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Channel Estimation for OFDM systems in the Presence of Carrier Frequency Offset and Phase Noise

Jun Tao†, Jingxian Wu§ and Chengshan Xiao‡
†Dept. of Electrical & Computer Eng., University of Missouri-Columbia, MO 65211, USA
§Dept. of Engineering Science, Sonoma State University, Rohnert Park, CA 94928, USA
‡Dept. of Electrical & Computer Eng., Missouri University of Science & Technology, Rolla, MO 65409, USA

Abstract— **Channel estimation for orthogonal frequency division multiplexing (OFDM) system at the presence of carrier frequency offset (CFO) and phase noise is discussed in this paper. A CFO estimation algorithm is developed by exploiting the time-frequency structure of training symbols, and it provides a very accurate estimation of CFO at the presence of both unknown frequency selective fading and phase noise. Based on the estimated CFO, the phase noise and frequency selective fading are jointly estimated by employing the maximum *a posteriori* (MAP) criterion. Specifically, the fading channel is estimated in the form of frequency domain channel transfer function (CTF). The estimation of CTF eliminates the requirement of the priori knowledge of channel length, and it is simpler compared to the time domain channel impulse response (CIR) estimation method in the literature. Theoretical analysis with Cramer-Rao lower bound demonstrates that the joint phase noise and CTF estimation can achieve near optimum performance.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) system has emerged as one of the most promising data transmission technologies for both wireless and wireline communication systems. By multiplexing data streams over mutually orthogonal subcarriers, OFDM has high spectral efficiency and is robust against intersymbol interference (ISI). However, the performance of OFDM system is very sensitive to both carrier frequency offset (CFO), which is caused by the frequency mismatch between oscillators at transmitter and receiver, and phase noise from local oscillators (LO). The CFO and phase noise, if not properly estimated and compensated, will cause amplitude reduction and phase drift at the receiver, and introduce inter-carrier interference (ICI), thus seriously degrade the performance of OFDM system [1], [2].

Channel estimation in the presence of CFO and phase noise is a challenging task for OFDM system design, and it usually involves simultaneous CFO estimation [1], [3], [4], and phase noise estimation/suppression [2], [5], [6]. In [7], the CFO is estimated and compensated before channel estimation, and the phase noise is suppressed when passing the estimated channel through a filter. This phase noise suppression method is just a byproduct of additive noise cancelation process. A joint CFO/phase noise/channel impulse response (CIR) estimator (JCPCE) is presented in [8]. The JCPCE gives a nonclosed-form estimation of CFO, thus requires high complexity numerical search to find the optimum solution. To reduce the complexity of JCPCE, a modified JCPCE (MJCPCE) algorithm with closed-form CFO estimation is also developed in [8] by adopting a special training symbol structure as proposed in [1]. The MJCPCE method, however, suffers in three aspects: first, it requires the knowledge of channel length; second, the resulting phase noise estimator has a very complex form; third, it can only estimate CFO with value less than subcarrier space.

To adress the problems in the MJCPCE method, we present in this paper an enhanced channel estimation algorithm for OFDM system at the presence of both CFO and phase noise. The CFO estimation is developed by exploiting the time-frequency properties of two consecutive training symbols with structures similar to those used in [4]. With the new method, CFO with arbitrary value can be accurately estimated at the presence of both unknown frequency selective fading and phase noise. With the estimated CFO, the phase noise and frequency selective fading are jointly estimated based on the maximum *a posteriori* (MAP) criterion. In particular, the channel is estimated in terms of the frequency domain channel transfer function (CTF), and it is different from the time domain CIR estimation used in MJCPCE. The adoption of CTF instead of CIR leads to an estimator with lower complexity while higher accuracy. In addition, it eliminates the requirement for priori knowledge of channel length, which is usually unavailable at receiver before channel estimation. The Cramer-Rao lower bound (CRLB) for the mean square error (MSE) of channel estimation is derived to benchmark the performance of the proposed algorithm. Simulation results show that the enhanced channel estimator can achieve a performance close to CRLB.

II. System Model and Assumptions

Baseband OFDM signal can be obtained by performing normalized inverse discrete Fourier transform (IDFT) on a set of modulated data, $s = [s_0, s_1, \cdots, s_{N-1}] \in \mathcal{C}^{1\times N}$, at the transmitter, as

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} s_k e^{j2\pi kn/N}, -N_p \leq n < N \tag{1}$$

where N is the number of subcarriers, and $N_p \geq L$ is the length of cyclic prefix (CP), with L being the length of the equivalent discrete-time CIR, $\{h_l\}_{l=0}^{L-1}$.

We consider a slow frequency selective fading channel in this paper. The CIR is assumed to be constant over one slot duration, which contains two OFDM training symbols followed by multiple OFDM data symbols [8].

