On the Error Performance of Linearly Modulated Systems with Doubly selective Rayleigh Fading channels

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***Abstract*— Theoretical error performances of communication systems with doubly selective (time-varying and frequency-selective) fadings and fractionally spaced (oversampled) receivers are analyzed. Closed-form error probability expressions of MPSK, MASK and MQAM systems are derived as tight lower bounds of the symbol error probabilities. The effects of receiver oversampling, Doppler spread and fading power delay profile are quantified in the error probability expressions. Simulation results show that the new analytical results can accurately predict the error performances of MLSE and MAP equalizers in a wide range of SNR. Moreover, it is discovered that significant performance gain can be achieved by fractionally spaced receivers over symbol spaced receivers for systems experiencing fast time-varying fading, whereas the effects of Doppler spread are overlooked by most previous works.**

I. Introduction

Error performance analysis of wireless communication systems experiencing frequency-selective fading has been a field of long time interests [1]-[7]. One of the most popular analytical methods used for performance analysis of systems with frequency-selective fading channels is the union bound technique [1]-[3], which provides an effective way to evaluate the upper bounds of system performances. Most of the union bound results are for systems with symbol spaced equalizers, *i.e.*, the sampling period T_s equals to the symbol period T_{sym}. More efficient matched filter bounds are derived in [4]-[6] by assuming that the receive filter is perfectly matched to the combined impulse response of the transmit filter and the frequency-selective channels, and the effects of receiver oversampling are not considered in the matched filter bounds.

All of the aforementioned results are based on the assumption that the fading channel is quasi-static or slowly time-varying. Moreover, some of the works [3] [4] have the frequency-selective fading channels modeled as T_{sym}-spaced tapped-delay-line filter with independent tap coefficients. However, it is shown in [9] that the independent tap coefficient assumption is not valid for most wireless communication systems.

In this paper, error performance analysis is performed for systems experiencing doubly selective Rayleigh fadings, and new, tight, closed-form error performance lower bounds are derived for linearly modulated systems with both symbol spaced receiver and fractionally spaced receiver. The effects of the transmit filter, receive filter and the physical doubly selective channel fadings are represented as a discrete-time tapped-delay-line filter with *correlated* tap coefficients, with the correlation information determined by the maximum Doppler spread and power delay profile of the physical channel. Instead of resorting to the pairwise error probability, the new performance bounds are evaluated on a symbol by symbol basis. Compared to the frequency domain analysis utilized by the matched filter bound [5], a much simpler time-domain analysis technique with an equivalent single-input multiple-output (SIMO) system representation is used in the derivation of the new bounds. It is shown by simulations that our new analytical results can accurately predict the error performances of MAP and MLSE equalizers at both low SNR and high SNR.

More importantly, it is shown in this paper by both theoretical analysis and simulations that the uncoded error performances of communication systems with doubly selective fading channels are affected by both the power delay profile and the Doppler spread of the system, whereas the effects of Doppler spread are overlooked by most of the previous works. For systems experiencing fast time-varying fading, significant performance gain can be achieved by fractionally spaced receivers over symbol spaced receivers.

II. Discrete-time System Model

Let $p_{_T}(t)$ and $p_{_R}(t)$ be the time-invariant impulse response of the transmit filter and the receive filter, respectively, and both are normalized with energy unity. Let $g(t, \tau)$ be the time-varying impulse response of the doubly selective fading channel. If we define a combined impulse response (CIR) of the doubly selective channel fading as

$$h(t, \tau) = p_{_T}(\tau) \circledast g(t, \tau) \circledast p_{_R}(\tau), \tag{1}$$

where $\circledast$ is the convolution operator, then the output of the receive filter is

$$y(t) = \sum_{n=-\infty}^{+\infty} s(n) h(t, t - nT_{sym}) + v(t) \circledast p_{_R}(t), \tag{2}$$

where $s(n)$ is the information symbol with symbol period T_{sym} and average energy E_s, $v(t)$ is the additive white Gaussian noise (AWGN) with variance N_0, and we use $z(t) = v(t) \circledast p_{_R}(t)$ to denote the noise component at the output of the receive filter.

