Estimation of Additive Noise Variance for Multiuser CDMA Systems

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Abstract—**In this paper, a novel additive noise variance estimation algorithm is proposed for multiuser CDMA systems experiencing time varying and frequency selective fading. The algorithm is developed by exploring the eigen-structure of the receive signal correlation matrix, which is characterized by the statistical properties of both the fading channel and the additive white Gaussian noise. Relying only on the knowledge of the receiver signal samples, the algorithm can accurately estimate the variance of the additive noise present at the receiver of multiuser CDMA systems. The estimated noise variance can be employed along with other receiver algorithms to improve system performance.**

I. Introduction

In communication systems, additive noise is primarily introduced by thermal agitation in electronic components as well as interferences encountered during transmission. As a result of central limit theorem, the additive noise is usually modeled as zero-mean Gaussian distributed with white spectrum, *i.e.*, additive white Gaussian noise (AWGN). Due to the white spectrum nature of AWGN, it's almost impossible to obtain the actual values of the additive noise at the communication receiver. However, the variance of the additive noise can be estimated at the receiver. Actually, a lot of advanced communication receiver algorithms rely on the knowledge of noise variance to improve system performance, *e.g.*, the minimum mean square error (MMSE) receiver [1], maximal *a posteriori* probability (MAP) algorithm implemented by turbo decoder [2], the belief propagation algorithm for low density parity check (LDPC) decoding [3], etc.

Several methods were presented for noise variance estimation at communication system receiver [2], [4], and [5]. In [2], an iterative noise variance estimation algorithm is implemented in conjunction with MAP decoder for a turbo coded system experiencing flat Rayleigh fading. A one shot noise variance estimation method is presented in [4] for code division multiple access (CDMA) system with quasi-static frequency selective channels.

In this paper, we present a new additive noise variance estimation algorithm for multiuser CDMA systems suffering from both time varying and frequency selective fading. The continuous-time CDMA system is equivalently represented as discrete-time tapped delay line filter with time varying tap coefficients. In addition, the tap coefficients are mutually correlated in both time domain and delay domain. A new rank inequality between the channel correlation matrix and receive signal correlation matrix is proposed by utilizing the temporal as well as delay domain correlation information of the fading channel. The results are exploited to facilitate the development of the noise variance estimation algorithm. The noise variance estimation algorithm requires only the knowledge of the received signal samples, and simulation results show that the new algorithm can accurately estimate the additive noise variance of multiuser CDMA systems.

The remainder of this paper is organized as follows. Section II presents a discrete-time representation of the multiuser CDMA system with correlated channel taps. In Section III, a rank inequality between channel and signal correlation matrices is derived based on the statistical properties of the discrete-time channel, and the results are used to formulate the additive channel estimation algorithm. Simulation examples are given in Section IV, and Section V concludes the paper.

II. Discrete-time System Representation

A. System Model

We consider the up-link of a multiuser CDMA system with M users. The baseband representation of the received signal at the basestation is

$$r(t) = \sum_{m=1}^{M} s_m(t-\tau_m) \circledast g_m(t,\tau) + v(t), \tag{1}$$

where $\circledast$ denotes the operation of convolution, $s_m(t)$ is the mth user's transmitted signal, τ_m is the relative propagation delay experienced by the mth user, $g_m(t,\tau)$ is the time varying channel impulse response, and $v(t)$ the additive white Gaussian noise (AWGN) with variance N_0 to be estimated. The channel impulse response $g_m(t,\tau)$ can be viewed as the response of the mth user's fading channel at time t to an impulse applied to the channel at time $t-\tau$ [6].

In a multiuser direct sequence CDMA system, each user is assigned a unique signature waveform. Let $q_m(t) = \frac{1}{N}\sum_{k=0}^{N-1} c_m(k)p(t-kT_c)$ be the normalized signature waveform of the mth user, where $\mathbf{c}_m = [c_m(0), c_m(1), \cdots, c_m(N-1)]^H \in \mathbb{C}^{N\times 1}$ is the mth user's spreading code, with $(\cdot)^t$ representing transpose operation, $p(t)$ is the normalized root raised cosine (RRC) filter with $\int p(t)p^*(t)dt = 1$, and T_c is the chip period. The processing gain of the system is N, and the code vector $\mathbf{c}_m$ satisfies $\mathbf{c}_m^H\mathbf{c}_m = N$, with $(\cdot)^H$ representing the Hermitian (complex transpose) operation. Thus the transmitted signal of the mth user can be written as

$$s_m(t) = \sum_{i=-\infty}^{+\infty} b_m(i)q_m(t-iT_s), \tag{2}$$

where $b_m(i)$ is the modulated symbol of the mth user with symbol period $T_s = NT_c$.