The OFDM training symbol is generated by transmitting $N/2$ pilot symbols on the even indexed subcarriers and zeros on the odd indexed subcarriers. The OFDM training symbol in the time domain can then be represented as

$$x_n = \frac{1}{\sqrt{N/2}} \sum_{k=0}^{N/2-1} s_{2k} e^{j2\pi(2k)n/N}, -N_p \le n < N \quad (2)$$

where the normalizing factor $1/\sqrt{N/2}$ is used to maintain constant signal energy. It's clear that (2) performs a $N/2$-point IDFT operation, and the time domain OFDM training symbol has two identical halves, *i.e.*, $\{x_n\}_{n=0}^{N/2-1}$ is the same as $\{x_n\}_{n=N/2}^{N-1}$. This property of the training symbol will be exploited to assist CFO estimation.

At the receiver, after removing CP, we have the time domain samples of the received OFDM training symbol as

$$\begin{aligned} y_n &= e^{j(2\pi n\epsilon/N+\phi_n)}(h_n \otimes x_n) + v_n \\ &= e^{j(2\pi n\epsilon/N+\phi_n)} \frac{1}{\sqrt{N/2}} \sum_{k=0}^{N/2-1} s_{2k} H_{2k} e^{j2\pi \frac{kn}{N/2}} + v_n, \\ &\quad \text{for } n = 0, 1, \cdots, N-1 \end{aligned} \quad (3)$$

where $\otimes$ denotes circular convolution, v_n is the additive white Gaussian noise (AWGN) with variance σ^2, ϵ is the CFO normalized with respect to subcarrier spacing $\frac{1}{NT_s}$, with T_s being the sampling period, ϕ_n is the phase noise, and the frequency domain CTF, H_{2k}, is defined as

$$H_{2k} = \sum_{l=0}^{L-1} h_l e^{-j2\pi(2k)l/N}, \; 0 \le k \le N/2-1 \quad (4)$$

Define $\mathbf{E} = \text{diag}([1, e^{j2\pi\epsilon/N}, \cdots, e^{j2\pi(N-1)\epsilon/N}]^T)$, $\mathbf{P} = \text{diag}([e^{j\phi_0}, e^{j\phi_1}, \cdots, e^{j\phi_{N-1}}]^T)$, $\mathbf{S} = \text{diag}([s_0, s_2, \cdots, s_{N-2}]^T)$, with $\text{diag}(\mathbf{a})$ being a diagonal matrix with column vector $\mathbf{a}$ on its diagonal, and $\mathbf{H} = [H_0, H_2, \cdots, H_{N-2}]^T$, then (3) can be represented in matrix format as

$$\mathbf{y} = \mathbf{E}\mathbf{P}\tilde{\mathbf{F}}^H \mathbf{S}\mathbf{H} + \mathbf{v} \quad (5)$$

where $\mathbf{y} = [y_0, y_1, \cdots, y_{N-1}]^T$, $\mathbf{v} = [v_0, v_1, \cdots, v_{N-1}]^T$, $(\cdot)^T$ and $(\cdot)^H$ stand for transpose and Hermitian transpose, respectively, and $\tilde{\mathbf{F}} = [\mathbf{F}, \mathbf{F}]$ with $\mathbf{F}$ being the $N/2$-point normalized DFT matrix. The (k, l)-th element of $\mathbf{F}$ is $(\mathbf{F})_{k,l} = \frac{1}{\sqrt{N/2}} e^{-j2\pi(k-1)(l-1)/(N/2)}$.

For phase-locked system, the phase noise can be modeled as a zero-mean, stationary, finite-power Gaussian distributed random process [9], *i.e.*, $\boldsymbol{\phi}=[\phi_0, \phi_1, \cdots, \phi_{N-1}]^T$ has a multivariate Gaussian distribution of $\boldsymbol{\phi} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_\phi)$, where $\mathbf{0}$ is an all zero column vector, and $\mathbf{R}_\phi$ is the covariance matrix of $\boldsymbol{\phi}$. The value of $\mathbf{R}_\phi$ can be calculated with the specifications of phase-locked voltage controlled oscillator (VCO).

III. Development of the Estimation Algorithm

A. CFO estimation in the presence of unknown fading and phase noise

Since $x_n = x_{n+N/2}$, the received time domain training samples, y_n and $y_{n+N/2}$, for $n = 0, 1, \cdots, N/2-1$, are the same except a phase difference, in the absence of additive noise and phase noise [c.f. (3)], that's

$$y_n^* y_{n+N/2} = |y_n|^2 e^{j\pi\epsilon} \quad (6)$$

Obviously, (6) is a periodic function of ϵ with period $2z$, where z is an integer. Thus, the CFO can be estimated by measuring the phase difference between $\mathbf{y}_1 = [y_0, y_1, \cdots, y_{N/2-1}]^T$ and $\mathbf{y}_2 = [y_{N/2}, y_{N/2+1}, \cdots, y_{N-1}]^T$, up to an ambiguity, $2z$.