The sampled output of the receive filter can be written as

$$y(k) = \sum_{l=0}^{L-1} x(k-l) h(k, l) + z(k), \tag{3}$$

where $y(k) = y(kT_s)$ and $z(k) = z(kT_s)$ are the T_s-spaced samples of the received signals and noise components, respectively, $h(k,l) = h(kT_s, lT_s)$ is the sampled version of the CIR $h(t,\tau)$, the sampling period satisfies $T_s = T_{sym}/\nu$ with the integer ν being the oversampling factor, and $x(k)$ is the oversampled version of the information symbol,

$$x(k) = \begin{cases} s(k), & \frac{k}{\nu} \text{ is integer,} \\ 0, & \text{otherwise.} \end{cases}$$

In the representation of (3), the CIR $h(k,l)$ is represented as a finite impulse response (FIR) filter in the delay domain l by discarding negligible channel taps. This FIR representation can be verified by the fact that the fading power delay profile (PDP) has finite time-domain support. Moreover, systems with non-causal CIR can always be made causal by appropriate delays at the receiver.

Equation (3) is a discrete-time representation of the communication systems, and the doubly selective fading channel is represented as a T_s-spaced tapped-delay-line filter. It is shown in [9] that the tap coefficients of $h(k,l)$ are correlated in both the time-domain k and the delay-domain l. If the channel experiences wide-sense stationary uncorrelated scattering (WSSUS) Rayleigh fadings, then the correlation function $\rho(k_1 - k_2; l_1, l_2) = E\left[h(k_1, l_1)h^*(k_2, l_2)\right]$ can be expressed as [9]

$$\rho(k_1 - k_2; l_1, l_2) = J_0\left[2\pi f_d(k_1 - k_2)T_s\right] c(l_1, l_2), \quad (4)$$

$$\text{with } c(l_1, l_2) = \int_{-\infty}^{+\infty} R_{p_T p_R}(l_1 T_s - \mu) R^*_{p_T p_R}(l_2 T_s - \mu) G(\mu) d\mu, \quad (5)$$

where $R_{p_T p_R}(t) = p_T(t) \circledast p_R(t)$, $J_0(x)$ is the zero-order Bessel function of the first kind, f_d is the maximum Doppler spread of the fading channel, and $G(\mu)$ is the normalized power delay profile with $\int_{-\infty}^{+\infty} G(\mu) d\mu = 1$.

The noise component $z(k)$ of the discrete-time system is a linear transformation of the AWGN $v(t)$, thus it is still Gaussian distributed with zero-mean, and the auto-correlation function is given by[9]

$$E\left[z(m+n)z^*(m)\right] = N_0 \cdot R_{p_R p_R}(nT_s), \quad (6)$$

where $R_{p_R p_R}(nT_s) = \int_{-\infty}^{+\infty} p_R(nT_s + \tau) p_R(\tau) d\tau$ is the autocorrelation function of the receive filter. If $R_{p_R p_R}(nT_s) = 0$ for $n \neq 0$, then the discrete-time noise component $z(k)$ is still white, and this is valid for T_{sym} spaced receivers with root raised cosine (RRC) filter. For fractionally spaced receivers, $z(k)$ becomes a colored Gaussian noise process, and the correlation among noise samples is introduced by the effects of oversampling and the time span of the receive filter.

It will show in this paper that this temporal-delay correlation information along with the noise correlation are critical to the performances of the communication systems.

III. Equivalent SIMO System Representation

Based on the discrete-time representation of the system given in (3), the input-output relationship of the system can be written in matrix format as

$$\mathbf{y}_k = \mathbf{h}_k \cdot x(k) + \widetilde{\mathbf{H}}_k \cdot \tilde{\mathbf{x}}_k + \mathbf{z}_k, \qquad \frac{k}{\nu} \text{ is integer,} \quad (7)$$