At the receiver, the chip matched filter $p^*(-t)$ is applied to $r(t)$, and the output of the matched filter is $y(t) = r(t) \circledast p^*(-t)$. If we define the composite channel impulse response (CIR) as follows

$$h_m(t,\tau) = \int_{-\infty}^{+\infty} R_{pp}(\tau - \tau_m - \alpha) g_m(t,\alpha) d\alpha, \quad (3)$$

$$\text{where } R_{pp}(t) = \int p(t+\tau)p^*(\tau)d\tau,$$

then the chip rate sampled output $y_j(n) = y(jT_s + nT_c)$ of the receiver filter can be represented by

$$y_j(n) = \sqrt{\frac{P_m}{N}} \sum_{m=1}^{M} \sum_{i=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} b_m(i) \times c_m[(j-i)N + (n-l)] \times h_m(j,l) + z_j(n), \quad \text{for } n = 0, 1, \cdots, N-1, \quad (4)$$

where $y_j(n)$ and $z_j(n)$ are the samples of $y(t)$ and $z(t) = v(t) \circledast p^*(-t)$ at the time instant $t = jT_s + nT_c$, respectively, and $h_m(j, jN+n) = h_m(jT_s, jNT_c + nT_c)$ is the discrete-time version of the CIR $h_m(t,\tau)$. It is assumed here that the fading channel varies slowly enough that it keeps nearly constant during one symbol duration.

To simplify the representation of (4), we note that the index k of $c_m(k)$ satisfies $0 \le k < N$. Combining this inequality with $k = (j-i)N + (n-l)$, we have $j + \frac{n-l}{N} - 1 < i \le j + \frac{n-l}{N}$. Since the index i of $b_m(i)$ can only take integer values, we can immediately get $i = j + \lfloor \frac{n-l}{N} \rfloor$, where $\lfloor \cdot \rfloor$ denotes rounding to the nearest smaller integer. Substituting the value of i into $k = (j-i)N + (n-l)$, we will have $k = -\lfloor \frac{n-l}{N} \rfloor N + (n-l) = (n-l)_N$, with $(x)_N$ denoting the residue of x/N with $0 \le (x)_N \le N-1$. The analysis above leads to a simplified representation of (4)

$$y_j(n) = \sqrt{\frac{P_m}{N}} \sum_{l=-\infty}^{+\infty} \sum_{m=1}^{M} b_m(j + \lfloor \frac{n-l}{N} \rfloor) \times c_m((n-l)_N) \times h_m(j,l) + z_j(n). \quad \text{for } n = 0, 1, \cdots, N-1. \quad (5)$$

Equation (5) is a discrete-time representation of the CDMA system, and the time varying frequency-selective fading is represented as a T_c-spaced tapped-delay line filter. The input-output relationship of the CDMA system are determined by the time varying channel tap coefficients $h_m(j,l)$ and the noise component $z_j(n)$.

B. Statistics of the Discrete-time System Model

The statistical properties of the discrete-time system model are analyzed here to facilitate the development of the noise variance estimation algorithm.

In the discrete time system representation, the original AWGN $v(t)$ is equivalently replaced by the discrete-time noise component $z_j(n)$. Since $z_j(n)$ is the linear transformation of the Gaussian process $v(t)$, it is still zero-mean Gaussian distributed with auto-correlation given by [7]

$$E\left[z_{j_1}(n_1) z_{j_2}^*(n_2)\right] = N_0 R_{pp}[(j_1 - j_2)T_s + (n_1 - n_2)T_c], \quad (6)$$

where $R_{pp}(t)$ is the auto-correlation function of the RRC filter $p(t)$ and it satisfies

$$R_{pp}(kT_c) = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0. \end{cases} \quad (7)$$

Equations (6) and (7) immediately leads to

$$E\left[z_{j_1}(n_1) \cdot z_{j_2}^*(n_2)\right] = \sigma_v^2 \cdot \delta(j_1 - j_2) \cdot \delta(n_1 - n_2), \quad (8)$$

which means $z_j(n)$ is still a white Gaussian process with variance of N_0. To this point, it's apparent that the problem of variance estimation of AWGN $v(t)$ is equivalent to estimate the variance of the discrete-time noise component $z(n)$.