In [8], CFO estimation with additive noise and phase noise rejection is performed as

$$\hat{\epsilon} = \frac{1}{\pi} \measuredangle[\mathbf{y}_1^H(\mathbf{Y}_1\mathbf{R}_\Delta\mathbf{Y}_1^H + 2\sigma^2\mathbf{I})^{-1}\mathbf{y}_2] \quad (7)$$

where $\measuredangle a \in (-\pi, \pi]$ returns the phase of the complex-valued number a, $\mathbf{Y}_1 = \text{diag}(\mathbf{y}_1)$, $\mathbf{I}$ is a size $N/2$ identity matrix, and $\mathbf{R}_\Delta = 2\mathbf{R}_{N/2} - \boldsymbol{\Upsilon} - \boldsymbol{\Upsilon}^T$, with $\mathbf{R}_{N/2} \in \mathcal{C}^{\frac{N}{2} \times \frac{N}{2}}$ and $\boldsymbol{\Upsilon} \in \mathcal{C}^{\frac{N}{2} \times \frac{N}{2}}$ being sub-matrices of $\mathbf{R}_\phi$ as follows

$$\mathbf{R}_\phi = \begin{bmatrix} \mathbf{R}_{N/2} & \boldsymbol{\Upsilon} \\ \boldsymbol{\Upsilon}^T & \mathbf{R}_{N/2} \end{bmatrix} \quad (8)$$

The validation of (7) implies $|\epsilon| < 1$. However, the actual CFO could be $\epsilon = \epsilon_0 + 2z$ with $|\epsilon_0| < 1$, since ϵ has a period of $2z$, as described in (6). We denote ϵ_0 as fractional CFO, and $2z$ as integer CFO. Obviously, (7) only provides an estimate for fractional CFO ϵ_0 while leaves the ambiguity of integer CFO $2z$. In the following, we will estimate the integer CFO, $2z$, in the frequency domain by utilizing two consecutive OFDM training symbols.

From (3), the received samples of the first and second OFDM training symbols can be written as

$$y_{1,n} = \frac{1}{\sqrt{N/2}} e^{j(2\pi n\epsilon/N+\phi_n)} \sum_{k=0}^{N/2-1} s_{1,2k} H_{2k} e^{j2\pi \frac{kn}{N/2}} + v_{1,n} \quad (9a)$$

$$y_{2,n} = \frac{1}{\sqrt{N/2}} e^{j\left[2\pi(n+N+N_p)\epsilon/N+\phi_{n+N+N_p}\right]} \times \sum_{k=0}^{N/2-1} s_{2,2k} H_{2k} e^{j2\pi \frac{kn}{N/2}} + v_{2,n} \quad (9b)$$

where $n = 0, 1, \cdots, N-1$ for both $y_{1,n}$ and $y_{2,n}$. The ratio of the two training sequences, $s_{1,2k}$ and $s_{2,2k}$, is set to be equal to a predefined pseudo-noise (PN) sequence $\{\alpha_k\}_{k=0}^{N/2-1}$, *i.e.*, $s_{2,2k}/s_{1,2k} = \alpha_k$, for $k = 0, \cdots, N/2-1$.

Fractional CFO ϵ_0 is first estimated from (9a) with (7), then corrected by multiplying $e^{-j2\pi n\hat{\epsilon}_0/N}$ and $e^{-j2\pi(n+N+N_p)\hat{\epsilon}_0/N}$ to $y_{1,n}$ and $y_{2,n}$, respectively. The result can be written as

$$\hat{y}_{1,n} = \frac{1}{\sqrt{N/2}} e^{j\tilde{\phi}_n} \sum_{k=0}^{N/2-1} s_{1,2k} H_{2k} e^{j2\pi \frac{(k+z)n}{N/2}} + \hat{v}_{1,n} \quad (10a)$$

$$\hat{y}_{2,n} = \frac{1}{\sqrt{N/2}} e^{j4\pi z N_p/N} e^{j\tilde{\phi}_{n+N+N_p}} \times \sum_{k=0}^{N/2-1} s_{2,2k} H_{2k} e^{j2\pi \frac{(k+z)n}{N/2}} + \hat{v}_{2,n} \quad (10b)$$

where $\hat{v}_{1,n}$ and $\hat{v}_{2,n}$ are the noise components after fractional CFO compensation, and $\tilde{\phi}_n$ and $\tilde{\phi}_{n+N+N_p}$ are the effective phase noises resulted from the combination of the original phase noises, and phase rotation caused by residual fractional CFO, $\Delta\epsilon_0 = \epsilon_0 - \hat{\epsilon}_0$. To estimate integer CFO, we first use the approximations, $e^{j\tilde{\phi}_n} \approx 1$ and $e^{j\tilde{\phi}_{n+N+N_p}} \approx 1$, since the effective phase noises are usually small in practice, and then perform N-point DFT on $\hat{y}_{1,n}$ and $\hat{y}_{2,n}$. We therefore have