where the vectors $\mathbf{y}_k = \left[y(k), \cdots, y(k+L-1)\right]^T \in \mathbb{C}^{L\times 1}$, $\mathbf{z}(k) = \left[z(k), \cdots, z(k+L-1)\right]^T \in \mathbb{C}^{L\times 1}$ comprises all the signal samples and noise samples related to the transmitted symbol $x(k)$, with $\mathbf{A}^T$ representing matrix transpose, $\mathbf{h}_k = \left[h(k,0),\ h(k+1,1), \cdots,\ h(K+L-1, L-1)\right]^T \in \mathbb{C}^{L\times 1}$ is the CIR vector related to $x(k)$, $\tilde{\mathbf{x}}_k = [x(k-L+1), \cdots, x(k-1), x(k+1), \cdots, x(k+L-1)]^T \in \mathbb{C}^{2(L-1)\times 1}$ is the interference vector relative to $x(k)$, and $\widetilde{\mathbf{H}}_k$ is the corresponding interference CIR matrix defined at the top of next page.

In the representation of (7), $x(k)$ is treated as the desired information symbol being transmitted in L parallel frequency flat fading channels, and the single-input single-output (SISO) system with doubly selective fading channels are equivalently represented as a SIMO system with mutually correlated flat fading channels. With such system configurations, the system error performances can be analyzed on a symbol-wise basis without resorting to the trellis structure as utilized by union bound techniques. Moreover, we are going to show by simulations that the results obtained by this method is more accurate than those obtained from union bounds.

If the interference components can be fully canceled by the receiver, *i.e.*, $\widetilde{\mathbf{H}}_k \tilde{\mathbf{x}}_k = 0$, then the error probability of the SIMO system can be minimized. It is well known that MLSE equalizers and MAP equalizers are optimum in the sense of maximizing the likelihood functions or *a posteriori* probabilities of the transmitted symbols. In this paper, we are going to show by simulations that the MLSE equalizers and MAP equalizers are also asymptotic optimum in the sense of interference cancellation, *i.e.*, the interference components $\widetilde{\mathbf{H}}_k \tilde{\mathbf{x}}_k$ will tend to 0 if MAP or MLSE equalization algorithms are employed to systems with long enough decoding length. Therefore, tight error probability lower bounds can be obtained by assuming $\widetilde{\mathbf{H}}_k \tilde{\mathbf{x}}_k = 0$.

From (7), the interference-free SIMO system can be represented as

$$\mathbf{y}_k = \mathbf{h}_k \cdot x(k) + \mathbf{z}_k, \quad (9)$$

where $\mathbf{z}_k$ is a zero-mean colored Gaussian vector with covariance matrix $E\left[\mathbf{z}_k \mathbf{z}_k^H\right] = N_0 \cdot \mathbf{R}_p$, with $\mathbf{A}^H$ denoting the Hermitian operation, $\mathbf{R}_p \in \mathbb{C}^{L\times L}$ is the receive filter correlation matrix with the (m,n)th elements being $(\mathbf{R}_p)_{m,n} = R_{p_R p_R}[(m-n)T_s]$. For most cases, the matrix $\mathbf{R}_p$ is rank deficient, thus non-invertible.

The Rayleigh fading channel vector $\mathbf{h}_k$ comprises zero-mean complex Gaussian random variables (CGRVs) with the covariance matrix $\mathbf{R}_h = E\left[\mathbf{h}_k \mathbf{h}_k^H\right]$ given by

$$\mathbf{R}_h = \begin{bmatrix} \rho(0;0,0) & \rho(1;0,1) & \cdots & \rho(L-1;0,L-1) \\ \rho(1;1,0) & \rho(0;1,1) & \cdots & \rho(L-2;1,L-1) \\ \vdots & \vdots & \vdots & \vdots \\ \rho(L-1;L-1,0) & \rho(L-2;L-1,1) & \cdots & \rho(0;L-1,L-1) \end{bmatrix}. \quad (10)$$

The correlation coefficient $\rho(k; l_1, l_2)$ (c.f. (4)) contains the information of both the temporal correlation $J_0(2\pi f_d k T_s)$ and the delay-domain correlation $c(l_1, l_2)$, which are in turn determined by the Doppler spread f_d and power delay profile $G(\mu)$ of the fading channel.

$$\widetilde{\mathbf{H}}_k = \begin{bmatrix} h_k(L-1) & \cdots & h_k(1) & 0 & \cdots & 0 \\ 0 & h_{k+1}(L-1) & \cdots & h_{k+1}(2) & h_{k+1}(0) & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & h_{k+L-1}(L-2) & \cdots & h_{k+L-1}(0) \end{bmatrix} \in \mathbb{C}^{L\times 2(L-1)} \quad (8)$$

IV. Error Performance Analysis

The symbol error rate (SER) of linearly modulated systems are derived based on an optimum decision rule of the interference free SIMO system, and SERs obtained by this methods are tight lower performance bounds of the corresponding SISO system.