Another important component of the discrete-time system model is the time varying discrete-time channel coefficient $h_m(j,l)$, which is sampled from $h_m(t,\tau)$. For Rayleigh fading channel, the physical channel impulse response $g_m(t,\tau)$ is a complex Gaussian process with zero-mean. Based on (3), it is easy to conclude that $h_m(t,\tau)$ and its sampled version $h_m(j,l)$ are zero-mean Gaussian distributed. Moreover, the channel coefficients $h_m(j,l)$ are correlated in both the time domain j and the delay domain l, and the tap correlation $\rho_m(j_1, j_2; l_1, l_2) = E\left[h_m(j,l_1) h_m^*(j,l_2)\right]$ is [7]

$$c_m(j_1, j_2; l_1, l_2) = J_0[2\pi f_d (j_1 - j_2) T_s] \times \rho_m(l_1, l_2), \quad (9)$$

where

$$\rho_m(l_1, l_2) = \int_{-\infty}^{+\infty} R_{pp}(l_1 T_c - \tau_m - \mu) R_{pp}^*(l_2 T_c - \tau_m - \mu) G_m(\mu) d\mu, \quad (10)$$

the function $J_0(x)$ is the zero-order Bessel function of the first kind, and $G_m(\mu)$ is the normalized power delay profile of the channel. The channel inter-tap correlation information will be explored in the next section to facilitate the development of the noise variance estimation algorithm.

With the correlation information given in (10), the power of the lth channel tap is $\rho(l,l)$, which is jointly determined by $G_m(\mu)$ and $R_{pp}(t)$. Since the time-domain tails of $p(t)$ falls off rapidly, $\rho(l,l)$ will decrease quickly with the increase of $|l|$. When $\rho(l,l)$ is smaller than a pre-defined threshold, it has very little impact on the output signal, and thus can be discarded. Without loss of generality, we use $\mathbf{l}_m = [-L_{m1}, \cdots, L_{m2}]^T \in \mathbb{I}^{\lambda_m \times 1}$ to represent the tap delay index vector of the mth user, where L_{m1} and L_{m2} are non-negative integers.

With $\mathbf{l}_m$, we define the tap coefficients vector of the mth user as

$$\mathbf{h}_m(j) = [h_m(j, -L_{m1}), \cdots, h_m(j, L_{m2})]^T, \quad (11)$$

then the tap correlation matrix $\mathbf{R}_{hm} = E[\mathbf{h}_m(j)\mathbf{h}_m^H(j)]$ of the mth user can be written as

$$\mathbf{R}_{hm} = \begin{bmatrix} \rho_m(-L_{m1}, -L_{m1}) & \cdots & \rho_m(-L_{m1}, L_{m2}) \\ \vdots & \ddots & \vdots \\ \rho_m(L_{m2}, -L_{m1}) & \cdots & \rho_m(L_{m2}, L_{m2}) \end{bmatrix}, \quad (12)$$

where $\rho_m(l_1, l_2)$ is the tap correlation coefficient defined in (10).

III. A New Noise Variance Estimation Algorithm

In this section, we present a new algorithm for the estimation of additive white Gaussian noise variance by exploring the statistical properties of both the additive noise and the time varying frequency selective channel.