$$\hat{Y}_{1,k} \approx \sqrt{2}s_{1,(k-2z)_N} H_{(k-2z)_N} + \hat{V}_{1,k} \quad (11a)$$

$$\hat{Y}_{2,k} \approx \sqrt{2}e^{j4\pi z N_p/N} s_{2,(k-2z)_N} H_{(k-2z)_N} + \hat{V}_{2,k} \quad (11b)$$

where $(\cdot)_N$ denotes modulus N operation, $\hat{Y}_{i,k}$ and $\hat{V}_{i,k}$ are DFTs of $\hat{y}_{i,n}$ and $\hat{v}_{i,n}$, respectively, for $i = 1, 2$. There is a phase difference, $e^{j4\pi z N_p/N}$, between $\hat{Y}_{1,k}$ and $\hat{Y}_{2,k}$ in the frequency domain, and the phase difference is independent of the subcarrier index k. It should be noted that the approximation used in (11) is only for the convenience of integer CFO estimation, and the estimation of phase noise will be discussed in the next subsection. It'll be shown in simulation that the integer CFO $2z$ can be accurately estimated even with the approximation used in (11).

To estimate $2z$, define the cost function as

$$\mathcal{M}(z) = \left| \sum_{k=0}^{N/2-1} \hat{Y}^*_{1,2k+2z} \alpha^*_k \hat{Y}_{2,2k+2z} \right| \quad (12)$$

where $(\cdot)^*$ denotes complex conjugate, and only even indexed subcarriers are considered since zeros are transmitted over odd indexed subcarriers. With $\mathcal{M}(z)$ defined in (12), the estimated value of z is obtained as

$$\hat{z} = \arg\max_z \mathcal{M}(z) \quad (13)$$

The estimation of the CFO, $\epsilon = \epsilon_0 + 2z$, can then be expressed as

$$\hat{\epsilon} = \hat{\epsilon}_0 + 2\hat{z} \quad (14)$$

B. Joint Phase Noise and CTF Estimation

With $\hat{\epsilon}$ in (14), we are able to construct the CFO compensation matrix as $\hat{\mathbf{E}}$=diag$([1, e^{j2\pi\hat{\epsilon}/N}, \cdots, e^{j2\pi\hat{\epsilon}(N-1)/N}]^T)$. Multiplying both sides of (5) with $\hat{\mathbf{E}}^H$, we get

$$\tilde{\mathbf{y}} = \mathbf{P}_{\text{eff}}\tilde{\mathbf{F}}^H\mathbf{S}\mathbf{H} + \tilde{\mathbf{v}} \quad (15)$$

where $\tilde{\mathbf{y}} = \hat{\mathbf{E}}^H\mathbf{y}$, and $\mathbf{P}_{\text{eff}} = (\Delta\mathbf{E})\mathbf{P}$ is the effective phase noise matrix after CFO compensation with $\Delta\mathbf{E} =$ diag$([1, e^{j2\pi\Delta\epsilon/N}, \cdots, e^{j2\pi\Delta\epsilon(N-1)/N}]^T)$ as the additional phase rotation matrix due to CFO estimation error $\Delta\epsilon = \epsilon - \hat{\epsilon}$. The equivalent noise $\tilde{\mathbf{v}} = \hat{\mathbf{E}}^H\mathbf{v}$ is still AWGN with covariance matrix $\sigma^2\mathbf{I}$.

The effective phase noise matrix can be alternatively represented as $\mathbf{P}_{\text{eff}} = \text{diag}(\boldsymbol{\phi}_{\text{eff}})$, where $\boldsymbol{\phi}_{\text{eff}} = \left[\phi_0, \phi_1 + 2\pi\frac{\Delta\epsilon}{N}, \cdots, \phi_{N-1} + 2\pi\frac{\Delta\epsilon(N-1)}{N}\right]^T$ is the effective phase noise vector. The effective phase noise vector $\boldsymbol{\phi}_{\text{eff}}$ has a multivariate Gaussian distribution of $\boldsymbol{\phi}_{\text{eff}} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_{\phi_{\text{eff}}})$. The covariance matrix, $\mathbf{R}_{\phi_{\text{eff}}}$, depends on the variance of residual CFO $\Delta\epsilon$. At high SNR, the variance of $\Delta\epsilon$ can be approximated by [1], [4]

$$\sigma^2_{\Delta\epsilon} = \frac{1}{\pi^2 \cdot (N/2) \cdot \gamma} \quad (16)$$