A. Optimum Combining

Due to the rank deficiency of the receive filter correlation matrix $\mathbf{R}_p$, the statistical properties of the SIMO system cannot be directly evaluated since the probability density function (pdf) of the CGRV vector $\mathbf{z}_k$ involves the inverse of the covariance matrix $\mathbf{R}_z = N_0 \cdot \mathbf{R}_p$. To avoid the inverse operation of a rank deficient matrix, we define a new matrix $\mathbf{\Psi}_p$ based on the non-zero eigen values of $\mathbf{R}_p$,

$$\mathbf{\Psi}_p = \bar{\mathbf{V}}\bar{\mathbf{\Omega}}_p^{-1}\bar{\mathbf{V}}^H \in \mathbb{C}^{L\times L}, \quad (11)$$

$$\text{with } \bar{\mathbf{V}} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_{L_p} \end{bmatrix} \in \mathbb{C}^{L\times L_p}, \quad (12a)$$

$$\bar{\mathbf{\Omega}}_p = \text{diag}\begin{bmatrix} \omega_1 & \omega_2 & \cdots & \omega_{L_p} \end{bmatrix} \in \mathbb{R}^{L_p\times L_p}, \quad (12b)$$

where L_p is the number of non-zero eigen values of $\mathbf{R}_p$, and the matrices $\bar{\mathbf{\Omega}}_p$ and $\bar{\mathbf{V}}$ contain the non-zero eigen values ω_l of $\mathbf{R}_p$ and the corresponding eigen vectors $\mathbf{v}_l$, for $l = 1, 2, \cdots, L_p$. With these definitions, the error probability minimizing decision rule of the interference free SIMO systems can be stated as follows.

Theorem: For SIMO systems with colored Gaussian noise, if the transmitted symbols are equiprobable, then the decision rule that minimizes the system error probability is

$$\hat{x}(k) = \underset{s_m\in\mathcal{S}}{\text{argmin}}\, |\eta_k - Q_k \cdot s_m|^2, \quad \forall \ \frac{k}{\nu} \text{ is integer.} \quad (13)$$

where $\hat{x}(k)$ is the detected symbol at the time instant k, $\mathcal{S}$ is the modulation alphabet set with Cardinality M, $\eta_k = \mathbf{h}_k^H \mathbf{\Psi}_p \mathbf{y}_k$ is the decision variable, $Q_k = \mathbf{h}_k^H \mathbf{\Psi}_p \mathbf{h}_k$ is a quadratic form of the CGRV vector $\mathbf{h}_k$.

Proof: Multiplying both sides of the SIMO system equation (9) with the reduced eigen vector matrix $\bar{\mathbf{V}}^H$, we have

$$\bar{\mathbf{y}}_k = \bar{\mathbf{V}}^H \mathbf{h}_k \cdot x(k) + \bar{\mathbf{z}}_k, \quad (14)$$

where $\bar{\mathbf{y}}_k = \bar{\mathbf{V}}^H \mathbf{y}_k \in \mathbb{C}^{L_p\times 1}$, $\bar{\mathbf{z}}_k = \bar{\mathbf{V}}^H \mathbf{z}_k \in \mathbb{C}^{L_p\times 1}$. The noise vector $\bar{\mathbf{z}}_k$ is obtained from linear transformation of the colored Gaussian vector $\mathbf{z}_k \sim \mathcal{N}(\mathbf{0}, N_0\mathbf{R}_p)$, thus $\bar{\mathbf{z}}_k$ is still Gaussian distributed with zero-mean and the covariance matrix $\mathbf{R}_{\bar{z}} = N_0 \bar{\mathbf{\Omega}}_p$, where $\bar{\mathbf{\Omega}}_p$ contains the non-zero eigen values of $\mathbf{R}_p$ as defined in (12b). Since the covariance matrix $\mathbf{R}_{\bar{z}}$ is diagonal, the elements of $\bar{\mathbf{z}}_k$ are uncorrelated, and the system with colored noise $\mathbf{z}_k$ is converted to an equivalent system with white Gaussian noise $\bar{\mathbf{z}}_k$ as described in (14).