For system with pilot symbol assisted modulation, the transmitted symbols are divided into slots with known pilot symbols inserted in the data stream. For convenience of representation, the symbol '1' is used as pilot symbols. From (5), the input-output relationship of the CDMA system at pilot symbol position j_p can be written in matrix format as

$$\mathbf{y}(j_p) = \mathbf{C}\mathbf{h}(j_p) + \mathbf{z}(j_p), \tag{13}$$

where

$$\begin{aligned} \mathbf{y}(j_p) &= [y_{j_p}(0), y_{j_p}(1), \cdots, y_{j_p}(N-1)]^T, & (14a)\\ \mathbf{z}(j_p) &= [z_{j_p}(0), z_{j_p}(1), \cdots, z_{j_p}(N-1)]^T, & (14b)\\ \mathbf{h}(j_p) &= [\mathbf{h}_1^T(j_p), \mathbf{h}_2^T(j_p), \cdots, \mathbf{h}_M^T(j_p)]^T. & (14c) \end{aligned}$$

The multiuser code matrix $\mathbf{C}$ is defined as

$$\mathbf{C} = [\mathbf{C}_1 \vdots \mathbf{C}_2 \vdots \cdots \vdots \mathbf{C}_M], \tag{15}$$

where

$$\mathbf{C}_m = \begin{bmatrix} c_m(L_{m1}) & \cdots & c_m(0) & \cdots & c_m(N-L_{m2}) \\ c_m(L_{m1}+1) & \cdots & c_m(1) & \cdots & c_m(N-l_{m2}+1) \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ c_m(L_{m1}-1) & \cdots & c_m(N-1) & \cdots & c_m(N-L_{m2}-1) \end{bmatrix}$$

with the ith column of $\mathbf{C}_m$ being obtained from circular shift i symbols of the original code vector $\mathbf{c}_m$.

From (13), the correlation matrix of the received pilot symbols $\mathbf{R}_{y_p} = \mathbb{E}[\mathbf{y}(j_p)\mathbf{y}^H(j_p)]$ can be written by

$$\mathbf{R}_{y_p} = \mathbf{C}\mathbf{R}_h\mathbf{C}^H + N_0\mathbf{I}_N, \tag{16}$$

where $\mathbf{R}_h = \mathbb{E}[\mathbf{h}(j_p)\mathbf{h}^H(j_p)]$ is the multi-user inter-tap correlation matrix defined as

$$\mathbf{R}_h = \begin{bmatrix} \mathbf{R}_{h_1} & 0 & \cdots & 0 \\ 0 & \mathbf{R}_{h_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{R}_{h_M} \end{bmatrix}, \tag{17}$$

with $\mathbf{R}_{h_m}$ being the channel correlation matrix of the mth user defined in (12).

Within one slot duration, the correlation matrix $\mathbf{R}_{y_p}$ and $\mathbf{R}_h$ can be approximated by their respective sample means

$$\begin{aligned} \widehat{\mathbf{R}}_{y_p} &= \frac{1}{P}\sum_{p=1}^{P}\mathbf{y}(j_p)\mathbf{y}^H(j_p), & (18a)\\ \widehat{\mathbf{R}}_h &= \frac{1}{P}\sum_{p=1}^{P}\mathbf{h}(j_p)\mathbf{h}^H(j_p), & (18b) \end{aligned}$$

where P is the total number of pilot symbols in one slot.

Correspondingly, the correlation matrix of the received pilot symbols can be approximated by (c.f. eqn. (16))

$$\widehat{\mathbf{R}}_{y_p} \approx \mathbf{C}\widehat{\mathbf{R}}_h\mathbf{C}^H + N_0\mathbf{I}_N. \tag{19}$$

In order to estimate the noise variance N_0 from (19), we need to exploit the eigen structure of $\widehat{\mathbf{R}}_h$, which is the sum of P correlated random matrices as described in (18a). According to the physical properties of the doubly-selective fading channel, we have the following theorem about the rank of the random matrix $\widehat{\mathbf{R}}_h$.

Theorem 1: If we define the temporal correlation matrix $\mathbf{R}_t \in \mathbb{C}^{P\times P}$ as at the top of the next page, where $J_0(\cdot)$ is the zero-order Bessel function of the first kind, f_d is the maximum Doppler frequency of the time varying fading channel, T_s is the symbol period, and j_p is the index of the pth pilot symbol of a slot with P pilot symbols, then we have the following rank inequality between the random matrix $\widehat{\mathbf{R}}_h = \frac{1}{P}\sum_{p=1}^{P}\mathbf{h}(j_p)\mathbf{h}^H(j_p)$ and the deterministic matrix $\mathbf{R}_t$

$$\operatorname{rank}(\widehat{\mathbf{R}}_h) \le \operatorname{rank}(\mathbf{R}_t). \tag{21}$$