where γ denotes SNR in linear scale. Therefore, the covariance matrix of $\boldsymbol{\phi}_{\text{eff}}$ can be accurately approximated as $\mathbf{R}_{\phi_{\text{eff}}} = \mathbf{R}_\phi + \frac{8}{N^3\cdot\text{SNR}}\mathbf{T}$, where $\mathbf{T} = \mathbf{b}^T\mathbf{b}$ with $\mathbf{b} = [0, 1, \cdots, N-1]$. Noting the fact that the scaling factor $\frac{8}{N^3\cdot\text{SNR}}$ of $\mathbf{T}$ is inversely proportional to N^3, while the maximum element in $\mathbf{T}$ is in the order of N^2, we conclude that the effect of residual CFO on phase noise is negligible. As a result, it's reasonable to assume that $\boldsymbol{\phi}_{\text{eff}}$ has the same distribution as $\boldsymbol{\phi}$, *i.e.* $\boldsymbol{\phi}_{\text{eff}} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_\phi)$. Simulation results show that the assumption of $\boldsymbol{\phi}_{\text{eff}} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_\phi)$ is valid under both low SNR and high SNR, and it doesn't apparently affect the accuracy of the proposed channel estimation method.

The MAP criterion is adopted for the joint estimation of $\boldsymbol{\phi}_{\text{eff}}$ and $\mathbf{H}$ from (15). The *a posteriori* probability (APP) density of $\boldsymbol{\phi}_{\text{eff}}$ and $\mathbf{H}$ can be written as

$$p(\boldsymbol{\phi}_{\text{eff}}, \mathbf{H}|\tilde{\mathbf{y}}) = p(\tilde{\mathbf{y}}|\boldsymbol{\phi}_{\text{eff}}, \mathbf{H})p(\boldsymbol{\phi}_{\text{eff}})p(\mathbf{H})/p(\tilde{\mathbf{y}}) \quad (17)$$

where it is assumed that $\boldsymbol{\phi}_{\text{eff}}$ and $\mathbf{H}$ are mutually independent. During the estimation, the CTF vector, $\mathbf{H}$, is treated as an unknown constant, thus $p(\mathbf{H}) = 1$. From (17), the negative log-likelihood function is calculated as

$$\mathcal{L}(\boldsymbol{\phi}_{\text{eff}}, \mathbf{H}) = \frac{1}{\sigma^2}\left\|\tilde{\mathbf{y}} - \mathbf{P}_{\text{eff}}\tilde{\mathbf{F}}^H\mathbf{S}\mathbf{H}\right\|^2 + \frac{1}{2}\boldsymbol{\phi}_{\text{eff}}{}^T\mathbf{R}_\phi^{-1}\boldsymbol{\phi}_{\text{eff}} + \log p(\tilde{\mathbf{y}}) \quad (18)$$

where $\|\mathbf{a}\|^2 = \mathbf{a}^H\mathbf{a}$ for a column vector $\mathbf{a}$.

Solving $\partial\mathcal{L}(\boldsymbol{\phi}_{\text{eff}}, \mathbf{H})/\partial\mathbf{H}^* = 0$ and assuming that $p(\tilde{\mathbf{y}})$ is irrelevant to specific $\boldsymbol{\phi}_{\text{eff}}$ and $\mathbf{H}$ lead to the optimal estimation of the CTF vector, $\mathbf{H}$, as

$$\hat{\mathbf{H}} = \frac{1}{2}\mathbf{S}^{-1}\tilde{\mathbf{F}}\mathbf{P}^H_{\text{eff}}\tilde{\mathbf{y}} \quad (19)$$

Substituting (19) into (18) and simplifying yield

$$\mathcal{L}(\boldsymbol{\phi}_{\text{eff}}) = \frac{1}{\sigma^2}\mathbf{p}^T\mathbf{B}\mathbf{p}^* + \frac{1}{2}\boldsymbol{\phi}_{\text{eff}}{}^T\mathbf{R}_\phi^{-1}\boldsymbol{\phi}_{\text{eff}} \quad (20)$$

where $\mathbf{p} = e^{j\boldsymbol{\phi}_{\text{eff}}}$, $\mathbf{B} = \tilde{\mathbf{Y}}^H(\mathbf{I} - \frac{1}{2}\tilde{\mathbf{F}}^H\tilde{\mathbf{F}})\tilde{\mathbf{Y}}$, and $\tilde{\mathbf{Y}} = \text{diag}(\tilde{\mathbf{y}})$. Using the approximation of $\mathbf{p} = e^{j\boldsymbol{\phi}_{\text{eff}}} \approx \mathbf{1}_N + j\boldsymbol{\phi}_{\text{eff}}$ for small $\boldsymbol{\phi}_{\text{eff}}$, and by solving $\partial\mathcal{L}(\boldsymbol{\phi}_{\text{eff}})/\partial\boldsymbol{\phi}_{\text{eff}} = 0$, we have the optimal estimation of $\boldsymbol{\phi}_{\text{eff}}$ as

$$\hat{\boldsymbol{\phi}}_{\text{eff}} = [\text{Re}(\mathbf{B}) + (\sigma^2/2)\mathbf{R}_\phi^{-1}]^{-1}\text{Im}(\mathbf{B})\mathbf{1}_N \quad (21)$$

where $\mathbf{1}_N$ denotes a $N \times 1$ all-one column vector. Obviously, the estimation of $\hat{\boldsymbol{\phi}}_{\text{eff}}$ is independent of the modulation data matrix $\mathbf{S}$, as is different from [8].