The sample vector $\bar{\mathbf{y}}_k$ conditioned on $\mathbf{h}_k$ and the transmitted symbol $x(k)$ is Gaussian distributed, *i.e.*, $\bar{\mathbf{y}}_k|(\mathbf{h}_k, x(x)) \sim \mathcal{N}(\bar{\mathbf{V}}^H \mathbf{h}_k x(k),\ \bar{\mathbf{\Omega}}_p)$. If the transmitted symbol are equiprobable, the error probability of (14) can be minimized by the maximum *a posteriori* (MAP) rule,

$$\begin{aligned} \hat{x}_k &= \underset{s_m\in\mathcal{S}}{\text{argmin}}\ \left(\bar{\mathbf{y}}_k - \bar{\mathbf{V}}^H \mathbf{h}_k s_m\right)^H \bar{\mathbf{\Omega}}_p^{-1} \left(\bar{\mathbf{y}}_k - \bar{\mathbf{V}}^H \mathbf{h}_k s_m\right), \\ &= \underset{s_m\in\mathcal{S}}{\text{argmin}}\ \left[Q_k \cdot |s_m|^2 - 2\Re(\eta_k \cdot s_m^*)\right], \end{aligned} \quad (15)$$

Noting the fact that Q_k is a real-valued scalar, it's straightforward that (15) is equivalent to (13), and this completes the proof. ■

B. Symbol Error Rates

Based on the decision rule given in (13), symbol error probabilities are derived for linearly modulated systems with correlated channel fadings and colored Gaussian noise.

From (9), (11)-(13), the decision variable η_k conditioned on $Q_k = \mathbf{h}_k^H \mathbf{\Psi} \mathbf{h}_k$ and s_m is Gaussian distributed with mean $\mu_{\eta_k|Q_k,s_m} = Q_k s_m$ and variance $\sigma^2_{\eta_k|Q_k,s_m} = Q_k N_0$, *i.e.*, $\eta_k|(Q_k, s_m) \sim \mathcal{N}(Q_k s_m, Q_k N_0)$.

With the help of a polar coordinate system with origin $\mu_{\eta_k|Q_k,s_m}$ [11] and the decision rules given in (13), we can get a unified solution of the conditional error probabilities (CEP) $P(E|Q_k)$ for MPSK, MASK and square MQAM modulated systems

$$P(E|Q_k) = \sum_{i=1}^{2} \frac{\beta_i}{\pi} \int_0^{\psi_i} \exp\left\{-\alpha \cdot \frac{\gamma Q_k}{\sin^2(\phi)}\right\} d\phi, \quad (16)$$

where $\gamma = \frac{E_s}{N_0}$ is the average SNR without fading, and the values of α, β_i and ψ_i for various modulation schemes are listed in Table 1. The derivations of (16) are omitted here for brevity.

The unconditional error probability $P(E)$ can be evaluated from $P(E|Q_k)$ with the help of the characteristic function (CHF) of Q_k, which is a quadratic form of the zero-mean CGRV vector $\mathbf{h}_k$, and the CHF $\Phi_Q(w)$ is [13]

$$\Phi_Q(w) = E_Q\left(e^{jwQ_k}\right) = \left[\det\left(\mathbf{I}_L - jw\mathbf{R}_h\mathbf{\Psi}_p\right)\right]^{-1}, \quad (17)$$

where w is a dumb variable, and $\mathbf{I}_L$ is an $L\times L$ identity matrix.