Proof: Define the branch CIR vector $\mathbf{h}_{m,l} \in \mathbb{C}^{P\times 1}$ as

$$\mathbf{h}_{m,l} = \begin{bmatrix} h_m(j_1,l) & h_m(j_2,l) & \cdots & h_m(j_P,l) \end{bmatrix}^T, \quad \text{for } l \in \mathbf{l}_m,\ m = 1, 2, \cdots, M, \tag{22}$$

where P is the number of pilot symbols of one slot. For WSSUS Rayleigh fading channel, the vectors $\mathbf{h}_{m,l}$ and $\mathbf{h}_{n,k}$ are zero-mean Gaussian distributed, and their cross-correlation matrix can be obtained from (9) as

$$\mathbf{R}_{ml,nk} = \mathbb{E}[\mathbf{h}_{m,l}\mathbf{h}_{n,k}^H] = \delta(m,n)\cdot\rho_m(l,k)\cdot\mathbf{R}_t, \tag{23}$$

where $\delta(m,l)$ is the Kronecker delta function, $\rho_m(l,k)$ is the tap correlation given in (10), and $\mathbf{R}_t$ is defined in (20).

With (23), $\mathbf{h}_{m,l}$ can be equivalently represented as the linear transformation of Gaussian vector $\mathbf{w}_{m,l}$ with independent and identically distributed (i.i.d.) elements

$$\mathbf{h}_{m,l} \sim \mathbf{R}_t^{1/2}\cdot\mathbf{w}_{m,l}, \tag{24}$$

where the symbol $\sim$ defines an equivalence relation between two random variables if they have the same statistical distributions, the vector $\mathbf{w}_{m,l} = [w_m(1,l), w_m(2,l), \cdots, w_m(P,l)]^T \sim \mathcal{N}(0, \rho_m(l,l)\mathbf{I}_P)$ is Gaussian distributed with zero-mean and covariance matrix $\rho_m(l,l)\cdot\mathbf{I}_P$, and the cross correlation matrix between $\mathbf{w}_{m,l}$ and $\mathbf{w}_{n,k}$ is $\mathbb{E}[\mathbf{w}_{m,l}\mathbf{w}_{n,k}^H] = \delta(m,n)\cdot\rho_m(l,k)\cdot\mathbf{I}_P$. The matrix $\mathbf{R}_t^{1/2}$ is the square root of $\mathbf{R}_t$ defined as

$$\mathbf{R}_t^{1/2} = \mathcal{U}_t\cdot\Lambda_t^{1/2}, \tag{25}$$

where $\Lambda_t = \operatorname{diag}\{\lambda_{t1}, \cdots, \lambda_{tP}\}$ is a diagonal matrix with λ_{tp}, for $p = 1, 2, \cdots, P$, being the eigen values of $\mathbf{R}_t$ in decreasing order, and $\mathcal{U}_t = [\mathbf{u}_{t1}, \cdots, \mathbf{u}_{tP}] \in \mathbb{C}^{P\times P}$ are the corresponding orthonormal eigen vectors.

$$\mathbf{R}_t = \begin{bmatrix} 1 & J_0[2\pi f_d(j_1-j_2)T_s] & \cdots & J_0[2\pi f_d(j_1-j_P)T_s] \\ J_0[2\pi f_d(j_2-j_1)T_s] & 1 & \cdots & J_0[2\pi f_d(j_2-j_P)T_s] \\ \vdots & \vdots & \ddots & \vdots \\ J_0[2\pi f_d(j_P-j_1)T_s] & J_0[2\pi f_d(j_P-j_2)T_s] & \cdots & 1 \end{bmatrix}, \quad (20)$$