The estimated value of $\hat{\boldsymbol{\phi}}_{\text{eff}}$ can then be substituted back into (19) to obtain the estimation of the CTF vector $\hat{\mathbf{H}}$. Equation (19) provides estimation of $\mathbf{H} \in \mathcal{C}^{\frac{N}{2}\times 1}$, which is the CTF on even indexed subcarriers. The estimation of normalized CTF on all subcarriers can be obtained from $\hat{\mathbf{H}}$ as

$$\hat{\mathbf{H}}_{\text{full}} = \sqrt{2/N}\mathbf{F}_N[(\mathbf{F}^H\hat{\mathbf{H}})^T\mathbf{0}^T_{N/2}]^T \quad (22)$$

where $\mathbf{F}_N$ is N-point normalized DFT matrix with (k,l)-th element as $(\mathbf{F}_N)_{k,l} = \frac{1}{\sqrt{N}} e^{-j2\pi(k-1)(l-1)/N}$, and $\mathbf{0}_{N/2}$ is an all zero column vector with size $N/2$.

Furthermore, the time domain channel CIR $\mathbf{h} = [h_0, h_1, \cdots, h_{L-1}]^T$ can be estimated by performing IDFT over the estimated CTF vector $\hat{\mathbf{H}}$ as [c.f. (4)]

$$\hat{\mathbf{h}} = \sqrt{2/N}\mathbf{F}_{1:L}^H \hat{\mathbf{H}} \tag{23}$$

where $\mathbf{F}_{1:L} \in \mathcal{C}^{\frac{N}{2}\times L}$ contains the first L columns of the $\frac{N}{2}$-point DFT matrix $\mathbf{F}$.

Compared to the MJCPCE method, the new algorithm has four main advantages. First, the new method can accurately estimate CFO with arbitrary value, while MJCPCE can only estimate CFO with value less than subcarrier spacing. Second, estimating CTF instead of CIR leads to a simpler estimators with lower computational complexity. Third, the phase noise estimator of (21) is independent of transmitted data. Therefore, the estimation can be performed also on OFDM data symbols. In MJCPCE, the estimation relies on transmitted data, thus can only be applied to training symbols. Fourth, the knowledge of channel length L is not required during CTF estimation. The matrix $\mathbf{F}_{1:L}$ used in (23) can also be replaced by $\mathbf{F}^H$ when L is unknown.

IV. CRLB FOR OFDM CHANNEL ESTIMATION

The CRLBs for the estimation of the frequency domain CTF, $\mathbf{H}$, $\mathbf{H}_{\text{full}}$, and the time domain CIR, $\mathbf{h}$, are evaluated in this section.

In the absence of CFO and phase noise, the log likelihood function, $\log p(\mathbf{y}|\mathbf{H})$, can be calculated from (5) as

$$\log p(\mathbf{y}|\mathbf{H}) = c - \frac{1}{\sigma^2}(\mathbf{y} - \tilde{\mathbf{F}}^H\mathbf{S}\mathbf{H})^H(\mathbf{y} - \tilde{\mathbf{F}}^H\mathbf{S}\mathbf{H}) \tag{24}$$

with c being a constant independent of $\mathbf{y}$ and $\mathbf{H}$. Taking the derivative with respect to $\mathbf{H}^*$, we have

$$\frac{\partial}{\partial \mathbf{H}^*}[\log p(\mathbf{y}|\mathbf{H})] = \frac{1}{\sigma^2}\mathbf{S}^H\tilde{\mathbf{F}}(\mathbf{y} - \tilde{\mathbf{F}}^H\mathbf{S}\mathbf{H}) = \frac{1}{\sigma^2}\mathbf{S}^H\tilde{\mathbf{F}}\mathbf{v} \tag{25}$$

and the Fisher information matrix is evaluated as

$$\begin{aligned}\mathbf{I}(\mathbf{H}) &= E\left\{\left[\frac{\partial}{\partial \mathbf{H}^*}[\log p(\mathbf{y}|\mathbf{H})]\right]\left[\frac{\partial}{\partial \mathbf{H}^*}[\log p(\mathbf{y}|\mathbf{H})]\right]^H\right\} \\ &= \frac{1}{(\sigma^2)^2}E\left[\mathbf{S}^H\tilde{\mathbf{F}}\mathbf{v}\mathbf{v}^H\tilde{\mathbf{F}}^H\mathbf{S}\right] = \frac{2}{\sigma^2}E\left[\mathbf{S}^H\mathbf{S}\right]\end{aligned} \tag{26}$$

Then, the CRLB is calculated as

$$\text{CRLB}(\mathbf{H}) = \text{tr}\left\{[\mathbf{I}(\mathbf{H})]^{-1}\right\} = \frac{\sigma^2}{2}\text{tr}\left\{\left\{E\left[\mathbf{S}^H\mathbf{S}\right]\right\}^{-1}\right\} \tag{27}$$

From the relationship in (22), it's easy to prove that $\text{CRLB}(\mathbf{H}_{\text{full}}) = \frac{2}{N} \times \text{CRLB}(\mathbf{H})$.