With the CHF defined in (17) and the unified CEP $P(E|Q_k)$ given in (16), the unconditional error probability $P(E) = E_Q\left[P(E|Q_k)\right]$ in Rayleigh fading channels can be computed as

$$\begin{aligned} P(E) &= \sum_{i=1}^{2} \frac{\beta_i}{\pi} \int_0^{\psi_i} \left[\det\left(\mathbf{I}_L + \frac{\alpha\gamma}{\sin^2\theta}\mathbf{R}_h\mathbf{\Psi}_p\right)\right]^{-1} d\theta, \\ &= \sum_{i=1}^{2} \frac{\beta_i}{\pi} \int_0^{\psi_i} \prod_{l=1}^{\tilde{L}} \left[1 + \gamma \cdot \frac{\alpha\lambda_l}{\sin^2\theta}\right]^{-1} d\theta, \end{aligned} \quad (18)$$

Table 1. CEP Parameters of (16) for Various Modulation Schemes.

Modulation	α	β_1	β_2	ϕ_1	ϕ_2
MPSK	$\sin^2\frac{\pi}{M}$	1	0	$\pi-\frac{\pi}{M}$	0
MASK	$\frac{3}{M^2-1}$	$2(1-\frac{1}{M})$	0	$\frac{\pi}{2}$	0
MQAM	$\frac{3}{2(M-1)}$	$4\left(1-\frac{1}{\sqrt{M}}\right)$	$4\left(1-\frac{1}{\sqrt{M}}\right)^2$	$\frac{\pi}{2}$	$\frac{\pi}{4}$

where λ_l is the eigen value of the product matrix $\mathcal{R} = \mathbf{R}_h\mathbf{\Psi}_p$, and $\tilde{L}$ is the number of non-zero eigen values of $\mathcal{R}$. The values of $\tilde{L}$ and λ_l, for $l = 1, 2, \cdots, \tilde{L}$ are determined by both $\mathbf{\Psi}_p$ and the temporal-delay correlation matrix $\mathbf{R}_h$ (c.f. (10, 11)), which are in turn related to the statistical properties of the colored Gaussian noise and the doubly selective fading channels.

The closed-form expressions of the SER given in (18) can be obtained by partial fraction expansion. For all systems with practical PDPs, *e.g.*, the exponential profile [5], the Typical Urban profile [12], it can be easily shown that the non-zero eigen values λ_l are different from each other, and the SER can be expressed as

$$P(E) = \sum_{i=1}^{2}\sum_{l=1}^{\tilde{L}} \frac{\beta_i d_l}{\pi}\int_0^{\psi_i}\left[1+\gamma\cdot\frac{\alpha\lambda_l}{\sin^2\theta}\right]^{-1} d\theta \qquad (19)$$

where the coefficient d_l can be computed as

$$\begin{aligned} d_l &= \prod_{\substack{j=1\\ j\neq l}}^{\tilde{L}}\left[1+\gamma\cdot\frac{\alpha\lambda_j}{\sin^2\theta}\right]^{-1}_{\sin^2\theta=-\gamma\cdot\alpha\cdot\lambda_l} \\ &= \prod_{\substack{j=1\\ j\neq l}}^{\tilde{L}}\frac{\lambda_l}{\lambda_j-\lambda_l}, \qquad \text{for } l=1,2,\cdots,\tilde{L}. \qquad (20)\end{aligned}$$

Replacing (20) in (19), and changing the integration variable of (19) to $z = \cot\theta$, we can get the closed-form solutions of the unconditional error probabilities for MPSK, MASK and MQAM systems.

$$P_{\text{MPSK}}(E) = \sum_{l=1}^{\tilde{L}}\prod_{\substack{j=1\\ j\neq l}}^{\tilde{L}}\frac{\lambda_l}{\lambda_j-\lambda_l}\left\{\frac{M-1}{M}-\sqrt{\frac{\gamma\lambda_l\sin^2(\frac{\pi}{M})}{1+\gamma\lambda_l\sin^2(\frac{\pi}{M})}}\times\left[\frac{1}{2}+\frac{1}{\pi}\tan^{-1}\left(\sqrt{\frac{\gamma\lambda_l\sin^2(\frac{\pi}{M})}{1+\gamma\lambda_l\sin^2(\frac{\pi}{M})}}\cot\left(\frac{\pi}{M}\right)\right)\right]\right\} \qquad (21)$$

