^{-2}, the required SSNR for the CD/CT schemes is 14 dB lower than that of the DS scheme. In addition, the CD/CT methods can actually achieve a better distortion performance with less transmission power. For example, the NMSD of the CD/CT schemes at $\gamma_c = 20$ dB is less than the NMSD of the DS scheme at $\gamma_c = 30$ dB.

V. Conclusions

A cross-layer, ultra-low power, high fidelity compressive wireless sensing mechanism for infrastructure monitoring was proposed in this paper. The wireless nodes transmit information through an asynchronous EI-MAC protocol, which can a.a.s. achieve collision-free communications. Enabled by the EI-MAC protocol, two ultra-low power wireless sensing schemes, CD and CT, were developed by leveraging on the the correlation among the data collected by spatially distributed sensors. Theoretical analysis demonstrates that, when the inter-sensor distance is negligible compared to the sensor-FC distance, the two ultra-low power sensing schemes have similar distortion behaviors scaling with $\mathcal{O}\left(\frac{\log n}{n}\right)$. Numerical examples demonstrated that the newly proposed CD and CT sensing schemes with EI-MAC can achieve a power saving of as much as 14 dB.

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