Similarly, from (23), (26), and (27), the Fisher information matrix and CRLB for the estimation of the time domain CIR vector $\mathbf{h}$ can be written as

$$\mathbf{I}(\mathbf{h}) = \frac{N}{\sigma^2}\mathbf{F}_{1:L}^H E\left[\mathbf{S}^H\mathbf{S}\right]\mathbf{F}_{1:L} \tag{28}$$

$$\text{CRLB}(\mathbf{h}) = \frac{\sigma^2}{N}\text{tr}\left\{\left\{\mathbf{F}_{1:L}^H E\left[\mathbf{S}^H\mathbf{S}\right]\mathbf{F}_{1:L}\right\}^{-1}\right\} \tag{29}$$

Assume that the modulation symbols are equiprobable and independent, *i.e.*, $E\left[\mathbf{S}^H\mathbf{S}\right] = \sigma_s^2\mathbf{I}$, then we have

$$\text{CRLB}(\mathbf{H}_{\text{full}}) = \sigma^2/2\sigma_s^2 \tag{30}$$

$$\text{CRLB}(\mathbf{h}) = L\sigma^2/N\sigma_s^2 \tag{31}$$

Comparing (30) and (31), we find $\text{CRLB}(\mathbf{H}_{\text{full}}) \neq \text{CRLB}(\mathbf{h})$. This is due to the fact that a low pass filter operation is implied in (23) by using prior knowledge of channel length L, while in (22), L is not used. If we transform $\hat{\mathbf{h}}$ in (23) back to CTF, then $\text{CRLB}(\mathbf{H}_{\text{full}}) = \text{CRLB}(\mathbf{h})$.

V. SIMULATION

Simulation results are presented in this section. System parameters similar to those used in [8] are adopted here for comparison purpose: the number of subcarriers is $N = 64$, and the system sampling rate is $f_s = 20$MHz leading to a subcarrier spacing of $\Delta f = f_s/N = 312.5$KHz. Phase noise is simulated by passing a white Gaussian process through a one-pole Butterworth low pass filter with 3dB bandwidth $f_o = 100$KHz. The covariance matrix of phase noise $\mathbf{R}_\phi$ is calculated as $(\mathbf{R}_\phi)_{m,n} = (\pi\phi_{\text{rms}}/180)^2 \exp\{-2\pi f_o|m-n|/f_s\}$. Fractional CFO ϵ_0 is generated as a uniform distribution over (-1,1), and integer CFO z is taken randomly from [-4,4]. The frequency selective fading has a power delay profile (PDP) of $1.2257 \times e^{-0.8l}(0 \le l < 10)$, which is normalized to unit energy. QPSK modulation is selected and ϕ_{rms} is set as 6 degrees in the simulations. The PN sequence $\{\alpha_k\}_{k=0}^{N/2-1}$ is generated randomly with the set $\{1, j, -j, -1\}$.

We first investigate the accuracy of CFO estimation. Fig. 1 plots the residual CFO, $\Delta\epsilon$, at different SNRs. For each SNR, 300 independent CFO estimations are performed at the presence of phase noise. From the figure, it's obvious that the residual CFO, $\Delta\epsilon$, is consistently close to zero. This observation indicates that the proposed algorithm can accurately estimate the integer part of the CFO without error, *i.e.*, $\hat{z} = z$, at the presence of unknown frequency selective fading and phase noise.

The performance of joint phase noise and CTF estimation algorithm is studied in the next example, where we focus on the case that $|\epsilon| < 1$. Fig. 2 illustrates the MSE and its corresponding CRLB of the estimated normalized CTF, $\hat{\mathbf{H}}_{\text{full}}$, at the presence of phase noise. The results from MJCPCE method and the proposed algorithm neglecting phase noise are also shown in the figure for comparison. Obviously, the new algorithm achieves a performance that is very close to CRLB. As expected, the estimation performance degrades when phase noise is ignored. For the MJCPCE method, it has been shown in [8] that its MSE performance is very close to the CRLB when $|\epsilon| < 0.4$. However, its performance degrades when the range of ϵ is extended to $(-1, 1)$, as shown in Fig. 2. The performance degradation of MJCPCE method is caused by phase flipping at the stage of CFO estimation. Phase flipping is referring to the case that $-\pi$ is estimated