$$P_{\text{MASK}}(E) = \sum_{l=1}^{\tilde{L}}\prod_{\substack{j=1\\ j\neq l}}^{\tilde{L}}\frac{\lambda_l}{\lambda_j-\lambda_l}\left[\frac{M-1}{M}\left(1-\sqrt{\frac{3\gamma\lambda_l}{M^2-1+3\gamma\lambda_l}}\right)\right] \qquad (22)$$

$$P_{\text{MQAM}}(E) = \sum_{l=1}^{\tilde{L}}\prod_{\substack{j=1\\ j\neq l}}^{\tilde{L}}\frac{\lambda_l}{\lambda_j-\lambda_l}\left\{2\left(1-\frac{1}{\sqrt{M}}\right)\times\left(1-\sqrt{\frac{3\gamma\lambda_l}{3\gamma\lambda_l+2M-2}}\right)+\left(1-\frac{1}{\sqrt{M}}\right)^2\times\left[\frac{4}{\pi}\sqrt{\frac{3\gamma\lambda_l}{3\gamma\lambda_l+2M-2}}\left(\frac{\pi}{2}-\tan^{-1}\sqrt{\frac{3\gamma\lambda_l}{3\gamma\lambda_l+2M-2}}\right)-1\right]\right\} \qquad (23)$$

For the special case of frequency-flat fading channel, we have $\tilde{L} = 1$, and (21) and (23) agree with the exact error probability expressions previously obtained in [10, eqn. (36), (43)] for systems with flat fading channels.

In some special cases, such as the equal gain PDP with symbol spaced sampling, the matrix $\mathcal{R} = \mathbf{R}_h\mathbf{\Psi}_p$ may have identical eigen values. To avoid the complexity of partial fraction expansion of expressions with roots multiplicity, an approximation method is presented in [13], where identical eigenvalues are slightly modified to be different from each other without apparently affecting the system performance. Moreover, exact values of $P(E)$ can still be computed from numerical integration of (18), which can be easily evaluated since it has finite integration limits and the integrand contains only elementary functions.

In the SER expressions given in (18) and (21) - (23), the effects of oversampling, Doppler spread f_d, and fading power delay profile $G(\mu)$ are quantified as the eigenvalues λ_l, for $l = 1, 2, \cdots, \tilde{L}$ of the product matrix $\mathcal{R} = \mathbf{R}_h\mathbf{\Psi}_p$.

It should be noted that the dependence of $P(E)$ on f_d is introduced by the relative time delay among the elements of the fading vector $\mathbf{h}_k$. For conventional uncoded SIMO systems with flat fading channels, *i.e.*, a system with one transmit antenna and L receive antennas, the uncoded performances are usually not affected by the Doppler spread of the channel.

V. Numerical Examples

In the first example, we are going to compare our new analytical results with the well-known union Chernoff bounds and true union bounds (TUB) [8]. Since it's difficult to apply the union bounds technique to systems with inter-tap correlation, a simple two tap equal-gain T_{sym}-spaced power delay profile with uncorrelated channel gains are used in this example. The analytical results along with the corresponding simulation results obtained with MLSE and MAP equalizers are shown in Fig. 1. The new SER lower bound can accurately predict the performances of MLSE and MAP equalizers at both

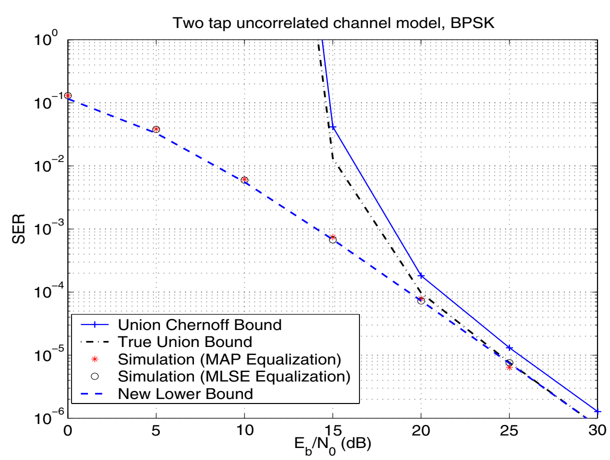

Fig. 1. Comparison of performance bounds of systems with two tap equal gain channel model. Decoding length for the equalizers: 1024 symbols.