From the analysis above, we can see that the family of vectors $\{\mathbf{h}_{m,l} | l \in \mathbf{l}_m, m = 1, \cdots, M\}$ has exactly the same distribution as $\left\{\mathbf{R}_t^{1/2} \cdot \mathbf{w}_{m,l} | l \in \mathbf{l}_m, m = 1, \cdots, M\right\}$. Therefore we have the following equivalent relation about the inner product of two branch CIR vectors

$$\mathbf{h}_{m,l}^H \cdot \mathbf{h}_{n,k} \sim \mathbf{w}_{m,l}^H (\mathbf{R}_t^{1/2})^H \cdot \mathbf{R}_t^{1/2} \mathbf{w}_{n,k}, \quad (26a)$$

$$\sim \sum_{p=1}^{R} \lambda_p w_m(p,l) w_n^*(p,k), \quad (26b)$$

where $R = \text{rank}(\mathbf{R}_t)$, and the equality $(\mathbf{R}_t^{1/2})^H \mathbf{R}_t^{1/2} = \Lambda_t$ from (25) is used in (26b).

Noting the fact that each element of the matrix $\widehat{\mathbf{R}}_h$ defined in (18b) can be written as the normalized inner product of two branch CIRs as

$$\frac{1}{P}\mathbf{h}_{m,l}^H \cdot \mathbf{h}_{n,k} = \frac{1}{P}\sum_{p=1}^{P} h_m(j_p,l) h_n^*(j_p,k), \quad (27)$$

we can replace all the elements of $\widehat{\mathbf{R}}_h$ as described in (27) corresponding to each value of m, n, l and k without altering the statistical property of $\widehat{\mathbf{R}}_h$, and the obtained matrix is

$$\widehat{\mathbf{R}}_h \sim \mathbf{S}_h = \sum_{p=1}^{R} \lambda_p \mathbf{w}(p)\mathbf{w}^H(p), \quad (28)$$

where $\mathbf{w}(p) = [\mathbf{w}_1^T(p) \;\; \mathbf{w}_2^T(p) \;\; \cdots \;\; \mathbf{w}_M^T(p)]^T$, and $\mathbf{w}_m(p) = [w_m(p, -L_{m1}), \cdots, w_m(p, L_{m2})]^T \in \mathbb{C}^{(L_{m1}+L_{m2}+1)\times 1}$ is the branch channel impulse response vector of the mth user. Therefore the random matrix $\widehat{\mathbf{R}}_h$ is statistically equivalent to the sum of R rank 1 matrices. According to the inequality $\text{rank}(\mathbf{A}+\mathbf{B}) \le \text{rank}(\mathbf{A}) + \text{rank}(\mathbf{B})$ [8, p.13], we have

$$\text{rank}(\mathbf{S}_h) \le \sum_{p=1}^{R} \text{rank}\left[\lambda_p \mathbf{w}(p)\mathbf{w}^H(p)\right] \le R, \quad (29)$$

where $R = \text{rank}(\mathbf{R}_t)$. Since $\widehat{\mathbf{R}}_h$ and $\mathbf{S}_h$ have the same stochastic property, the supermum of their ranks should be the same, and this completes the proof. ■

It should be noted that the value of the actual multiuser correlation matrix $\mathbf{R}_h$ is determined by the frequency-selective property of the fading channel, while the matrix $\mathbf{R}_t$ reflects the time varying property of the channel. According to (9), there should be no interaction between $\mathbf{R}_h$ and $\mathbf{R}_t$. However, the approximation matrix $\widehat{\mathbf{R}}_h$ is the average of P time-domain samples, which are correlated with each other due to the temporal correlation of the fading channel. Therefore the properties of $\widehat{\mathbf{R}}_h$ depend on both $\mathbf{R}_h$ and $\mathbf{R}_t$, and the interactions between $\widehat{\mathbf{R}}_h$ and $\mathbf{R}_t$ are employed here for the estimation of the noise variance.

With the rank inequality given in (21) and the fact that $\text{rank}(\mathbf{A}\cdot\mathbf{B}) \le \min[\text{rank}(\mathbf{A}), \;\; \text{rank}(\mathbf{B})]$ [8, p.13], we will have

$$\text{rank}(\mathbf{C}\widehat{\mathbf{R}}_h\mathbf{C}^H) \le \text{rank}(\mathbf{R}_t). \quad (30)$$

Combining (19) and (30), we can see that the noise variance N_0 is equal to the average of the smallest $N-\text{rank}(\mathbf{R}_t)$ eigen values of the correlation matrix $\widehat{\mathbf{R}}_{y_p}$.