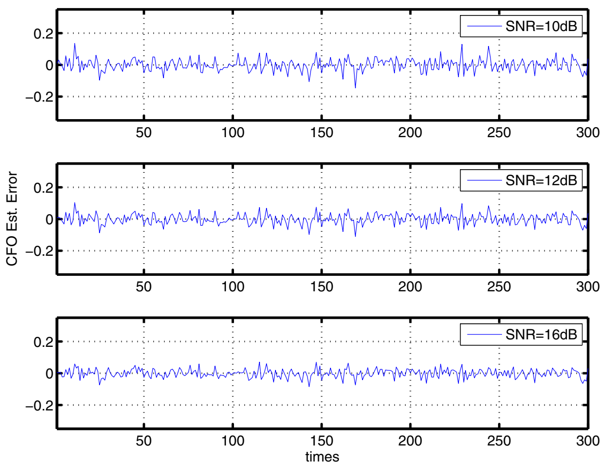

Fig. 1. CFO estimation Error ($\epsilon_0 \in (-1, 1), z \in [-4, 4]$).

as π, or vice versa, when the phase difference $\pi\epsilon$ in (6) approaches $-\pi$ or π. The performance of MJCPCE method suffers greatly from phase flipping, even in system with only fractional CFO. Due to phase flipping, the inclusion of CFO estimation and compensation in MJCPCE results in worse performance compared to the case that CFO is not estimated at all. Phase flipping also happens in the proposed method. However, the incorrectly estimated fractional CFO caused by phase flipping can be easily corrected at the stage of integer CFO estimation. Therefore, the performance of the proposed method is not affected by phase flipping.

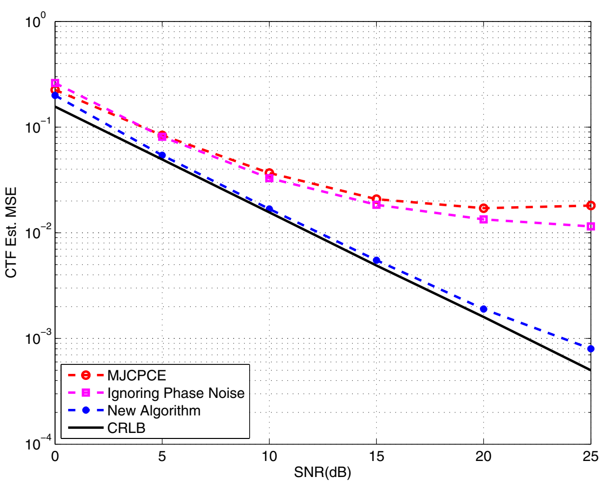

Fig. 2. CTF estimation MSE versus SNR (QPSK, $\epsilon_0 \in (-1, 1), z = 0$).

The next example demonstrates the channel estimation performance when integer CFO z is introduced in addition to fractional CFO ϵ_0. The MSE results along with the CRLB of CIR estimation are presented in Fig. 3. As expected, the MJPCE method, which neglects the integer CFO, doesn't function properly in such system configuration. The proposed scheme, on the other hand, consistently works well regardless of the presence of integer CFO.

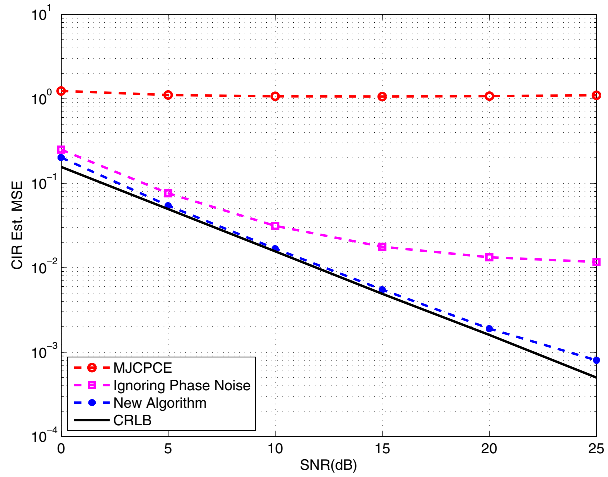

Fig. 3. CIR estimation MSE versus SNR (QPSK, $\epsilon_0 \in (-1, 1), z \in [-4, 4]$).

VI. Conclusion

An enhanced channel estimation algorithm operating at the presence of both CFO and phase noise was proposed for OFDM systems in slow frequency selective fading environment. The CFO was estimated by a hybrid time-frequency estimation method, with which both fractional and integer CFOs could be estimated accurately. With the CFO-compensated signal, a joint phase noise and CTF estimation algorithm was developed by employing the MAP criterion over time domain samples. Compared to the CIR estimation algorithm in the literature, the new algorithm had lower complexity, and it didn't require the knowledge of channel length. Simulation results showed that the joint phase noise and CTF estimation algorithm achieved a MSE performance close to CRLB. In addition, the proposed channel estimation scheme could be easily extended to single input multiple output (SIMO) system.

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