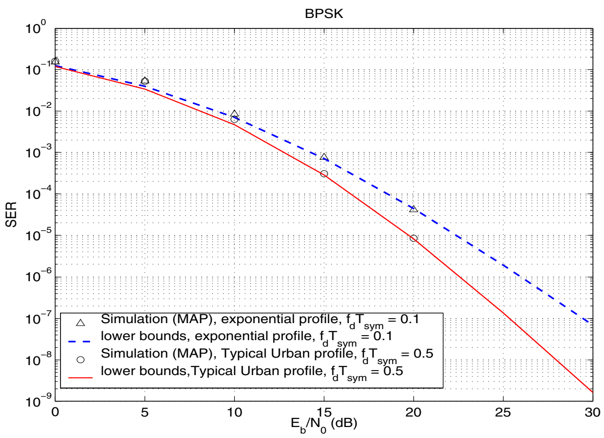

Fig. 2. Performances of doubly selective fading channels with different power delay profiles and Doppler spreads. Decoding length of the equalizers: 1024 symbols.

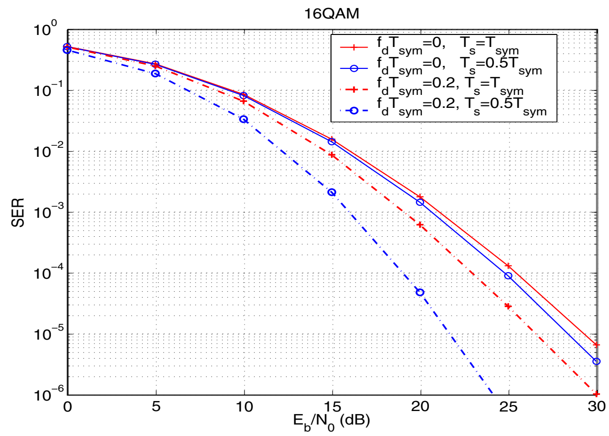

Fig. 3. Performances of fractionally spaced receivers with different values of Doppler spread.

low SNR and high SNR. On the other hand, the union Chernoff bound and TUB converges only when E_b/N_0 is higher than 20dB. Moreover, Since the error probabilities of the newly proposed methods are analyzed on a symbol by symbol basis, considerable computation efforts can be saved compared to the computations of the union bounds.

The next example is used to illustrate the error performances of doubly selective fading channel with practical power delay profiles. Fig. 2 shows the theoretical performances of fading channels with the exponential power profile [5] with the maximum delay spread being 5μs, and Typical Urban profile [12]. The symbol period of the system is $T_{sym} = 3.69\mu s$. Excellent agreements between the theoretical results and simulation results obtained from MAP equalization are observed from the figure for $E_b/N_0 \geq 10$dB, which means the MAP algorithm is asymptotic optimum in the sense of interference cancellation. Even at lower SNR, the lower bounds are still very tight compared with the simulation results.

The performances of fractionally spaced receivers are analyzed in the last example. The theoretical error performances of 16QAM systems with exponential PDP are shown in Fig. 3. It is interesting to note that oversampling has no apparent effects on the error performance for systems with quasi-static channels. On the other hand, if the system experiences fast time-varying fadings ($f_dT_{sym} = 0.2$), a 5dB performance gain at the SER level of 10^{-5} is observed for the $T_{sym}/2$-spaced system compared to the symbol spaced system. This performance improvement is due to an extra dimension of Doppler diversity achieved by the process of oversampling in fast time-varing fading channels.

VI. Conclusions

New, tight theoretical performance bounds were derived for wireless communication systems with doubly selective Rayleigh fading channels based on an equivalent SIMO system method, and the new equivalent system method is much simpler than both the frequency-domain analysis method used by the matched filter bound and the trellis structure analysis utilized in union bound technique. More importantly, it was discovered in this paper that the Doppler spreads of the channel fadings have important effects on the uncoded performances of systems with practical power delay profiles, and significant performance gain can be achieved by fractionally-spaced receivers over symbol spaced receivers for systems with fast time-varying fading. Simulation results showed that our new analytical results can accurately predict the error performances of MLSE and MAP equalizers at a wide range of SNR.

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