It is shown in [9] that $\text{rank}(\mathbf{R}_t) \approx \lfloor 2Nf_dT_s \rfloor + 1$. For CDMA systems, we always have $f_dT_s \ll 1$, which means $\lfloor 2Nf_dT_s \rfloor + 1 \ll N$. Therefore, an estimation of N_0 can be obtained by averaging over the smallest $N - \lfloor 2Nf_dT_s \rfloor - 1$ eigen values of $\widehat{\mathbf{R}}_{yp}$. Since the maximum Doppler frequency f_d is not available at the receiver, we can replace f_d by its maximum possible value for practical CDMA systems, *e.g.*, 200 Hz, without losing the estimation accuracy. It should be noted that the value of $\widehat{\mathbf{R}}_h$ and $\mathbf{R}_t$ are not required during the estimation of the noise variance, although the estimation method is derived based on the properties of these two matrices.

We conclude this section by summarizing the noise variance estimation process as follows:

Step 1 : Formulate the matrix $\widehat{\mathbf{R}}_{y_p}$ based on $\mathbf{y}(j_p)$ as described in (19).
Step 2 : Perform eigenvalue decomposition of $\widehat{\mathbf{R}}_{y_p}$. The N eigenvalues are stored in decreasing order as $[\lambda_1, \lambda_2, \cdots, \lambda_N]$.
Step 3 : Based on the results from Step 2, the noise variance can be estimated by

$$\hat{N}_0 = \frac{1}{I}\sum_{n=I+1}^{N} \lambda_n, \quad (31)$$

where $I = \lfloor 2Nf_dT_s \rfloor + 1$.

IV. Simulation Examples

Simulations are carried out in this section to demonstrate the performance of the proposed noise variance estimation algorithm.

The simulation is performed over a 5-user CDMA system operating in the Vehicular A channel profile [10]. The slot duration used in the simulation is T_{slot} = 2ms, and every 2 slots are combined as a hyper-slot during the estimation process. In each of the slot, there are 56 data symbols and 6 pilot symbols. Gold sequences with N = 127 are used as the spreading codes, and the chip rate is 3.84 Mcps. The transmitted data are QPSK modulated. RRC filter with rolloff factor of 0.22 is used as the transmit filter and receiver filter. The normalized Doppler frequency is $f_d T_{slot} = 0.1$.

The noise variance estimated from the algorithm proposed in this paper is shown in Fig. 1 under various values of signal to noise ratio. For the purpose of comparison, the actual noise variance is also plotted in the figure. Good agreement can be observed between the estimated noise variance and actual noise variance. The simulation results verify that the proposed algorithm is very accurate for a large range of signal to noise ratio.

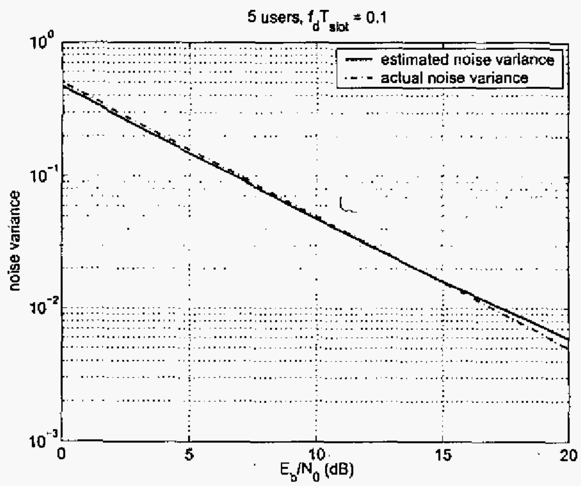

Fig. 1. Estimated noise variance v.s. actual noise variance.

V. Conclusions

In this paper, we presented a novel and simple noise variance estimation algorithm for multiuser CDMA systems experiencing time varying and frequency selective fading. The noise variance estimation algorithm was developed by exploring both the eigen-structure of the channel inter-tap correlation matrix and the time domain correlation information of the fading channel. During the estimation process, the only knowledge required by the algorithm is the received signal samples. Therefore, the noise variance estimation algorithm can be placed immediately after sampler at communication receiver, and the results can be used by other receiver algorithms to improve system performance. Simulation results show that the new algorithm can accurately estimate the variance of additive noise present at the channel.

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